(*) QUESTION 4:

(a) $f(x) = x^2 + 5x + 2$, $\dfrac{f(x+h) - f(x)}{h} = ?$

$f(x+h) = (x+h)^2 + 5(x+h) + 2$

$f(x+h) = x^2 + h^2 + 2xh + 5x + 5h + 2$

$f(x+h) - f(x) = (x^2 + h^2 + 2xh + 5x + 5h + 2) - (x^2 + 5x + 2)$

$f(x+h) - f(x) = x^2 + h^2 + 2xh + 5x + 5h + 2 - x^2 - 5x - 2$

$f(x+h) - f(x) = h^2 + 2xh + 5h$

$$\frac{f(x+h) - f(x)}{h} = \frac{h^2 + 2xh + 5h}{h}$$

$$= \frac{h(h + 2x + 5)}{h}$$

$$\boxed{\frac{f(x+h) - f(x)}{h} = 2x + h + 5} \quad \text{Ans.}$$

(b) $f(x) = 2x^2 - 3x$, $\dfrac{f(x+h) - f(x)}{h} = ?$

$f(x+h) = 2(x+h)^2 - 3(x+h)$

$f(x+h) = 2(x^2+h^2+2xh) - 3x - 3h$

$f(x+h) = 2x^2 + 2h^2 + 4xh - 3x - 3h$

$f(x+h) - f(x) = (2x^2+2h^2+4xh-3x-3h) - (2x^2-3x)$

$f(x+h) - f(x) = 2x^2 + 2h^2 + 4xh - 3x - 3h - 2x^2 + 3x$

$f(x+h) - f(x) = 2h^2 + 4xh - 3h$

$$\frac{f(x+h)-f(x)}{h} = \frac{2h^2+4xh-3h}{h}$$

$$\frac{f(x+h)-f(x)}{h} = \frac{h(2h+4x-3)}{h}$$

$$\boxed{\frac{f(x+h)-f(x)}{h} = 2h+4x-3} \quad \text{Ans.}$$

(c) $f(x) = -2x^2+4x+1$, $\dfrac{f(x+h)-f(x)}{h} = ?$

$f(x+h) = -2(x+h)^2 + 4(x+h) + 1$

$f(x+h) = -2(x^2+h^2+2xh) + 4x + 4h + 1$

$f(x+h) = -2x^2 - 2h^2 - 4xh + 4x + 4h + 1$

$f(x+h) - f(x) = (-2x^2 - 2h^2 - 4xh + 4x + 4h + 1) - (-2x^2 + 4x + 1)$

$f(x+h) - f(x) = -2x^2 - 2h^2 - 4xh + 4x + 4h + 1 + 2x^2 - 4x - 1$

$f(x+h) - f(x) = -2h^2 - 4xh + 4h$

$$\frac{f(x+h) - f(x)}{h} = \frac{-2h^2 - 4xh + 4h}{h}$$

$$= \frac{h(-2h - 4x + 4)}{h}$$

$$\boxed{\frac{f(x+h) - f(x)}{h} = -2h - 4x + 4} \quad \text{Ans.}$$

x ~~~ x ~~~ x

(*) QUESTION 5:

(a) $f(x) = x^3 + 2x^2 - 3x - 6$

One exact root is $x_2 = -2$

The other two roots are inexact which are

$x_1 = -1.732$, $x_3 = 1.732$.

