New York City College of Technology

MAT 1375/D575 - Spring 2019

Exam 3 - Version X - Total Points: 100

NAME: Key

Instructions: Write your solutions in the space provided after each question. You may use the back of each page for any scratch work that you need to do. To receive (partial) credit you must **show all your work in a clear and organized manner.**

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1. (7 points) Condense the following expression into a single logarithm by applying the properties of logarithms: $7\log x - \frac{1}{2}\log y + 3\log z$.

$\log x^7 - \log y^{1/2} + \log z^3$

$= \boxed{\log\left(\frac{x^7 z^3}{\sqrt{y}}\right)}$

2. (8 points) Let $u = \ln x$, $v = \ln y$, where $x, y > 0$. Write the expression $\ln\left(\frac{\sqrt[4]{x^3}}{y^5}\right)$ in terms of u and v.

$\ln(\sqrt[4]{x^3}) - \ln(y^5) = \ln(x^{3/4}) - \ln(y^5) = \frac{3}{4}\ln(x) - 5\ln(y)$

$= \boxed{\frac{3}{4}u - 5v}$

3. (10 points) Solve the equation $4 * e^{(3x-5)} = 48$. First find the exact answer, then approximate the answer to the nearest hundredth using the calculator.

$e^{3x-5} = \frac{48}{4} = 12$

$e^{3x-5} = 12$

$\ln(e^{3x-5}) = \ln(12)$

$(3x-5)\ln e = \ln(12)$ $\quad \ln e = 1$

$3x - 5 = \ln(12)$

$3x = \ln(12) + 5$

$\boxed{x = \frac{\ln(12)+5}{3}}$ exact $\approx 2.49496883 = \boxed{2.49}$ approximation

4. (12 points) The population of a city grows exponentially at a rate of 3% per year. If the population was 60,000 people in the year 2007, then what is the population size of this city in the year 2014? [Round your answer to the nearest integer.] In what year will the population be 100,000 people?

$P(t) = 60{,}000\,(1+.03)^t = 60{,}000\,(1.03)^t$ $\quad t=0$ is 2007

In 2014 $\quad P(7) = 60{,}000\,(1.03)^7 = 73{,}792.43193$ $\quad \boxed{73{,}792}$

$100{,}000 = 60{,}000\,(1.03)^t$

$\frac{10}{6} = \frac{5}{3} = (1.03)^t$

$\ln\left(\frac{5}{3}\right) = t \ln(1.03)$ $\quad t = \frac{\ln(5/3)}{\ln(1.03)} = 17.28167534$

$2007 + 17 = \boxed{2024}$

5. (10 points) Given that $\sin(\alpha) = -4/5$ and α is in quadrant 3, find the exact values of $\sin(2\alpha)$ and $\cos(2\alpha)$. [Use the formulas $\sin(2\alpha) = 2\sin(\alpha)\cos(\alpha)$ and $\cos(2\alpha) = \cos^2(\alpha) - \sin^2(\alpha)$].

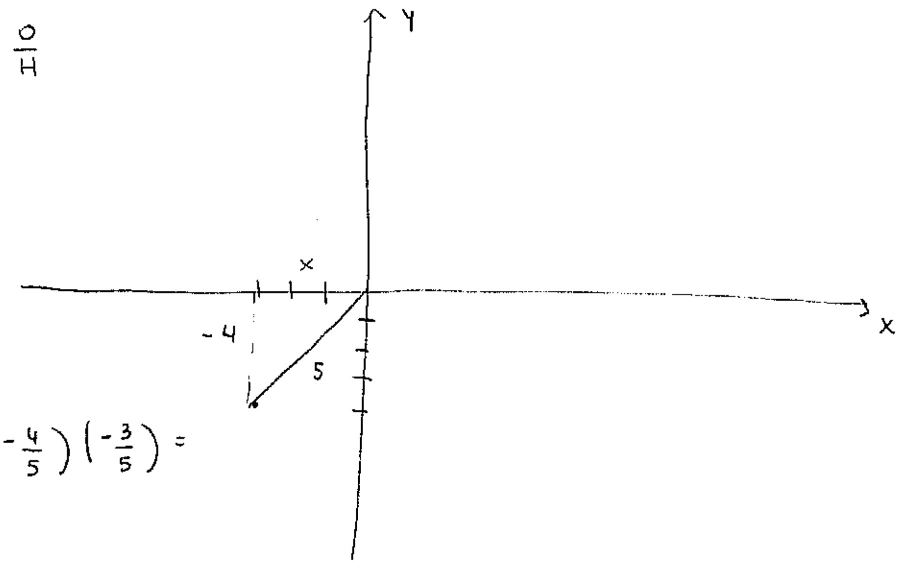
$\sin(\alpha) = -\frac{4}{5} = \frac{O}{H}$

