**Mathematics 1375/D572, Spring 2019** **Exam #2**
**Instructor: Suman Ganguli** **Tuesday, April 2** **Name:** ____________________

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1. (10 points) Consider the quadratic polynomial

$$q(x) = -x^2 + 2x + 1$$

(a) Find the roots of $q(x)$ algebraically, and express them in simplest radical form. (Hint: The function does *not* factor, so use the quadratic formula.)

**Solution:** To find the roots, we solve the equation $q(x) = 0$. By the quadratic formula:

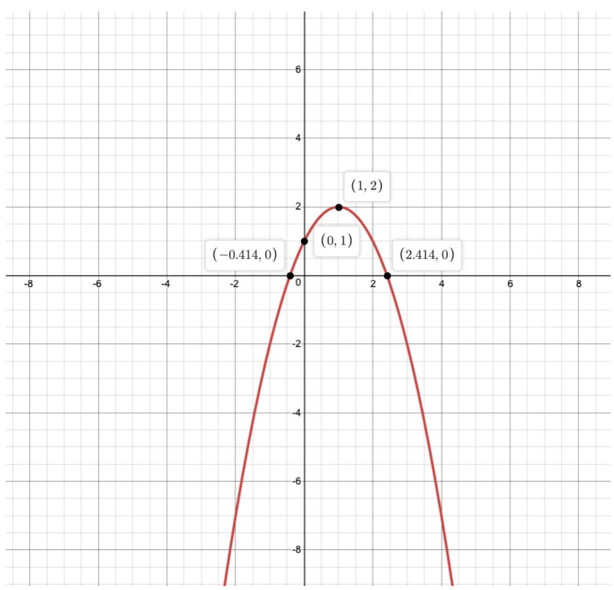
$$x = \frac{-2 \pm \sqrt{4 - 4(-1)(1)}}{-2} = \frac{-2 \pm \sqrt{8}}{-2} = \frac{-2 \pm 2\sqrt{2}}{-2} = 1 \pm \sqrt{2}$$

(b) What are the coordinates of the vertex of the parabola $y = q(x)$? (Recall that for a parabola $y = ax^2 + bx + c$, the $x$-coordinate of the vertex is given by $x = -\frac{b}{2a}$.)

**Solution:** The $x$-coordinate of the vertex is at $x = -\frac{b}{2a} = -\frac{2}{-2} = 1$ and so the $y$-coordinate of the vertex is $q(1) = -1 + 2(1) + 1 = 2$. Thus, the vertex of the parabola occurs at $(1, 2)$.

(c) Sketch the graph of $q(x)$, labelling the $x$-intercepts, the $y$-intercept, and the vertex with their coordinates:

**Solution:** From #1, we know the $x$-intercepts occur at $(-1, 0)$ and $(3, 0)$.
Since $f(0) = 0^2 - 2(0) - 3 = -3$, the $y$-intercepts occurs at $(0, -3)$.




(d) Use the graph to solve the following inequality; express the solution in interval notation:

$$-x^2 + 2x + 1 > 0$$

**Solution:** From the graph (or really just from the fact that we know the graph of $y = -x^2 + 2x + 1$ is an parabola opening downwards, with $x$-intercepts at $x = 1 - \sqrt{2}$ and $x = 1 + \sqrt{2}$), we see that the solution set of $-x^2 + 2x + 1 > 0$ is

