

①  $f(x) = (x+2)(x-5)(x-(3-2i))(x-(3+2i))$

(Recall that since  $3-2i$  is a root, its complex conjugate  $3+2i$  is also a root)

Now  $(x-(3-2i))(x-(3+2i)) = (x-3+2i)(x-3-2i) =$

$x^2 - 3x - 2ix - 3x + 9 + 6i + 2ix - 6i - 4i^2 = x^2 - 6x + 9 - 4(-1) = x^2 - 6x + 9 + 4 = x^2 - 6x + 13$ , which has real coefficients.

So  $f(x) = (x+2)(x-5)(x^2-6x+13)$

②  $f(x) = d x (x-1)(x-3)$

We need to find  $d$  using  $f(2) = 10$

$10 = d \cdot 2 \cdot (2-1) \cdot (2-3)$

$10 = -2d \quad d = -5$

$f(x) = -5x(x-1)(x-3)$

③  $f(x) = \frac{x-2}{x^2+x+1}$

domain: Solve  $x^2+x+1=0 \quad x = \frac{-1 \pm \sqrt{1-4}}{2} = \frac{-1 \pm \sqrt{-3}}{2}$  no real solutions.

The denominator is never 0, so domain is  $(-\infty, \infty)$

x-intercept  $x-2=0 \quad x=2 \quad (2, 0)$

y-intercept  $f(0) = -2 \quad (0, -2)$

Horizontal asymptote  $y=0$  (because the degree of the numerator (1) is less than the degree of the denominator (2)).

Vertical asymptote **None** (because the denominator is never zero)

④  $f(x) = \frac{3x-9}{2-x}$

a) domain  $2-x \neq 0 \quad x \neq 2$

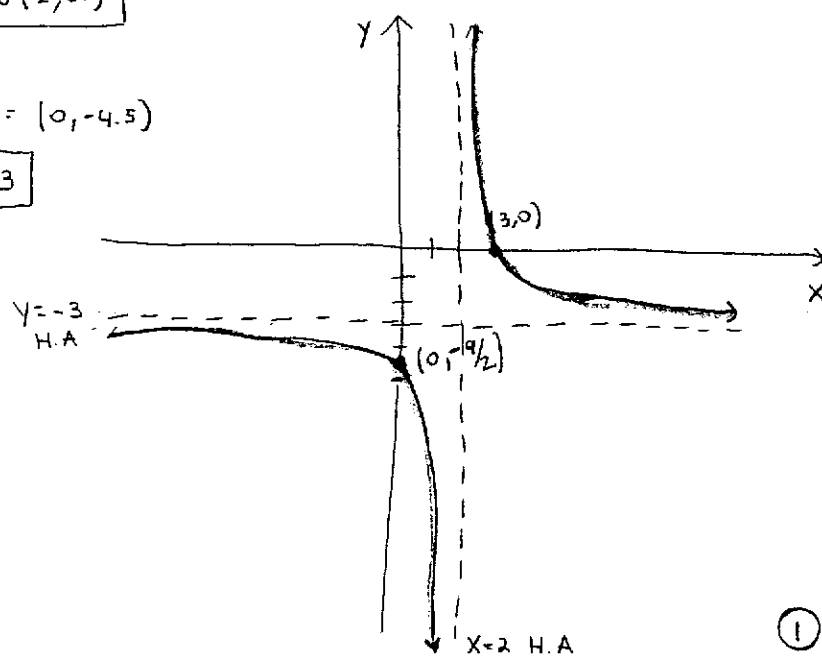
x-intercept  $3x-9=0 \quad x=3$

y-intercept  $f(0) = -9/2$

Horizontal asymptote  $y = \frac{3}{-1} \quad y = -3$

Vertical asymptote  $x=2$

b) Graphing paper will be provided



c) Identify roots and V.A:  $x=2, x=3$ . Test a point in each interval.

NEGATIVE	POSITIVE	NEGATIVE
$x=1$	$x=2.5$	$x=4$
$\frac{3(1)-9}{2-1} = -6 < 0$	$\frac{3(2.5)-9}{2-2.5} = 3 > 0$	$\frac{3(4)-9}{2-4} = -\frac{3}{2} < 0$

Answer  $(2, 3]$

(Notice that  $x=2$  is not included since it is not in the domain)

⑤  $25 - 9x^2 < 0$

Identify roots:  $25 = 9x^2$      $x^2 = \frac{25}{9}$      $x = \pm \frac{5}{3}$

Test a point in each interval

NEGATIVE	POSITIVE	NEGATIVE
$x=-2$	$x=0$	$x=2$
$25 - 9(-2)^2 = -11 < 0$	$25 > 0$	$25 - 9(2)^2 = -11 < 0$

Answer:  $(-\infty, -\frac{5}{3}) \cup (\frac{5}{3}, \infty)$

⑥

$$\begin{array}{r}
 2x^2 + 3x + 6 \\
 x-1 \overline{) 2x^3 + x^2 + 3x + 5} \\
 \underline{-2x^3 + 2x^2} \phantom{+ 5} \\
 3x^2 + 3x + 5 \\
 \underline{-3x^2 + 3x} \phantom{+ 5} \\
 6x + 5 \\
 \underline{-6x + 6} \\
 11
 \end{array}$$

Answer  $2x^2 + 3x + 6 + \frac{11}{x-1}$

or: Quotient =  $2x^2 + 3x + 6$   
Remainder = 11

⑦ From the graph of  $f(x)$  (or a table) we "guess" that  $x=-1$  is a root.