$x^2 + 16 = 25$

$x^2 = 9$

$x = -3$

$\cos(\alpha) = \frac{A}{H} = -\frac{3}{5}$



$\sin(2\alpha) = 2 * \left(-\frac{4}{5}\right)\left(-\frac{3}{5}\right) =$

$\boxed{\frac{24}{25}}$

$\cos(2\alpha) = \left(-\frac{3}{5}\right)^2 - \left(-\frac{4}{5}\right)^2 = \frac{9}{25} - \frac{16}{25} = \boxed{\frac{-7}{25}}$

6. Find all exact solutions in radians.

a) (6 points) $\cos(x) = -\frac{\sqrt{2}}{2}$ $\qquad \cos^{-1}\left(-\frac{\sqrt{2}}{2}\right) = \frac{3\pi}{4}$ $\qquad (135°)$

$x = \pm \cos^{-1}\left(-\frac{\sqrt{2}}{2}\right) + 2\pi N$

$\boxed{x = \pm \frac{3\pi}{4} + 2\pi N \text{ where } N \text{ is an integer}}$

b) (12 points) $2\sin^2(x) = \sin(x)$

$2\sin^2(x) - \sin(x) = 0$

$\sin(x)\,[2\sin(x) - 1] = 0$

$\sin(x) = 0 \qquad x = 0, \pi$

$\sin(x) = \frac{1}{2} \qquad x = \frac{\pi}{6}, \frac{5\pi}{6}$

$$\boxed{\begin{array}{l} x = 0 + 2\pi N \\ x = \pi + 2\pi N \\ x = \frac{\pi}{6} + 2\pi N \\ x = \frac{5\pi}{6} + 2\pi N \\ \text{where } N \text{ is an integer} \end{array}}$$

7. (10 points) Solve the equation $\log_5(x) + \log_5(x - 24) = 2$.

$\log_5(x(x-24)) = 2$

$\log_5(x^2 - 24x) = 2$

$x^2 - 24x = 5^2 = 25$

$x^2 - 24x - 25 = 0$

$(x-25)(x+1) = 0$

$\boxed{x = 25}$

$\cancel{x = -1}$

X

8. (10 points) Find the domain, asymptotes, and x-intercepts of the function $f(x) =$ ~~[illegible]~~. Sketch its graph on graphing paper.