$$x^2 - 3$$

$$x+2 \;\big|\; x^3 + 2x^2 - 3x - 6$$

$$\oplus x^3 \oplus 2x^2$$

$$-3x - 6$$

$$\oplus 3x \oplus 6$$

$$0$$

$$x^3 + 2x^2 - 3x - 6 = (x+2)(x^2 - 3)$$

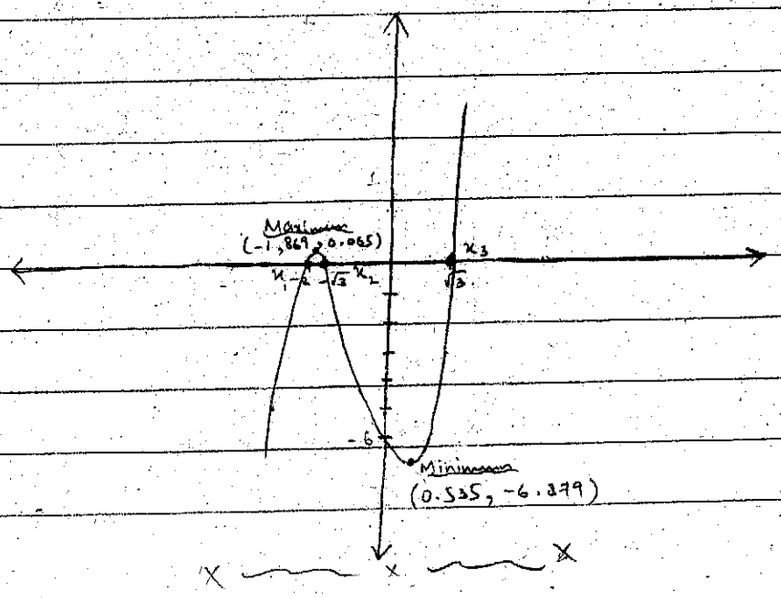
Solving $x^2 - 3 = 0$.

$$\sqrt{x^2} = \sqrt{3}$$

$$x = \sqrt{3} \quad , \quad x = -\sqrt{3}$$

3 exact roots:

$$\boxed{x_1 = -2 \;,\; x_2 = -\sqrt{3} \;,\; x_3 = \sqrt{3}}$$



(b) $f(x) = 2x^3 - 7x^2 - 3x + 8$

One root is exact which is $x_1 = 1$

The other two roots are inexact which are

$x_2 = -1.108$, $x_3 = 3.608$

$$
\begin{array}{r|l}
 & 2x^2 - 5x - 8 \\
\hline
x - 1 & 2x^3 - 7x^2 - 3x + 8 \\
 & \ominus 2x^3 \oplus 2x^2 \\
\hline
 & -5x^2 - 3x + 8 \\
 & \ominus 5x^2 \oplus 5x \\
\hline
 & -8x + 8 \\
 & \ominus 8x \oplus 8 \\
\hline
 & 0
\end{array}
$$

$$2x^3 - 7x^2 - 3x + 8 = (x - 1)(2x^2 - 5x - 8)$$

Solving $2x^2 - 5x - 8 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(2)(-8)}}{2(2)}$$

