3.2 Growth of Functions - Worksheet

For 1) - 3) to establish a big-O relationship, find witnesses C and k such that $|f(x)| \leq C|g(x)|$ whenever $x > k$.

1. Determine whether each of these functions is $O(x)$.
   (a) $f(x) = 10$
   (b) $f(x) = 3x + 7$
   (c) $f(x) = x^2 + x + 1$
   (d) $f(x) = 5\log(x)$
   (e) $f(x) = \lfloor x \rfloor$
   (f) $f(x) = \lceil x/2 \rceil$
2. Find the least integer n such that $f(x)$ is $O(x^n)$ for each of these functions.
   (a) $f(x) = 2x^2 + x^3\log(x)$
   (b) $f(x) = (x^4 + x^2 + 1)/(x^4 + 1)$
   (c) $f(x) = (x^3 + 5\log(x))/(x^4 + 1)$
3. Show that x^3 is $O(x^4)$ but that x^4 is not $O(x^3)$.