$\log(12-4x)$

domain

$12 - 4x > 0$

$-4x > -12$

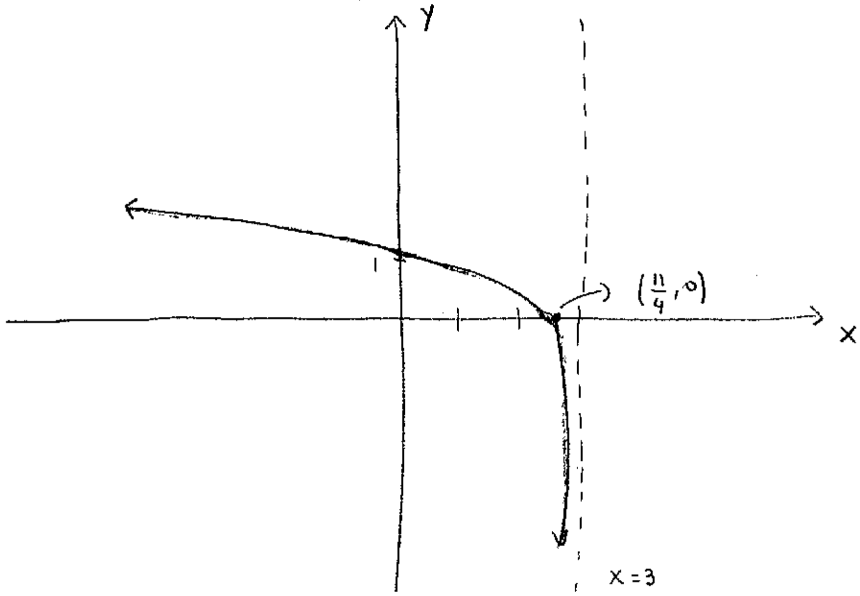
$x < 3$ $\boxed{(-\infty, 3)}$

V.A $\boxed{x = 3}$

x-int. $12 - 4x = 1$

$11 = 4x$ $x = \frac{11}{4}$ $\boxed{\left(\frac{11}{4}, 0\right)}$

$(2.75, 0)$



9. (15 points) Given $y = 4\sin(3x + \pi)$, state the amplitude, period and phase shift, and then sketch one complete cycle of the graph on graphing paper. Label all maxima, minima and x-intercepts, with exact answers in radians.

$y = 4\sin(3x + \pi)$ $A = 4$ $B = 3$ $C = \pi$

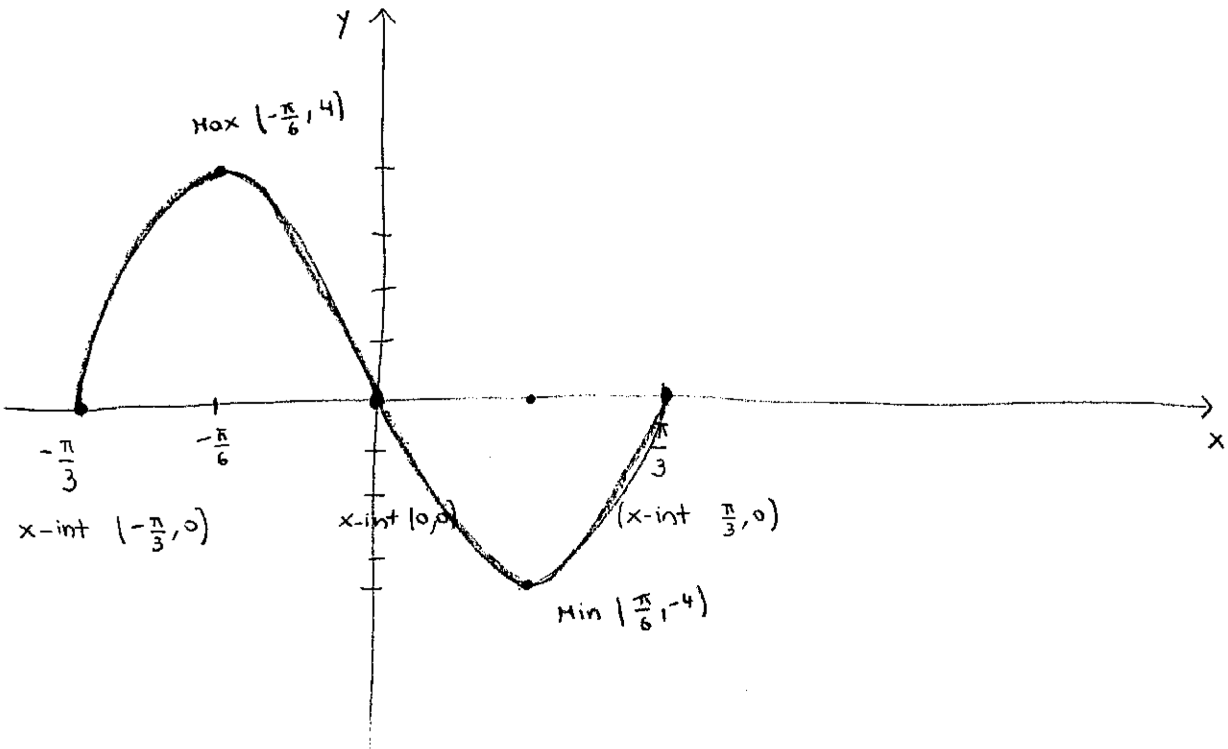
Amplitude $= \boxed{4}$

period $= \frac{2\pi}{B} = \boxed{\frac{2\pi}{3}}$

phase shift $= -\frac{C}{B} = \boxed{-\frac{\pi}{3}}$

wave starts at $-\frac{\pi}{3}$

ends at $-\frac{\pi}{3} + \frac{2\pi}{3} = \frac{\pi}{3}$



New York City College of Technology

MAT 1375/D575 - Spring 2019

Exam 3 - Version Z - Total Points: 100

NAME: Key

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1. (7 points) Condense the following expression into a single logarithm by applying the properties of logarithms: $7\log x - 3\log y - \frac{1}{2}\log z$.

