

Amplitude:

$$\boxed{\text{Amplitude} = 3}$$

Period:

$$\text{Period} = \left| \frac{2\pi}{B} \right|$$

$$= \left| \frac{2\pi}{12} \right|$$

$$\boxed{\text{Period} = \pi}$$

Phase Shift:

$$\text{Phase Shift} = -\frac{C}{B}$$

$$\boxed{\text{Phase Shift} = -\frac{\pi}{2}}$$

Start of the wave:

$$\text{Phase Shift} = \left[ \frac{-\pi}{2} \right]$$

End of the wave:

Phase Shift + Period

$$= \frac{-\pi}{2} + \pi$$

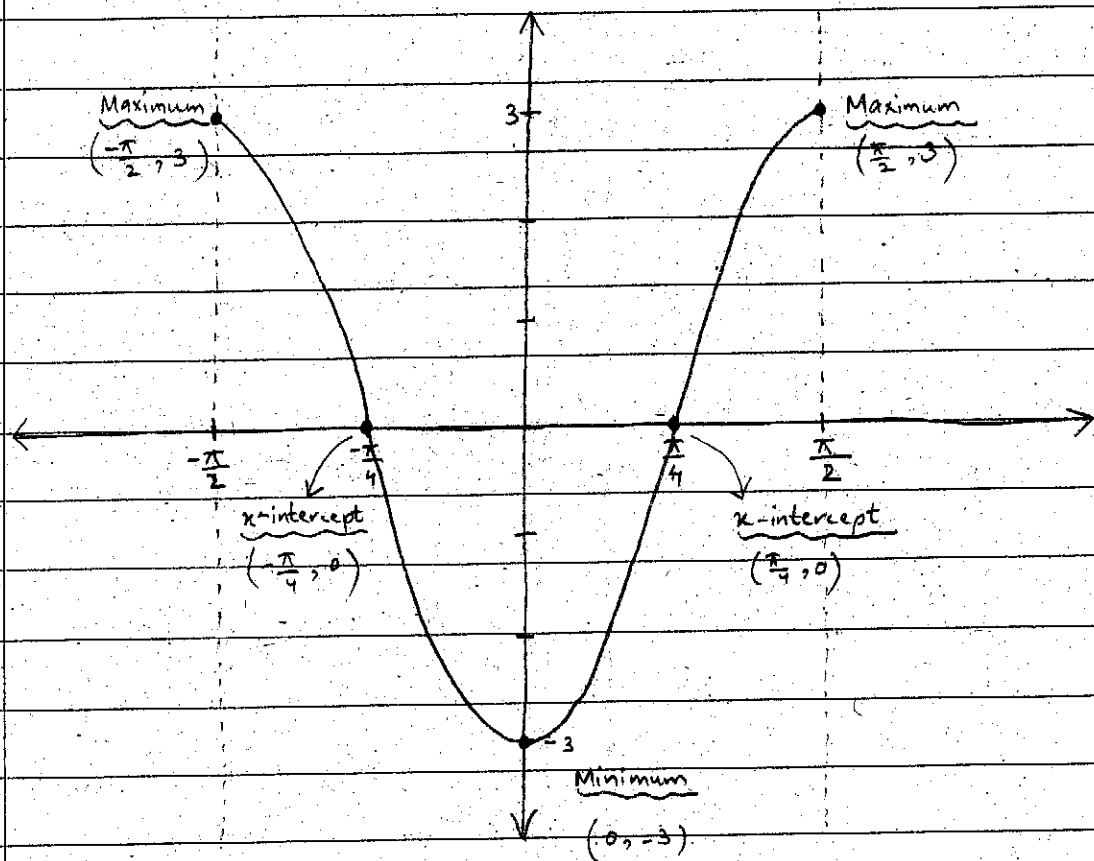
$$= \boxed{\frac{\pi}{2}}$$

R.W

$$\frac{-\frac{\pi}{2} + \frac{\pi}{2}}{1}$$

$$\frac{-\pi + 2\pi}{2}$$

$$\frac{\pi}{2}$$



$$(b) \quad y = 2 \sin(4x - \pi)$$

Amplitude:

$$A = 2$$

Period:

$$P = \frac{2\pi}{B}$$

$$P = \frac{2\pi}{4}$$

$$P = \frac{\pi}{2}$$

Phase Shift:

$$P.S. = -\frac{c}{B}$$

$$= -\frac{(-\pi)}{4}$$

$$P.S. = \frac{\pi}{4}$$

Start of the wave:

$$\text{Phase Shift} = \frac{\pi}{4}$$

End of the wave:

$$\text{Phase Shift} + \text{Period} = \frac{\pi}{4} + \frac{\pi}{2}$$

$$= \frac{3\pi}{4}$$

$$\frac{\frac{\pi}{4} + \frac{\pi}{2}}{\frac{\pi + 2\pi}{4}}$$

$$\frac{12\pi}{48}$$

$$\frac{22.5^\circ \times \pi}{2 + 10 \cdot 180^\circ}$$

$$\frac{12\pi}{4}$$

$$\frac{67.5^\circ \times \pi}{10 \cdot 180^\circ}$$

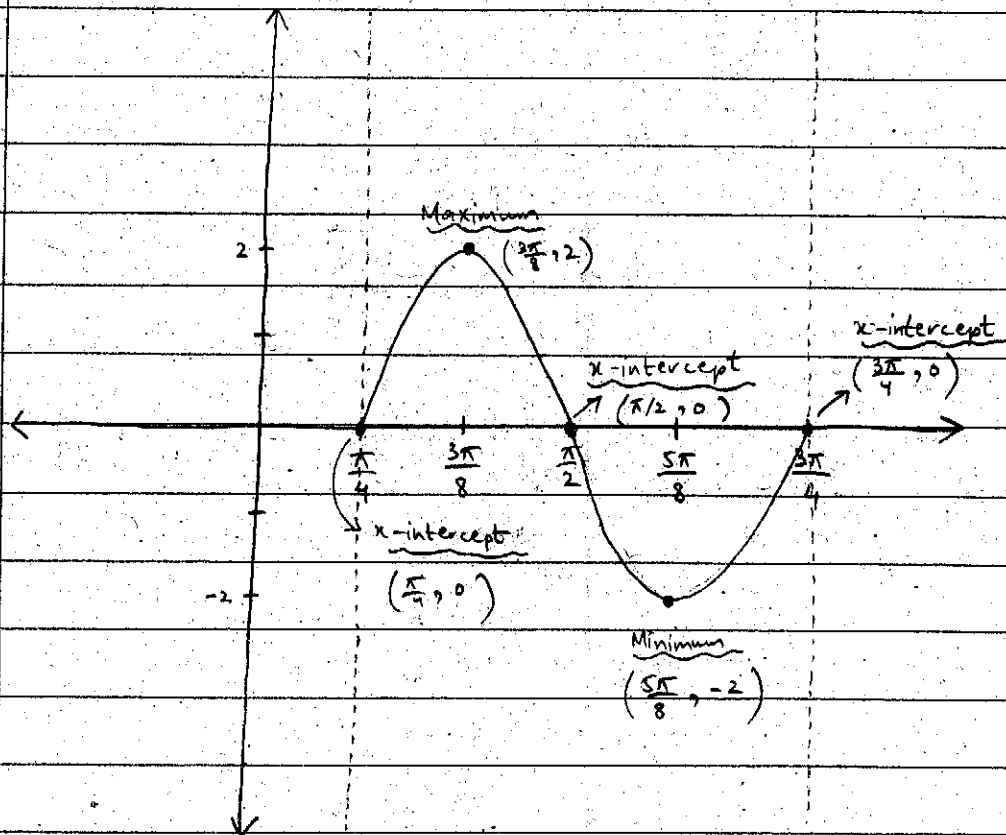
$$\frac{12\pi}{4}$$

$$\frac{3\pi}{4}$$

$$\frac{11.25^\circ \times \pi}{5 \cdot 180^\circ}$$

$$\frac{12\pi}{4}$$

$$\frac{5\pi}{8}$$



$$c) \quad y = -4 \sin\left(x - \frac{\pi}{2}\right)$$

Amplitude:

$$A = -4$$

Period:

$$P = \frac{2\pi}{B}$$

$$P = \frac{2\pi}{1}$$

$$P = 2\pi$$

Phase Shift:

$$P.S. = -\frac{c}{B}$$

$$= -\frac{(-\pi/2)}{1}$$

$$P.S. = \frac{\pi}{2}$$

Start of the wave:

$$\text{Phase Shift} = \frac{\pi}{2}$$

Row

$$\frac{\pi + 2\pi}{2}$$

$$\frac{\pi + 4\pi}{2}$$

$$\frac{5\pi}{2}$$

$$\frac{5\pi}{2} \times \frac{180^\circ}{\pi}$$

$$\frac{90}{2}$$

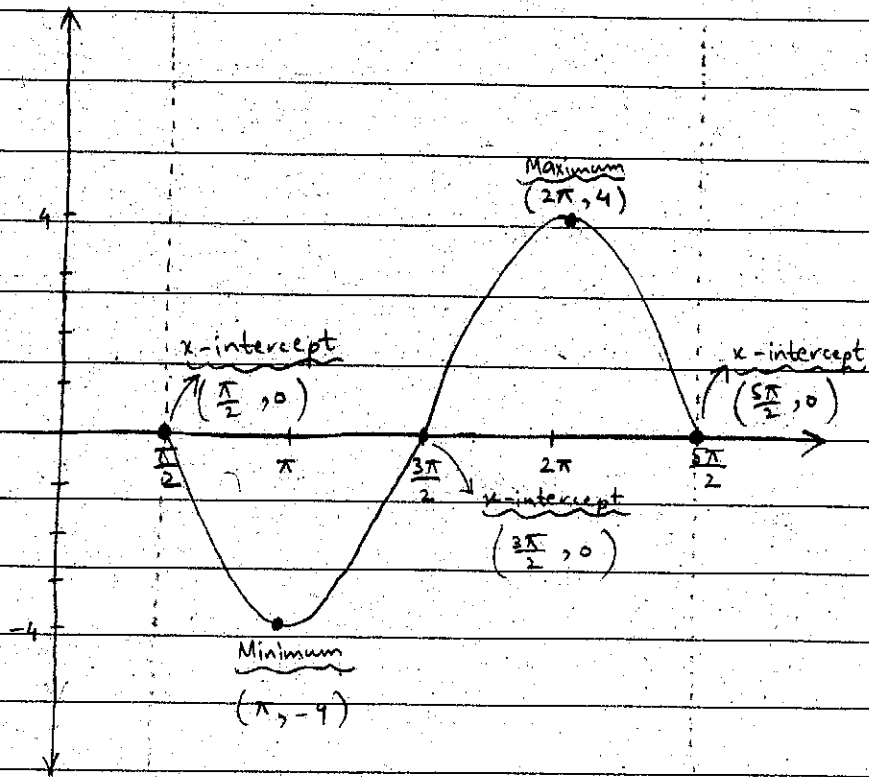
$$+ 5$$

$$= 45$$

End of the wave:

$$\text{Phase Shift} + \text{Period} = \frac{\pi}{2} + 2\pi$$

$$= \frac{5\pi}{2}$$



(34)

(\*)

QUESTION 11:

P.W

$\frac{\pi - \pi}{6}$

$\frac{6\pi - \pi}{6}$

$\frac{5\pi}{6}$

$\frac{5\pi}{6}$

(a)

$$2 \sin^2 x = \sin x$$

$$2 \sin^2(x) - \sin(x) = 0$$

$$\sin(x) \{ 2 \sin(x) - 1 \} = 0$$

$$\sin(x) = 0, \quad 2 \sin(x) - 1 = 0$$

$$x = \sin^{-1}(0), \quad x = \sin^{-1}\left(\frac{1}{2}\right)$$

$$x = (\pi - 0) + 2\pi n, \quad x = \left(\pi - \frac{\pi}{6}\right) + 2\pi n$$

$$x = \pi + 2\pi n$$

$$x = \frac{5\pi}{6} + 2\pi n$$

where  $n = 0, \pm 1, \pm 2, \dots$ , where  $n = 0, \pm 1, \pm 2, \dots$

x ~~~~~ x ~~~~~ x

(b)

$$2 \cos^2 x - \cos x - 1 = 0$$

$$\text{Let } \cos(x) = a$$

$$2a^2 - a - 1 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

2a

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(-1)}}{2(2)}$$

2(2)

$$x = \frac{1 \pm \sqrt{1+8}}{4}$$

$$x = \frac{1 \pm \sqrt{9}}{4}$$

$$x = \frac{1 \pm 3}{4}$$

$$x = \frac{1+3}{4}, \quad x = \frac{1-3}{4}$$

$$x = \frac{4}{4}, \quad x = \frac{-2}{4}$$

$$x = 1, \quad x = -\frac{1}{2}$$

$$\cos(x) = 1, \quad \cos(x) = -\frac{1}{2}$$

$$x = \cos^{-1}(1), \quad x = \cos^{-1}\left(-\frac{1}{2}\right)$$

$$x = \pm \pi + 2\pi n, \quad x = \pm \frac{2\pi}{3} + 2\pi n$$

where  $n = 0, \pm 1, \pm 2, \dots$

(36)



