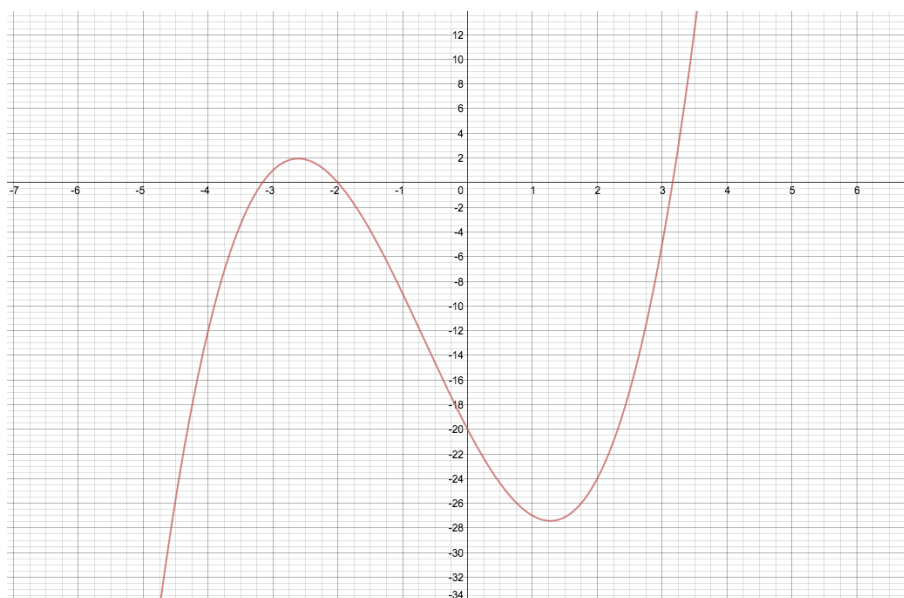


Shown below is the graph of the cubic polynomial $p(x) = x^3 + 2x^2 - 10x - 20$:



1. (1 point) From the graph, it *seems* that $x = -2$ is a root of $p(x)$. Verify that this is the case (i.e., show that $p(-2) = 0$).

Solution: $p(-2) = (-2)^3 + 2(-2)^2 - 10(-2) - 20 = -8 + 8 + 20 - 20 = 0$

2. (3 points) Use the root $c = -2$ to factor the polynomial $p(x)$:
- (a) Since we know from #1 that $c = -2$ is a root of p , we know $(x - c) = (x + 2)$ is a factor of $p(x)$. Use long division to compute $\frac{p(x)}{x + 2}$:

$$x + 2 \overline{) x^3 + 2x^2 - 10x - 20}$$

Solution:

$$\begin{array}{r} x^2 \quad - 10 \\ x + 2 \overline{) x^3 + 2x^2 - 10x - 20} \\ \underline{-x^3 - 2x^2} \\ - 10x - 20 \\ \underline{10x + 20} \\ \\ 0 \end{array}$$

- (b) Fill in the blank with your result from (a) to get the factorization of $p(x)$:

$$p(x) = x^3 + 2x^2 - 10x - 20 = (x + 2)(\underline{\hspace{2cm}})$$

Solution:

$$p(x) = x^3 + 2x^2 - 10x - 20 = (x + 2)(x^2 - 10)$$

3. (2 points) Use the factorization from #2(b) to algebraically solve for the other two roots of $p(x)$ in radical form (i.e., solve for the roots of the quadratic polynomial that results from factoring $x + 2$ out of $p(x)$).

Solution: The roots of $p(x) = x^3 + 2x^2 - 10x - 20 = (x + 2)(x^2 - 10)$ occur when $x + 2 = 0$ or $x^2 - 10 = 0$.

The equation $x + 2 = 0$ yields the root $x = -2$, which was identified from the graph and verified as a root in part (a). We solve the equation $x^2 - 10 = 0$ in order to find the other two roots of $p(x)$. You can use the quadratic formula, but in this case (when $b = 0$, i.e., there's no x term) it's easier to just solve directly:

$$x^2 - 10 = 0 \iff x^2 = 10 \iff x = \pm\sqrt{10}$$

4. (2 points) (a) Write down the coordinates of the 3 x -intercepts of the graph of $p(x)$, corresponding to the 3 roots:

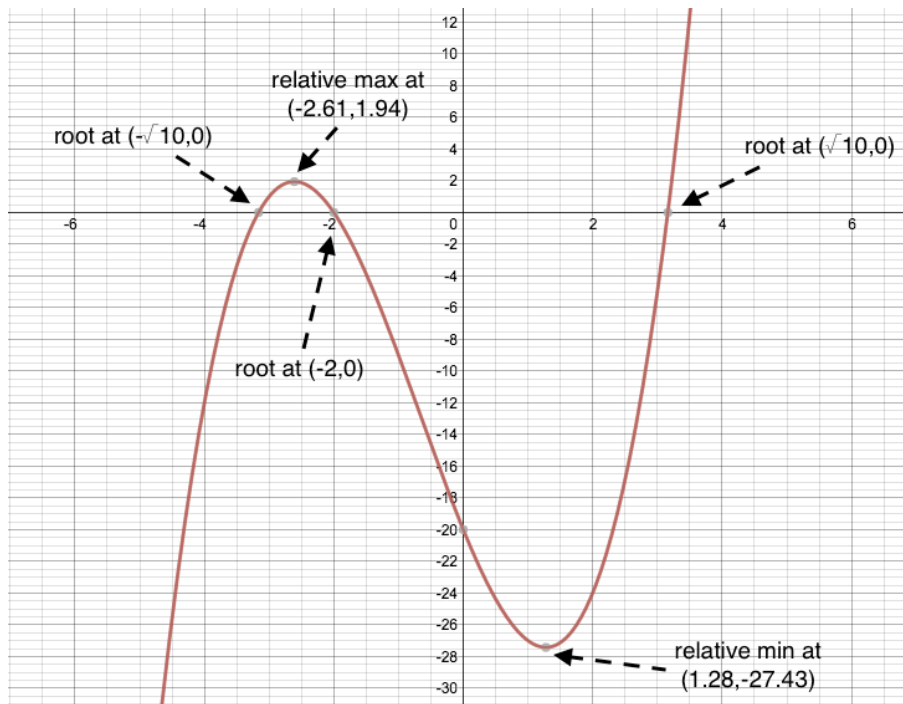
Solution:

$$(-\sqrt{10}, 0), (-2, 0), \sqrt{10}, 0$$

- (b) Algebraically calculate the y -intercept of the graph $y = p(x)$ and write down the coordinates of the y -intercept:

Solution: Since $p(0) = 0^3 + 2(0^2) - 10(0) - 20 = -20$, the y -intercept of the graph is at $(-20, 0)$.

5. (2 points) Finally, label the x -intercepts and the y -intercept on the graph with their coordinates (leave the x -coordinates corresponding to the 2 roots you found in #3 in radical form, i.e., in terms of square roots).



Extra credit (up to 3pts): Recreate the graph of $p(x)$ in Desmos, and then click on x -intercepts, the y -intercept, and also the local maximum and the local minimum (so that Desmos displays the coordinates of these 6 points).

Either (a) print out your graph (click on the “Share” button in the upper-right of the Desmos screen) and hand in your printout with this quiz; or (b) download or screenshot your graph to an image file, and email it to me at sganuli@citytech.cuny.edu (with subject line “MAT1375 Quiz 5 graph”)