$\log x^7 - \log y^3 - \log z^{1/2} = \log x^7 - \log y^3 - \log \sqrt{z}$

$= \boxed{\log\left(\frac{x^7}{y^3\sqrt{z}}\right)}$

2. (8 points) Let $u = \ln x$, $v = \ln y$, where $x, y > 0$. Write the expression $\ln\left(\frac{\sqrt[4]{x^5}}{y^3}\right)$ in terms of u and v.

$\ln\left(\sqrt[4]{x^5}\right) - \ln\left(y^3\right) = \ln\left(x^{5/4}\right) - \ln\left(y^3\right) = \frac{5}{4}\ln x - 3\ln y = \boxed{\frac{5}{4}u - 3v}$

3. (10 points) Solve the equation $\log_3(x+8) + \log_3(x) = 2$.

$\log_3\left(x^2+8x\right) = 2$

$x^2+8x = 9$

$x^2 + 8x - 9 = 0$

$(x+9)(x-1) = 0$

$\cancel{x = -9}$

$\boxed{x=1}$

4. (12 points) The population of a city decreases exponentially at a rate of 3% per year. If the population was 60,000 people in the year 2007, then what is the population size of this city in the year 2014? [Round your answer to the nearest integer.] In what year will the population be 10,000 people?

$P(t) = 60{,}000\ (1 - .03)^t = 60{,}000\ (.97)^t \quad t=0$ is 2007

In 2014 $\quad P(7) = 60{,}000\ (.97)^7 = 48{,}478.97069\ldots \quad \boxed{48{,}479}$

$10{,}\cancel{000} = 60{,}\cancel{000}\ (.97)^t$

$\frac{1}{6} = (.97)^t$

$\ln\left(\frac{1}{6}\right) = t \ln(.97)$

$t = \dfrac{\ln\left(\frac{1}{6}\right)}{\ln(.97)} = 58.82$

$2007 + 58 = \boxed{2065}$

5. (10 points) Given that $\sin(\alpha) = -4/5$ and α is in quadrant 4, find the exact values of $\sin(2\alpha)$ and $\cos(2\alpha)$. [Use the formulas $\sin(2\alpha) = 2\sin(\alpha)\cos(\alpha)$ and $\cos(2\alpha) = \cos^2(\alpha) - \sin^2(\alpha)$].

$\sin(\alpha) = -\frac{4}{5} = \frac{O}{H}$

$x^2 + 16 = 25$

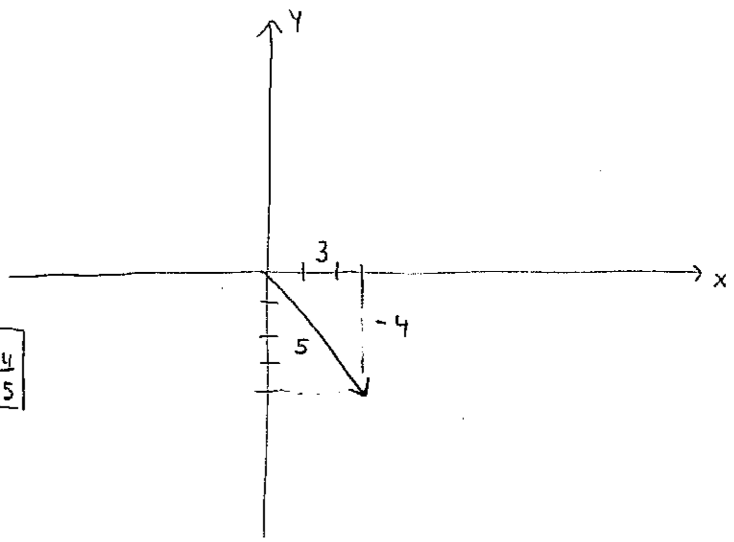
$x^2 = 9 \qquad x = 3$

$\cos(\alpha) = \frac{A}{H} = \frac{3}{5}$

$\sin(2\alpha) = 2\left(-\frac{4}{5}\right)\left(\frac{3}{5}\right) = \boxed{-\frac{24}{25}}$