We check that  $f(-1) = 0$ :  $f(-1) = (-1)^3 + 7(-1)^2 + 13(-1) + 7 = -1 + 7 - 13 + 7 = 0$ . So  $x=-1$  is a root.

The other roots are not exact, so we use algebra.

Since  $x=-1$  is a root,  $x+1$  is a factor of  $f(x)$ ,

$f(x) = (x+1)g(x)$ . To find  $g(x)$  we divide  $f(x)$  by  $x+1$  (notice that remainder must be zero)

$$\begin{array}{r}
 x^2 + 6x + 7 \\
 x+1 \overline{) x^3 + 7x^2 + 13x + 7} \\
 \underline{-x^3 - x^2} \phantom{+ 7} \\
 6x^2 + 13x + 7 \\
 \underline{-6x^2 - 6x} \phantom{+ 7} \\
 7x + 7 \\
 \underline{-7x - 7} \\
 0
 \end{array}$$

$f(x) = (x+1)(x^2 + 6x + 7)$

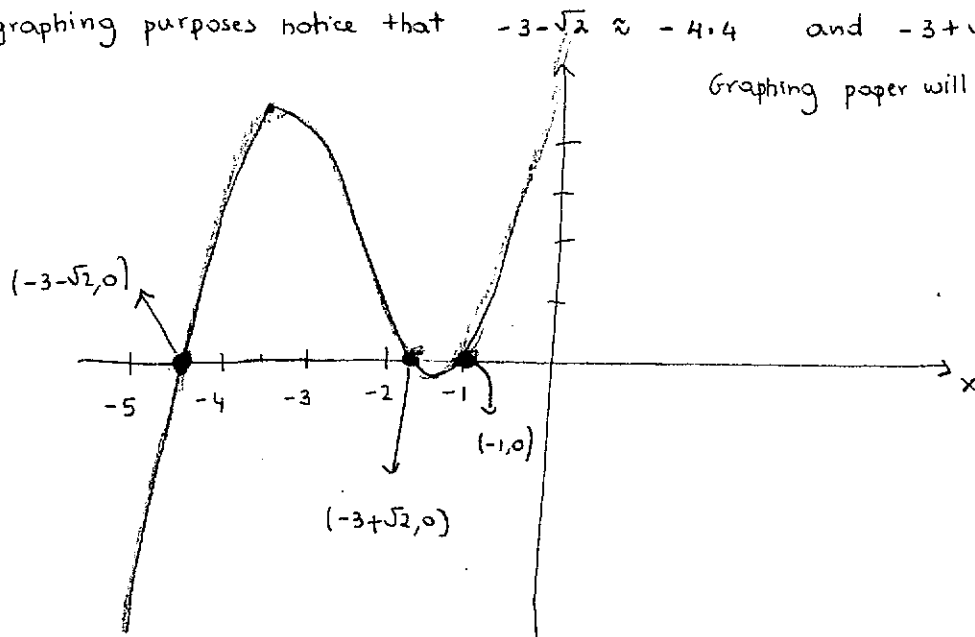
To find the other roots we solve  $x^2 + 6x + 7 = 0$  using the quadratic formula

$$x^2 + 6x + 7 = 0 \quad x = \frac{-6 \pm \sqrt{36 - 28}}{2} = \frac{-6 \pm \sqrt{8}}{2} = \frac{-6 \pm 2\sqrt{2}}{2} = -3 \pm \sqrt{2}$$

So the 3 roots of  $f(x)$  are:  $x = -1, x = -3 - \sqrt{2}, x = -3 + \sqrt{2}$

For graphing purposes notice that  $-3 - \sqrt{2} \approx -4.4$  and  $-3 + \sqrt{2} \approx -1.6$

Graphing paper will be provided



⑧  $f(x) = \frac{x^2 - 36}{x - 6}$

domain  $(-\infty, 6) \cup (6, \infty)$

y-intercept  $(0, 6)$

x-intercepts  $x^2 - 36 = 0 \quad x = \pm 6$  but  $x = 6$  is not in the domain, so

the only x-intercept is  $(-6, 0)$

Since  $\frac{(x-6)(x+6)}{(x-6)} = x+6$   $f(x)$  is the line  $y = x+6$  with a hole at  $x=6$

There are no horizontal or vertical asymptotes.

$$(9) \quad 15x^2 + 22x - 5 \leq 0$$

Identify roots:  $15x^2 + 22x - 5 = 0$

$$x = \frac{-22 \pm \sqrt{(22)^2 - 4(15)(-5)}}{30} = \frac{-22 \pm \sqrt{484 + 300}}{30} = \frac{-22 \pm \sqrt{784}}{30} =$$

$$\frac{-22 \pm 28}{30} \begin{cases} \frac{-22 + 28}{30} = \frac{6}{30} = \frac{1}{5} \\ \frac{-22 - 28}{30} = \frac{-50}{30} = -\frac{5}{3} \end{cases}$$

Test a point in each interval:

POSITIVE	NEGATIVE	POSITIVE
$x = -2$	$x = 0$	$x = 1$
$15(-2)^2 + 22(-2) - 5 = 60 - 44 - 5 = 11 > 0$	$-5 < 0$	$15 + 22 - 5 = 32 > 0$

Answer  $[-\frac{5}{3}, \frac{1}{5}]$