

Handout 2.3

- **Definition:** Let A and B be nonempty sets. A *function* f from A to B , denoted $f: A \rightarrow B$ is an assignment of each element of A to exactly one element of B . We write $f(a) = b$ if b is the unique element of B assigned by the function f to the element a of A .
- Given a function $f: A \rightarrow B$:
- We say f maps A to B or f is a mapping from A to B .
- A is called the *domain* of f .
- B is called the *codomain* of f .
- If $f(a) = b$,
- - then b is called the *image* of a under f .
 - a is called the *pre-image* of b .
- The range of f is the set of all images of points in A under f . We denote it by $f(A)$.
- Two functions are *equal* when they have the same domain, the same codomain and map each element of the domain to the same element of the codomain.
- **Definition:** A function f is said to be *one-to-one*, or *injective*, if and only if $f(a) = f(b)$ implies that $a = b$ for all a and b in the domain of f . A function is said to be an *injection* if it is one-to-one.
- **Definition:** A function f from A to B is called *onto* or *surjective*, if and only if for every element b in B , there is an element a in A with $f(a) = b$. A function f is called a *surjection* if it is onto.
- **Definition:** A function f is a *one-to-one correspondence*, or a *bijection*, if it is both one-to-one and onto (surjective and injective).

f^{-1} • **Definition:** Let f be a bijection from A to B . Then the *inverse* of f , denoted f^{-1} , is the function from B to A defined as

$$f^{-1}(y) = x \text{ iff } f(x) = y$$

• **Definition:** Let $f: B \rightarrow C$, $g: A \rightarrow B$. The *composition* of f with g , denoted $f \circ g$ is the function from A to C defined by

$$f \circ g(x) = f(g(x))$$

• The *floor* function, denoted $f(x) = \lfloor x \rfloor$ is the largest integer less than or equal to x .

• The *ceiling* function, denoted $f(x) = \lceil x \rceil$ is the smallest integer greater than or equal to x .