$\cos(2\alpha) = \left(\frac{3}{5}\right)^2 - \left(-\frac{4}{5}\right)^2 =$

$\frac{9}{25} - \frac{16}{25} = \boxed{-\frac{7}{25}}$



6. Find all exact solutions in radians.

a) (6 points) $\cos(x) = \frac{\sqrt{2}}{2}$

$\cos^{-1}\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$

$\boxed{x = \pm \frac{\pi}{4} + 2\pi N \text{ where } N \text{ is an integer}}$

b) (12 points) $2\sin^2(x) = -\sin(x)$

$2\sin^2(x) + \sin(x) = 0$

$\sin(x)\,[2\sin(x) + 1] = 0$

$\sin(x) = 0 \qquad x = 0, \pi$

$\sin(x) = -\frac{1}{2} \qquad x = \sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$

$\pi - \left(-\frac{\pi}{6}\right) = \frac{7\pi}{6}$

$x = 0 + 2\pi N$

$x = \pi + 2\pi N$

$x = -\frac{\pi}{6} + 2\pi N$

$x = \frac{5\pi}{6} + 2\pi N$

where N is an integer

7. (10 points) Solve the equation $4 * e^{(-3x+5)} = 48$. First find the exact answer, then approximate the answer to the nearest hundredth using the calculator.

$e^{-3x+5} = \frac{48}{4} = 12$

$e^{-3x+5} = 12$

$\ln(e^{-3x+5}) = \ln 12$

$(-3x+5)\underbrace{(\ln e)}_{=1} = \ln 12$

$-3x + 5 = \ln 12$

$-3x = \ln 12 - 5$

$x = \boxed{\frac{\ln 12 - 5}{-3}} = \underbrace{\boxed{\frac{5 - \ln 12}{3}}}_{\text{exact}} \approx .8383644501 \approx \boxed{.84}$

8. (10 points) Find the domain, asymptotes, and x-intercepts of the function $f(x) = \log(12 - 6x)$. Sketch its graph on graphing paper.

domain $12 - 6x > 0$

$12 > 6x$

$2 > x$ $\boxed{(-\infty, 2)}$

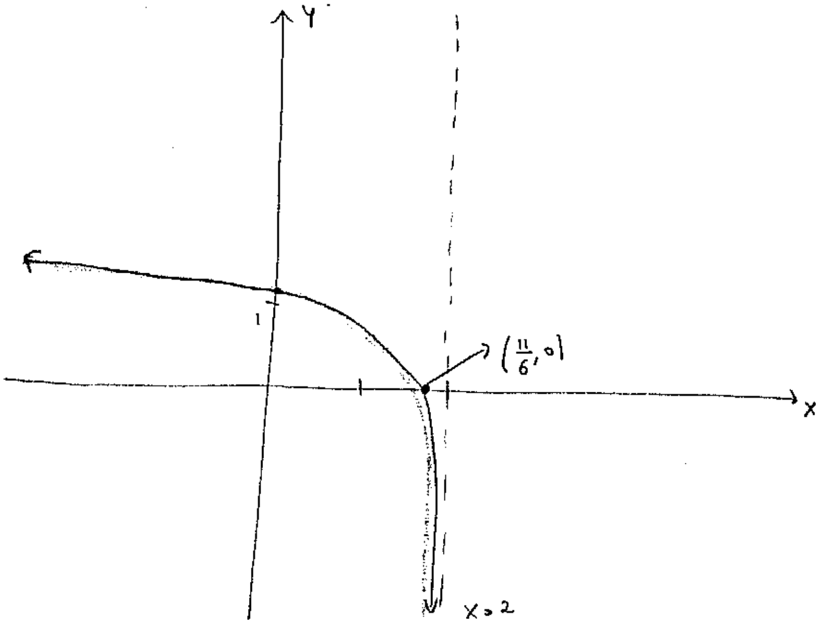
V.A $\boxed{x = 2}$

x-int. $12 - 6x = 1$

$11 = 6x$

$x = \frac{11}{6}$ $\left(\frac{11}{6}, 0\right)$

$\approx (1.8, 0)$



9. (15 points) Given $y = 4\sin(4x + \pi)$, state the amplitude, period and phase shift, and then sketch one complete cycle of the graph on graphing paper. Label all maxima, minima and x-intercepts, with exact answers in radians.

