1.5 Nested Quantifiers (more practice) and 1.6 Rules of Inference - Worksheet

1. Express the negations of each of these statements so that all negation symbols immediately precede predicates. Note: This is the rest of exercise # 8 from last class's worksheet.

   (a) $\exists x \exists y P(x, y) \wedge \forall x \forall y Q(x, y)$

   (b) $\exists x \exists y (Q(x, y) \leftrightarrow Q(y, x))$

   (c) $\forall y \exists x \exists z (T(x, y, z) \vee Q(x, y))$

2. What are the truth values of each of these? Assume that in each case the universe consists of all real numbers.

   (a) $\exists x \exists y (xy = 2)$

   (b) $\exists x \forall y (xy = 2)$

   (c) $\forall x \exists y (xy = 2)$

   (d) $\forall x \forall y (xy = 2)$

3. Write the following statements in English, using the predicate $S(x, y)$: "x shops in y," where x represents people and y represents stores:

   (a) $\exists y \forall x S(x, y)$.

   (b) $\forall x \exists y S(x, y)$.

4. What rule of inference is used in each of these arguments?

   (a) Kangaroos live in and Australia are marsupials. Therefore, kangaroos are marsupials.

   (b) It is either hotter than 100 degrees today or the pollution is dangerous. It is less than 100 degrees outside today. Therefore, the pollution is dangerous.

   (c) Linda is an excellent swimmer. If Linda is an excellent swimmer, then she can work as a lifeguard. Therefore, Linda can work as a lifeguard.

   (d) Steve will work at a computer company this summer. Therefore, this summer Steve will work at a computer company or he will be a beach bum.

   (e) If I work all night on this homework, then I can answer all the exercises. If I answer all the exercises, I will understand the material. Therefore, if I work all night on this homework, then I will understand the material.

5. Use rules of inference to show that the hypotheses:
   "If it does not rain or if it is not foggy, then the sailing race will be held and the lifesaving demonstration will go on,"
   "If the sailing race is held, then the trophy will be awarded," and
   "The trophy was not awarded" imply the conclusion
   "It rained."

6. Determine whether this argument is valid:
   Lynn works part time or full time.
   If Lynn does not play on the team, then she does not work part time.
   If Lynn plays on the team, she is busy.
   Lynn does not work full time.
   Therefore, Lynn is busy.

7. Suppose we have: Every student in this class is a Junior.
   Every Junior in this class passed the final exam.
   Allen is a student in this class.
   Explain why we can draw the conclusion: Allen passed the final exam.