$$\textcircled{c} \quad \tan^2 x - \tan x = 0$$

$$\tan(x) \{ \tan(x) - 1 \} = 0$$

$$\tan(x) = 0, \quad \tan(x) - 1 = 0$$

$$x = \tan^{-1}(0), \quad x = \tan^{-1}(1)$$

$$\boxed{x = \pi + \pi n}$$

$$\boxed{x = \frac{\pi}{4} + \pi n}$$

where  $n = 0, \pm 1, \pm 2, \dots$

x ~~~~~ x ~~~~~ x

\* QUESTION 12:

$$\textcircled{a} \quad f(t) = 10,000 (1 - 0.015)^t$$

$$f(t) = 10,000 (0.985)^t$$

$$f(5) = 10,000 (0.985)^5$$

$\boxed{f(5) = 9272}$  The population will be approximately 9272 after 5 years.

$$\textcircled{b} \quad 5,000 = 10,000 (0.985)^t$$

$$\cancel{15,000} = \cancel{10,000} (0.985)^t$$

$$\cancel{2 \cdot 10,000} \quad \cancel{10,000}$$

$$\frac{1}{2} = (0.985)^t$$

$$\ln\left(\frac{1}{2}\right) = \ln(0.985)^t$$

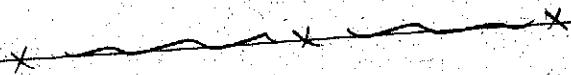
$$\ln\left(\frac{1}{2}\right) = t \ln(0.985)$$

$$t = \frac{\ln(1/2)}{\ln(0.985)}$$

$$t = 45.86236$$

$$1998 + 45 = \boxed{2043}$$

In 2043, there will be half of the population left



(\*) QUESTION 13:

(a)  $f(t) = 80,000 (1 + 0.04)^t$

$$f(t) = 80,000 (1.04)^t$$

$$f(12) = 80,000 (1.04)^{12}$$

$$\boxed{f(12) = 128,083}$$

The population will be approximately  
128,083 in 2015.

$$(b) \quad 240,000 = 80,000 (1.04)^t$$

$$3 \frac{240,000}{80,000} = \frac{80,000}{80,000} (1.04)^t$$

$$3 = (1.04)^t$$

$$\ln(3) = \ln(1.04)^t$$

$$\ln(3) = t \ln(1.04)$$

$$t = \frac{\ln(3)}{\ln(1.04)}$$

$$\ln(1.04)$$

$$t = 28.01102276$$

$$2003 + 28 = \boxed{2031}$$

The population will be tripled in 2031.

x ~~~~~ x ~~~~~ x

(\*) QUESTION 14:

$$(a) \quad y = 3 - 4x$$

$$x = 3 - 4y$$

$$x + 4y = 3$$

$$4y = 3 - x$$

$$4y = 4$$

$$y = \frac{3-x}{4}$$

$$f^{-1} = \frac{3-x}{4} \quad \text{Ans.}$$

x ~~~~~ x ~~~~~ x

$$(b) \quad y = \frac{4}{x-3}$$

$$x = \frac{4}{y-3}$$

$$x(y-3) = 4$$

$$xy - 3x = 4$$

$$\frac{xy}{x} = \frac{4+3x}{x}$$

$$y = \frac{4+3x}{x}$$

$$f^{-1} = \frac{4+3x}{x} \quad \text{Ans.}$$

(40)

x ~~~~~ x ~~~~~ x

$$c) \quad y = \frac{2}{8x+5}$$

$$x = \frac{2}{8y+5}$$

$$x(8y+5) = 2$$

$$8xy + 5x = 2$$

$$\frac{8xy}{8x} = \frac{2-5x}{8x}$$

$$y = \frac{2-5x}{8x}$$

$f^{-1} = \frac{2-5x}{8x}$	ans.
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x ~~~~~ x ~~~~~ x

$$d) \quad y = \frac{x-1}{x+2}$$

$$x = \frac{y-1}{y+2}$$

$$(x)(y+2) = y-1$$

$$xy + 2x = y - 1$$

$$xy = -1 - 2x + y$$

$$xy - y = -1 - 2x$$

$$\frac{y(\cancel{x-1})}{(\cancel{x-1})} = \frac{-1-2x}{x-1}$$

$$y = \frac{-2x-1}{x-1}$$

$$f^{-1} = \frac{-2x-1}{x-1} \quad \text{Ans.}$$

(\*) QUESTION 15:

(a) 22, 19, 16, 13, ...

$$a_1 = 22$$

$$d = -3$$

$$a_n = a_1 + d(n-1)$$

$$a_{70} = 22 - 3(70-1)$$

(b)  $a_{70} = 22 - 3(69)$