$y = 4\sin(4x + \pi)$ $A = 4$ $B = 4$ $C = \pi$

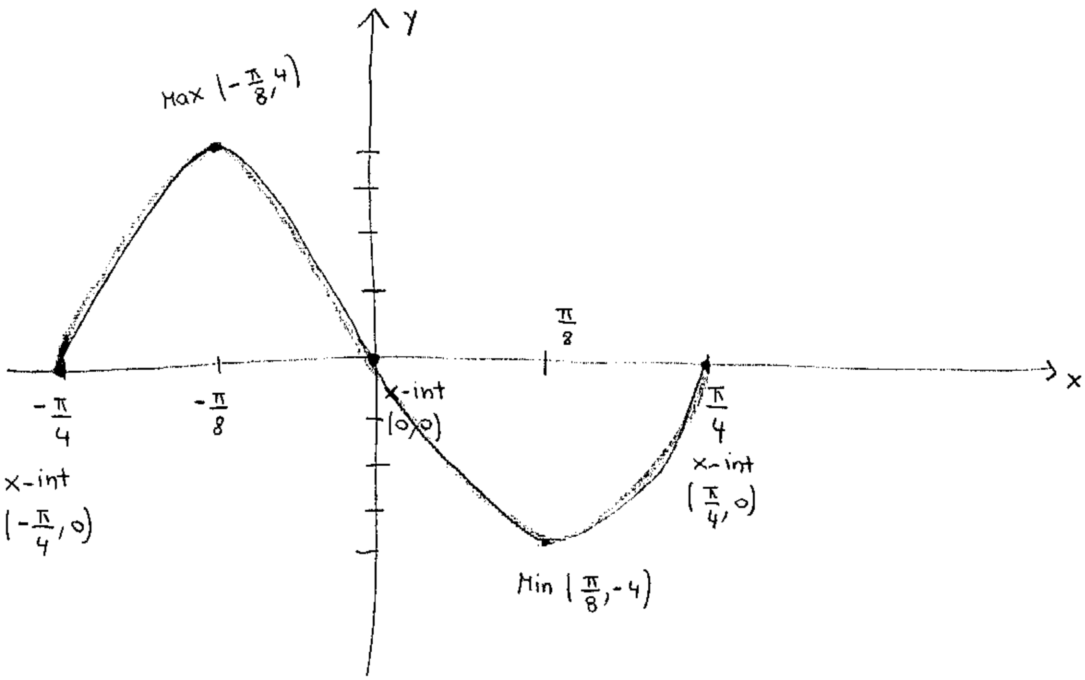
Amplitude $= \boxed{4}$

period $= \frac{2\pi}{B} = \frac{2\pi}{4} = \boxed{\frac{\pi}{2}}$

phase shift $= -\frac{C}{B} = \boxed{-\frac{\pi}{4}}$

wave starts at $-\frac{\pi}{4}$

wave ends at $-\frac{\pi}{4} + \frac{\pi}{2} = \frac{\pi}{4}$



New York City College of Technology
MAT 1375/D575 - Spring 2019
Exam 3 - Version Y - Total Points: 100

NAME: Key

Instructions: Write your solutions in the space provided after each question. You may use the back of each page for any scratch work that you need to do. To receive (partial) credit you must **show all your work in a clear and organized manner.**

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1. (8 points) Let $u = \ln x$, $v = \ln y$, where $x, y > 0$. Write the expression $\ln\left(\frac{y^4}{\sqrt[3]{x^2}}\right)$ in terms of u and v.

$\ln y^4 - \ln \sqrt[3]{x^2} = \ln y^4 - \ln x^{2/3} = 4 \ln y - \frac{2}{3} \ln x = \boxed{4v - \frac{2}{3} u}$

2. (7 points) Condense the following expression into a single logarithm by applying the properties of logarithms: $\frac{1}{2}\log x - 6\log y - 8\log z$.