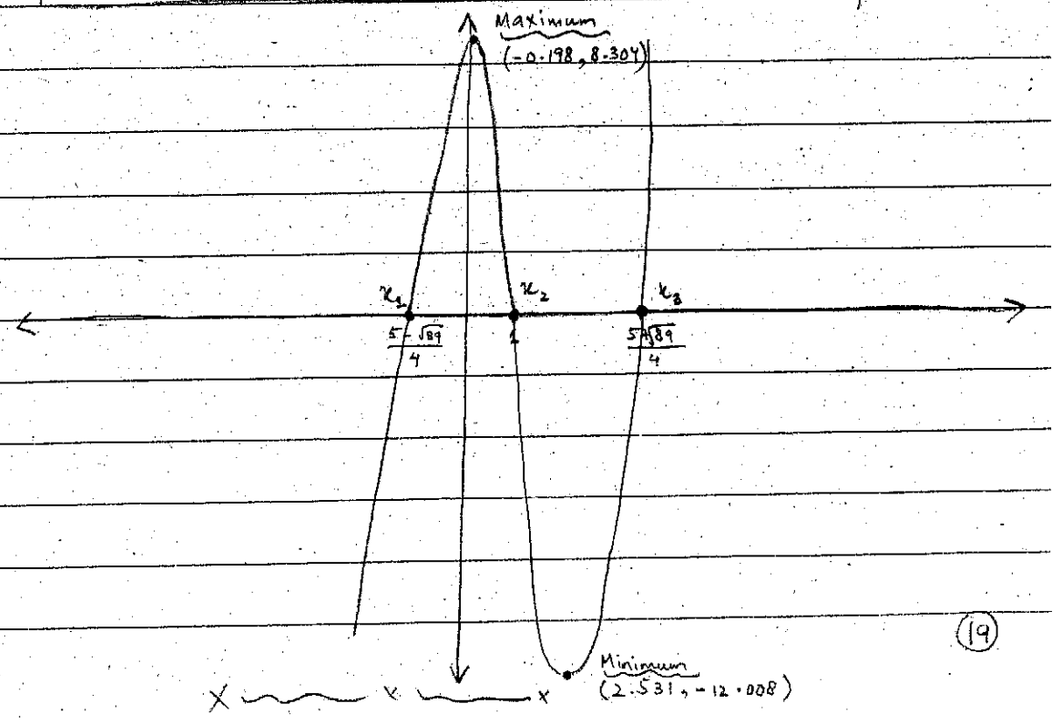
$$x = \frac{5 \pm \sqrt{25 + 64}}{4}$$

$$x = \frac{5 \pm \sqrt{89}}{4}$$

$$x = \frac{5 + \sqrt{89}}{4}, \quad x = \frac{5 - \sqrt{89}}{4}$$

3 exact roots:

$$x_1 = 1, \quad x_2 = \frac{5 + \sqrt{89}}{4}, \quad x_3 = \frac{5 - \sqrt{89}}{4}$$



(*) QUESTION 6:

(a) $v = \langle 2, -2\sqrt{3} \rangle$

$a = 2, \quad b = -2\sqrt{3}$

$\theta = \tan^{-1}\left(\frac{b}{a}\right)$

$\theta = \tan^{-1}\left(\frac{-2\sqrt{3}}{2}\right)$

$\theta = \tan^{-1}\left(-\sqrt{3}\right)$

$\boxed{\theta = -60^\circ}$

$\|\vec{v}\| = \sqrt{a^2 + b^2}$

$= \sqrt{(2)^2 + (-2\sqrt{3})^2}$

$= \sqrt{4 + 12}$

$= \sqrt{16}$

$\boxed{\|\vec{v}\| = 4}$

(b) $v = \langle -3, -3 \rangle$

$a = -3$, $b = -3$

$\theta = \tan^{-1}\left(\frac{b}{a}\right)$

$\theta = \tan^{-1}\left(\frac{-3}{-3}\right)$

$\theta = \tan^{-1}(1)$

$\theta = 45°$

$\theta = 45° + 180°$

$\boxed{\theta = 225°}$

$\|\vec{v}\| = \sqrt{a^2 + b^2}$

$= \sqrt{(-3)^2 + (-3)^2}$

$= \sqrt{9 + 9}$

$= \sqrt{18}$

$\boxed{\|\vec{v}\| = 3\sqrt{2}}$

(c) $v = \langle -5\sqrt{3}, 5 \rangle$

$a = -5\sqrt{3}, \quad b = 5$

$\theta = \tan^{-1}\left(\frac{8}{-8\sqrt{3}}\right)$

$\theta = \tan^{-1}\left(\frac{1}{-\sqrt{3}}\right)$

$\theta = -30^\circ$

$\theta = -30^\circ + 180^\circ$

$\boxed{\theta = 150^\circ}$

$\|\vec{v}\| = \sqrt{a^2 + b^2}$

$\|\vec{v}\| = \sqrt{(-5\sqrt{3})^2 + (5)^2}$

$\|\vec{v}\| = \sqrt{100}$

$\boxed{\|\vec{v}\| = 10}$

(*) QUESTION 7:

(a) $\frac{6\,(\cos 225^\circ + i \sin 225^\circ)}{3\,(\cos 135^\circ + i \sin 135^\circ)}$

$= \frac{8^2}{1\cancel{8}} [\cos(225° - 135°) + i\sin(225° - 135°)]$

$= \boxed{2\ [\cos(90°) + i\sin(90°)]} \Rightarrow$ In degrees.

$= 2\ [(0) + i(1)]$

$= 2\ (0 + i)$

$= 0 + 2i$

$= \boxed{2i} \Rightarrow$ In standard form.

(b) $2\ [\cos(120°) + i\sin(120°)] \times 4\ [\cos(90°) + i\sin(90°)]$

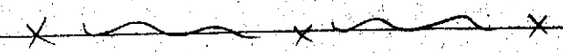
$= 2 \times 4\ [\cos(120° + 90°) + i\sin(120° + 90°)]$

$= \boxed{8\ [\cos(210°) + i\sin(210°)]} \Rightarrow$ In degrees.