$$(1 - \sqrt{2}, 1 + \sqrt{2})$$

2. (10 points) Consider the function

$$f(x) = (x-2)^3$$

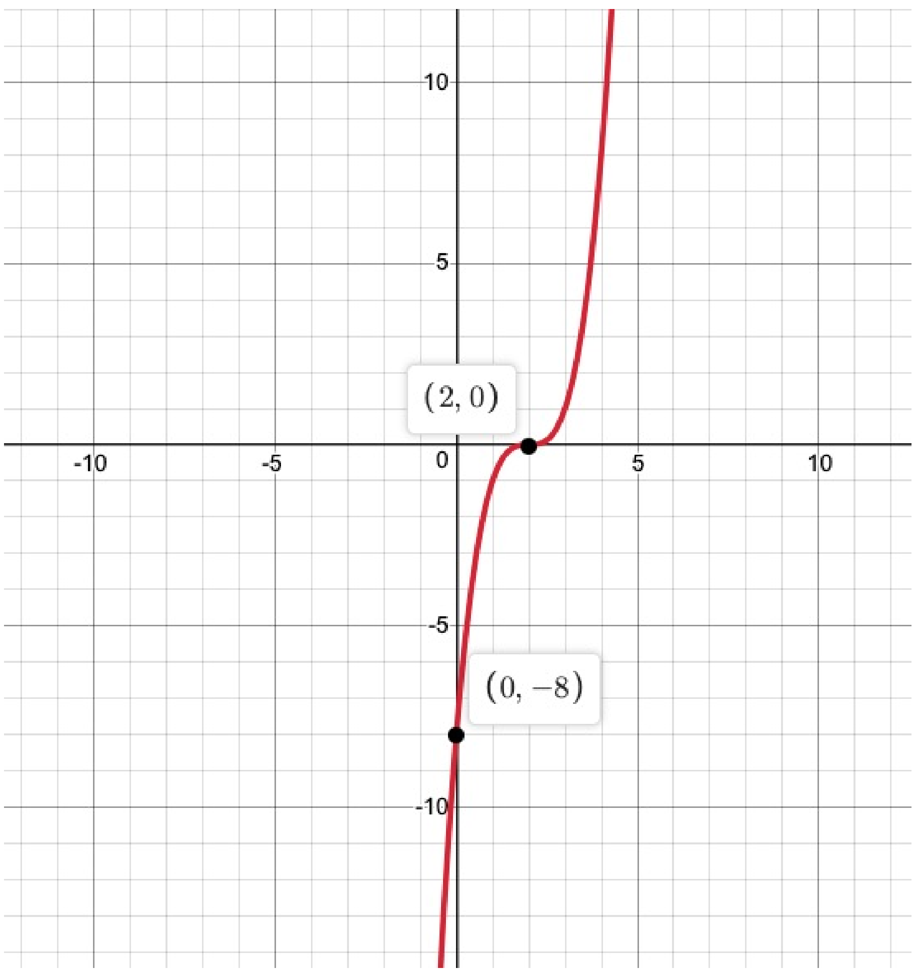
(a) Fill in the blanks:
"The only root of $f(x)$ is $x =$ ______, which is a root of multiplicity ______ ."

**Solution:** The only root of $f(x)$ is $x = 2$, which is a root of multiplicity 3.

(b) What is the $y$-intercept of the graph of $f(x)$? Show the necessary calculations.

**Solution:** $f(0) = (0-2)^3 = -8$, so the $y$-intercept is $(0, -8)$.

(c) Sketch the graph of $y = f(x)$. Label the $x$-intercept and $y$-intercept on your graph.



3. (15 points) Consider the cubic polynomial:

$$p(x) = x^3 + x^2 - x - 1$$

(a) Verify that $c = -1$ is a root of $p(x)$ (i.e., show that $p(-1) = 0$):

**Solution:** $p(-1) = (-1)^3 + (-1)^2 - (-1) - 1 = -1 + 1 + 1 - 1 = 0$

(b) Since we know from (a) that $c = -1$ is a root of $p$, we know $(x - c) = (x + 1)$ is a factor of $p(x)$. Use long division to compute $\dfrac{p(x)}{x+1}$:

**Solution:**

$$\begin{array}{r|l} & x^2 \qquad - 1 \\ x+1 & x^3 + x^2 - x - 1 \\ & -x^3 - x^2 \\ \hline & \qquad -x - 1 \\ & \qquad\ \ x + 1 \\ \hline & \qquad\qquad 0 \end{array}$$

(c) Fill in the blank with your result from (b), and then finish factoring $p(x)$:

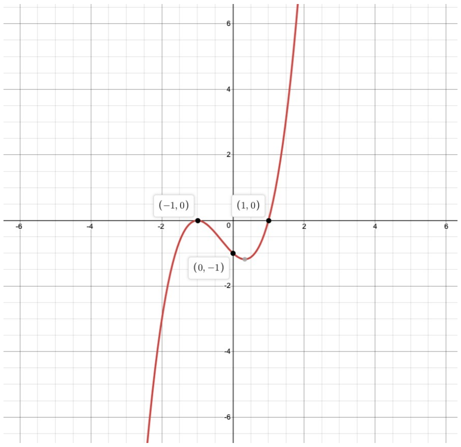
$$p(x) = x^3 + x^2 - x - 1 = (x + 1)(\underline{\qquad\qquad\qquad}) =$$

**Solution:** $p(x) = x^3 + x^2 - x - 1 = (x + 1)(x^2 - 1) = (x + 1)(x + 1)(x - 1) = (x + 1)^2(x - 1)$

(d) What are the roots of $p(x)$?

**Solution:** The roots of $p(x)$ are $x = -1$ and $x = 1$.

(e) Sketch a complete graph of the function below (with the help of Desmos or a graphing calculator). Label the $x$-intercepts and the $y$-intercept on the graph with their coordinates.




(f) Use the graph to solve the following inequality: **circle the parts of the graph corresponding to the solution set of the inequality**, and write down the solution set in interval notation:

$$x^3 + x^2 - x - 1 < 0$$

**Solution:**

$(-\infty, -1) \cup (-1, 1)$

4. (15 points) Consider the rational function: $f(x) = \dfrac{x-2}{x^2+2x-3}$:

(a) What is the domain of $f$? Show your calculations, and write the solution in interval notation:

**Solution:** Since the denominator of $f$ is $x^2+2x-3 = (x+3)(x-1)$, the function is undefined for $x = -3$ and $x = 1$. Hence, the domain of $f$ is

$$(-\infty, -3) \cup (-3, 1) \cup (1, \infty)$$

(b) What are the vertical asymptotes of this function?

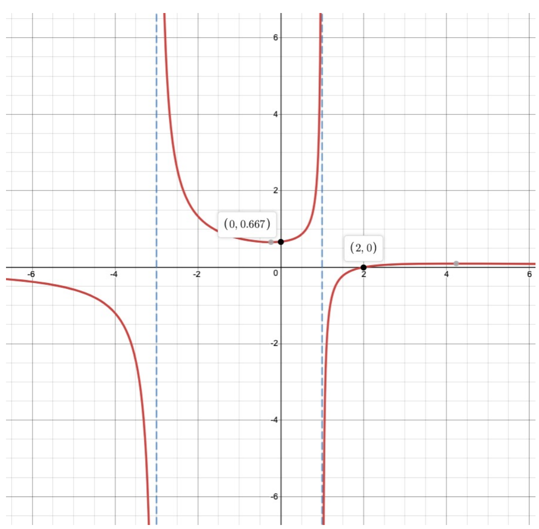
**Solution:** The vertical asymptotes occur at the $x$-values at which the denominator is 0, i.e., the vertical lines $x = -3$ and and $x = 1$.

(c) Algebraically calculate for the the $x-$ and $y-$intercepts of the graph of $f(x)$. Again, show the necessary calculations, and write the coordinates of the intercepts in $(x, y)$ form:

**Solution:**
The $x-$intercept occurs when $x - 2 = 0$, hence at $x = 2$, i.e., at the point $(2, 0)$
The $y-$intercept occurs at $f(0) = \dfrac{0-2}{0+0-3} = \dfrac{2}{3}$, i.e., at the point $(0, \dfrac{2}{3})$

(d) Sketch a complete graph of the function below (with the help of Desmos or a graphing calculator):

- Label the $x$- and $y$-intercepts with their coordinates
- Draw the vertical asymptotes as dashed lines, and label each with its equation




(e) Use the graph to solve the following inequality: **circle the parts of the graph corresponding to the solution set of the inequality**, and write down the solution set in interval notation.

$$\frac{x-2}{x^2+2x-3} \leq 0$$

**Solution:**

$$(-\infty, -3) \cup (1, 2]$$