$\log x^{1/2} - \log y^6 - \log z^8$

$= \boxed{\log\left(\frac{\sqrt{x}}{y^6 z^8}\right)}$

3. (10 points) Solve the equation $5 * e^{(5x-1)} = 45$. First find the exact answer, then approximate the answer to the nearest hundredth using the calculator.

$e^{5x-1} = \frac{45}{5} = 9$

$e^{5x-1} = 9$

$\ln e^{5x-1} = \ln 9$

$(5x-1) \underbrace{\ln e}_{=1} = \ln 9$

$5x - 1 = \ln 9$

$5x = \ln 9 + 1$

$\boxed{x = \frac{\ln 9 + 1}{5}}$ exact $\approx .6394449155 \approx \boxed{.64}$

4. (12 points) The population of a city grows exponentially at a rate of 4% per year. If the population was 50,000 people in the year 2010, then what is the population size of this city in the year 2016? [Round your answer to the nearest integer.] In what year will the population be 80,000 people?

$P(t) = 50{,}000\,(1+.04)^t = 50{,}000\,(1.04)^t$ $\quad t=0$ is 2010

In 2016 $\quad t=6 \quad P(6) = 50{,}000\,(1.04)^6 = 63265.95092$ $\quad \boxed{63{,}266}$

$80{,}000 = 50{,}000\,(1.04)^t$

$1.6 = (1.04)^t$

$\ln(1.6) = t\ln(1.04)$

$t = \dfrac{\ln(1.6)}{\ln(1.04)} = 11.98355643$

$2010 + 11 = \boxed{2021}$

(ok if 2022)

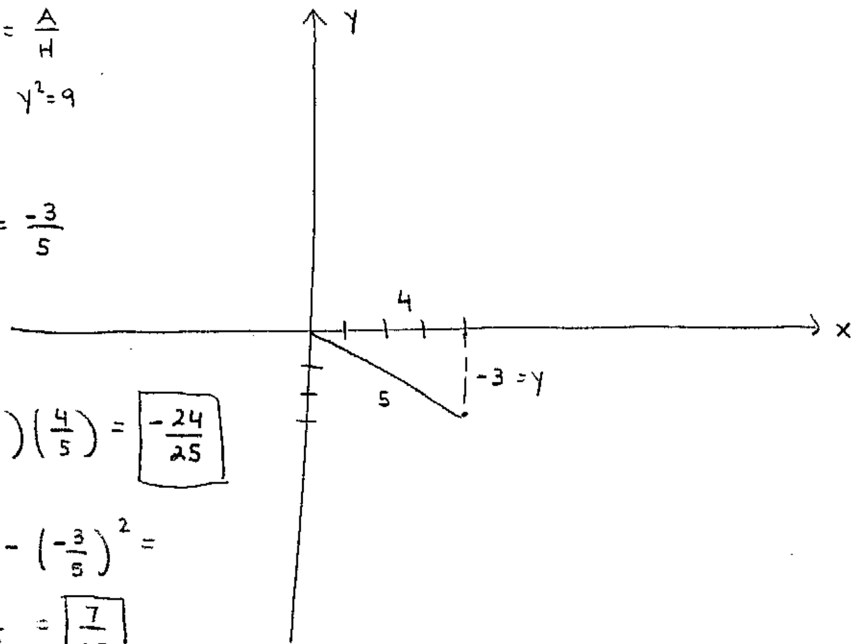
5. (10 points) Given that $\cos(\alpha) = 4/5$ and α is in quadrant 4, find the exact values of $\sin(2\alpha)$ and $\cos(2\alpha)$. [Use the formulas $\sin(2\alpha) = 2\sin(\alpha)\cos(\alpha)$ and $\cos(2\alpha) = \cos^2(\alpha) - \sin^2(\alpha)$].

$\cos(\alpha) = \frac{4}{5} = \frac{A}{H}$

$y^2 + 16 = 25 \quad y^2 = 9$

$y = -3$

$\sin(\alpha) = \frac{O}{H} = \frac{-3}{5}$



$\sin(2\alpha) = 2\left(-\frac{3}{5}\right)\left(\frac{4}{5}\right) = \boxed{-\frac{24}{25}}$