$= 8\left[\left(\frac{-\sqrt{3}}{2}\right) + i\left(\frac{-1}{2}\right)\right]$

$= \frac{-8\sqrt{3}}{1\cancel{2}} - \frac{\overset{4}{\cancel{8}}}{1\cancel{2}} i$

$= \boxed{-4\sqrt{3} - 4i}$ ⇒ In Standard form.



(*) QUESTION 8:

(a) $\ln\left(x^8 \sqrt[3]{y^4}\right)$

$= \ln x^8 + \ln \sqrt[3]{y^4}$

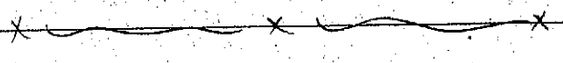
$= 8\ln(x) + \ln (y^4)^{1/3}$

$= 8\ln(x) + \ln y^{4/3}$

$= 8\ln(x) + \frac{4}{3}\ln(y)$

Taking $\ln(x) = u$ and $\ln(y) = v$.

$= \boxed{8u + \frac{4}{3}v}$ Ans.



(b) $\ln\left(\frac{\sqrt{x^5}}{y^3}\right)$

$= \ln \sqrt{x^5} - \ln y^3$

$= \ln (x^5)^{1/2} - 3 \ln(y)$

$= \ln x^{5/2} - 3 \ln(y)$

$= \frac{5}{2} \ln(x) - 3 \ln(y)$

Taking $\ln(x) = u$ and $\ln(y) = v$.

$= \boxed{\frac{5}{2} u - 3v}$ Ans.

× ~~~ × ~~~ ×

(c) $\ln\left(\sqrt{x^3 \sqrt[4]{y}}\right)$

$= \ln \sqrt{x^3} + \ln \sqrt{\sqrt[4]{y}}$

$= \ln (x^3)^{1/2} + \ln (y^{1/4})^{1/2}$

$= \ln x^{\frac{3}{2}} + \ln y^{\frac{1}{8}}$

$= \frac{3}{2} \ln(x) + \frac{1}{8} \ln(y)$

Taking $\ln(x) = u$ and $\ln(y) = v$.

$= \boxed{\frac{3}{2} u + \frac{1}{8} v}$ Ans.

× ~~~ × ~~~ ×

(*) QUESTION 9:

(a) $f(x) = \log(x-4)$

Domain:

$x - 4 > 0$

$x > 4$

$\boxed{(4, \infty)}$

V.A.:

$x - 4 = 0$

$\boxed{x = 4}$

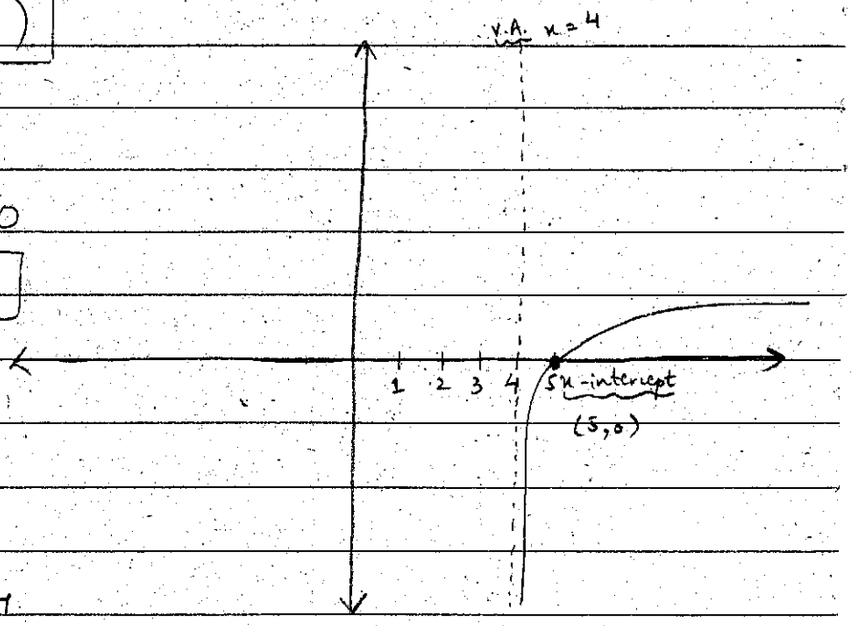
x-intercept:

$x - 4 = 1$

$x = 1 + 4$

$x = 5$

$\boxed{(5, 0)}$



(b) $f(x) = \log(5-x)$

Domain:

$5 - x > 0$

$-x > -5$

$x < 5$

$\boxed{(-\infty, 5)}$

V.A.:

$5 - x = 0$

$-x = -5$

$\boxed{x = 5}$

x-intercept:

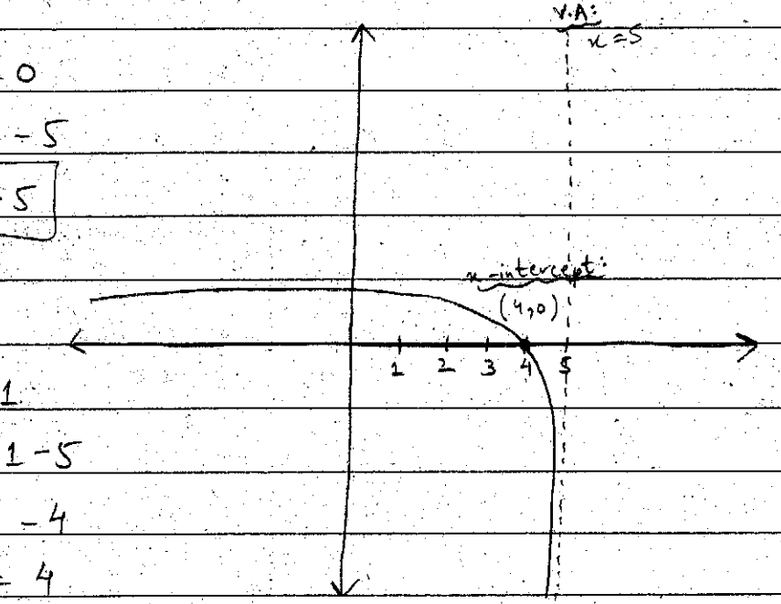
$5 - x = 1$

$-x = 1 - 5$

$-x = -4$

$x = 4$

$\boxed{(4, 0)}$



(c) $f(x) = -\ln(x+2)$

Domain:

$x + 2 > 0$

$x > -2$

$\boxed{(-2, \infty)}$

V.A.:

$x + 2 = 0$

$\boxed{x = -2}$

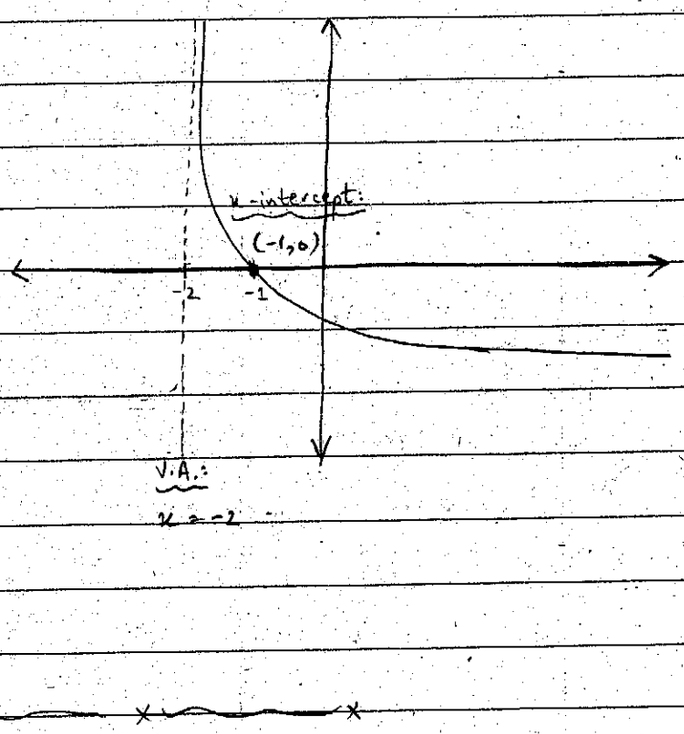
x-intercept:

$x + 2 = 1$

$x = 1 - 2$

$x = -1$

$(-1, 0)$



(*) QUESTION 10:

(a) $y = 3\cos(2x + \pi)$