MAT 1375 Review for EXAM I Solutions

① $(-\infty, -1)$

② $\left[-5, \frac{1}{2}\right]$

③ $\sqrt{2}$

④ 0

⑤ π

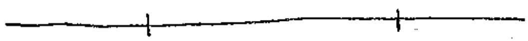
⑥ $f^{-1}(11) = 5$

⑦ $x = 8, -8$

⑧ No solution (the absolute value is never negative)

⑨ $(-\infty, -2] \cup [2, \infty)$

First solve $|x| = 2$ which gives $x = 2$ and $x = -2$. Then test a point in each interval.



Test $x = -3$ | -2 | Test $x = 0$ | 2 | Test $x = 3$

$|-3| \overset{?}{\geq} 2$ yes | $|0| \overset{?}{\geq} 2$ No | $|3| \overset{?}{\geq} 2$ yes

⑩ $y = (x+3)^2 - 5$ (graph with Desmos to check!)

⑪ a) $f(-1) = (-1)^2 - 2(-1) - 3 = 1 + 2 - 3 = \boxed{0}$

b) $g(-4) = 2(-4) - 8 = \boxed{-16}$

c) $f(a) = \boxed{a^2 - 2a - 3}$

d) f and g are both polynomials, so they both have domains $\boxed{(-\infty, \infty)}$

e) $(f+g)(5) = f(5) + g(5) = [25 - 10 - 3] + [10 - 8] = 12 + 2 = \boxed{14}$

[or: $(f+g)(x) = x^2 - 11$ so $(f+g)(5) = 25 - 11 = 14$]

f) $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \boxed{\frac{x^2 - 2x - 3}{2x - 8}}$

g) We need $2x - 8 \neq 0$ so $x \neq 4$ domain $\boxed{(-\infty, 4) \cup (4, \infty)}$

h) $\left(\frac{g}{f}\right)(x) = \frac{g(x)}{f(x)} = \boxed{\frac{2x - 8}{x^2 - 2x - 3}}$

i) We need $x^2 - 2x - 3 \neq 0$ Domain $\boxed{(-\infty, -1) \cup (-1, 3) \cup (3, \infty)}$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$ gives $x = 3$, $x = -1$

j) $(fg)(x) = f(x)\,g(x) = (x^2 - 2x - 3)(2x - 8) = 2x^3 - 8x^2 - 4x^2 + 16x - 6x + 24 = \boxed{2x^3 - 12x^2 + 10x + 24}$

k) $(g \circ f)(2) = g(f(2)) = g(-3) = 2(-3) - 8 = \boxed{-14}$

$f(2) = 4 - 4 - 3 = -3$

l) $(f \circ f)(0) = f(f(0)) = f(-3) = (-3)^2 - 2(-3) - 3 = 9 + 6 - 3 = \boxed{12}$

m) $(f \circ g)(x) = f(g(x)) = f(2x-8) = (2x-8)^2 - 2(2x-8) - 3 =$

$4x^2 - 32x + 64 - 4x + 16 - 3 = \boxed{4x^2 - 36x + 77}$

n) $(g \circ f)(x) = g(f(x)) = 2(x^2 - 2x - 3) - 8 = \boxed{2x^2 - 4x - 14}$

(12) $3 - 5x \geq 0 \quad -5x \geq -3 \quad \frac{-5x}{-5} \leq \frac{-3}{-5} \quad x \leq \frac{3}{5} \quad \left(-\infty, \frac{3}{5}\right]$

(13) $|5x+4| = 11$

$5x + 4 = 11$ | $5x + 4 = -11$

$5x = 7 \quad x = \frac{7}{5}$ | $5x = -15$

$x = -3$

$\boxed{x = -3, \; x = \frac{7}{5}}$

(14) $y = 9x - 3$

$x = 9y - 3$ (switch x and y)

Solve for y: $9y = x + 3$

$\boxed{y = \frac{x+3}{9}}$

(15) We need $2x - 8 > 0$ (it can not be zero since it is at the denominator)

$2x > 8$

$x > 4$

$\boxed{(4, \infty)}$

(16) $|6x+2| < 7$

$|6x+2| = 7$ gives $x = -\frac{3}{2}, \; x = \frac{5}{6}$

$6x + 2 = 7$ | $6x + 2 = -7$

$6x = 5$ | $6x = -9$

$x = \frac{5}{6}$ | $x = -\frac{9}{6} = -\frac{3}{2}$

$\boxed{\left(-\frac{3}{2}, \frac{5}{6}\right)}$

NO	YES	NO
Test $x = -2$	Test $x = 0$	Test $x = 1$
$\lvert 6(-2)+2 \rvert \overset{?}{<} 7$	$\lvert 2 \rvert \overset{?}{<} 7$ YES	$\lvert 6(1)+2 \rvert \overset{?}{<} 7$
$10 \overset{?}{<} 7$ NO		$8 \overset{?}{<} 7$ NO

$-\frac{3}{2} = -1.5 \qquad \frac{5}{6}$

(17) $\dfrac{f(x+h) - f(x)}{h} = \dfrac{(x+h)^2 - 4(x+h) - 2 - (x^2 - 4x - 2)}{h} =$

$\dfrac{\cancel{x^2} + 2xh + h^2 - \cancel{4x} - 4h - \cancel{2} - \cancel{x^2} + \cancel{4x} + \cancel{2}}{h} = \dfrac{h(2x + h - 4)}{h} = \boxed{2x + h - 4}$

(18) $y = \dfrac{4}{7x - 3}$

$x = \dfrac{4}{7y - 3}$

(switch x and y)

Now solve for y:

$x(7y - 3) = 4$

$7xy - 3x = 4$

$7xy = 3x + 4$

$\boxed{y = \dfrac{3x+4}{7x}}$ or $y = \dfrac{3}{7} + \dfrac{4}{7x}$