$\cos(2\alpha) = \left(\frac{4}{5}\right)^2 - \left(-\frac{3}{5}\right)^2 =$

$\frac{16}{25} - \frac{9}{25} = \boxed{\frac{7}{25}}$

6. Find all exact solutions in radians.

a) (6 points) $\sin(x) = -\frac{\sqrt{2}}{2}$

$x_1 = \sin^{-1}\left(-\frac{\sqrt{2}}{2}\right) = -\frac{\pi}{4}$

$x_2 = \pi - \left(-\frac{\pi}{4}\right) = \frac{5\pi}{4}$

$x = -\frac{\pi}{4} + 2\pi N$

$x = \frac{5\pi}{4} + 2\pi N$

where N is an integer

b) (12 points) $4\cos^2(x) = 2\cos(x)$

$4\cos^2(x) - 2\cos(x) = 0$

$2\cos(x)\,[2\cos(x) - 1] = 0$

$\cos(x) = 0$ $\quad x = \frac{\pi}{2}, -\frac{\pi}{2}$

$\cos^{-1}(0) = \frac{\pi}{2}$

$2\cos(x) - 1 = 0$

$\cos(x) = \frac{1}{2}$

$\cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$ $\quad x = \frac{\pi}{3}, -\frac{\pi}{3}$

$x = \pm\frac{\pi}{2} + 2\pi N$

$x = \pm\frac{\pi}{3} + 2\pi N$

where N is an integer

7. (10 points) Solve the equation $\log_4(x + 6) + \log_4(x) = 2$.

$\log_4(x^2 + 6x) = 2$

$x^2 + 6x = 4^2 = 16$

$x^2 + 6x - 16 = 0$

$(x+8)(x-2) = 0$

$x = \cancel{-8}$

$\boxed{x = 2}$

Y

8. (10 points) Find the domain, asymptotes, and x-intercepts of the function $f(x) = \log(10 - 5x)$. Sketch its graph on graphing paper.

domain $\quad 10 - 5x > 0$

$-5x > -10$

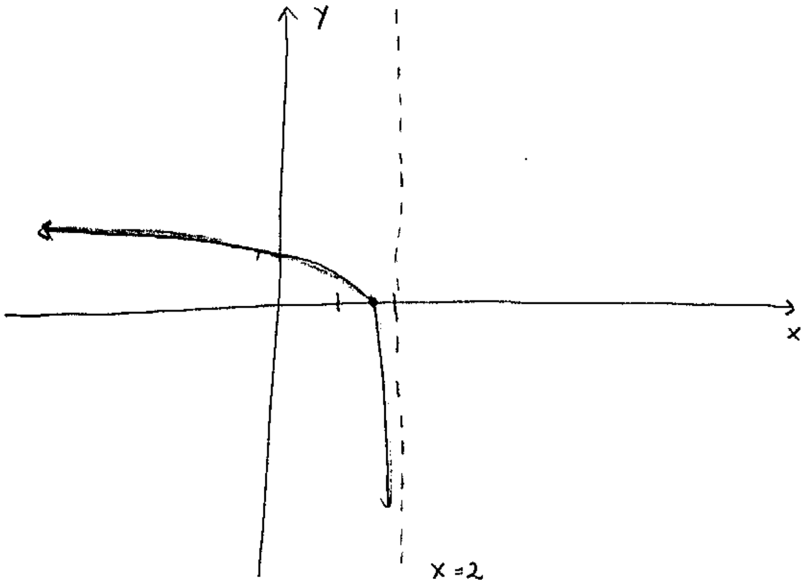
$x < 2 \quad \boxed{(-\infty, 2)}$

V.A. $\quad \boxed{x = 2}$

x-int $\quad 10 - 5x = 1$

$-5x = -9 \quad x = \frac{9}{5} \quad \boxed{\left(\frac{9}{5}, 0\right)}$

$(1.8, 0)$



9. (15 points) Given $y = 2\cos(3x + \pi)$, state the amplitude, period and phase shift, and then sketch one complete cycle of the graph on graphing paper. Label all maxima, minima and x-intercepts, with exact answers in radians.

$y = 2\cos(3x + \pi) \quad A = 2 \quad B = 3 \quad C = \pi$

amplitude $= 2$

period $= \frac{2\pi}{3}$

phase shift $= -\frac{C}{B} = -\frac{\pi}{3}$

graph starts at $-\frac{\pi}{3}$

ends at $-\frac{\pi}{3} + \frac{2\pi}{3} = \frac{\pi}{3}$