Mathematics 1375/D572, Spring 2019 **Exam #3**
Instructor: Suman Ganguli **Tuesday, April 30** **Name:** ______________________

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1. (5 points) Solve the following inequality algebraically, and write the solution set in interval notation:

$$|3x - 2| > 4$$

Solution:

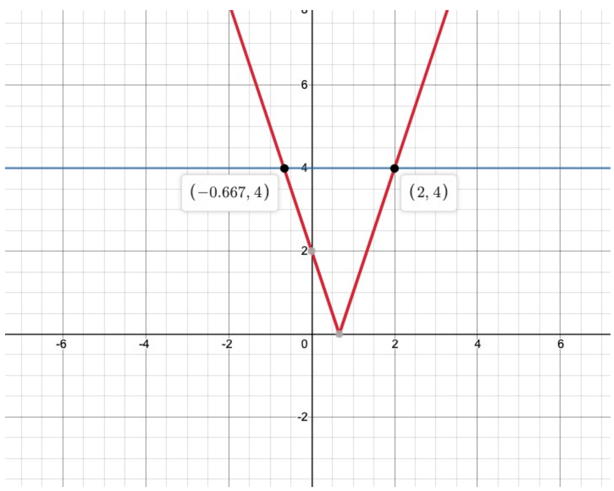
$3x - 2 < -4 \quad \text{or} \quad 3x - 2 > 4$

$3x < -2 \quad \text{or} \quad 3x > 6$

$x < -2/3 \quad \text{or} \quad x > 2$

$(-\infty, -2/3) \cup (2, \infty)$

Extra credit (up to 3 points): Check your solution by sketching the graphs of $y = |3x - 2|$ and $y = 4$ (you can use Desmos as a guide). Label the points where the two graphs intersect with their coordinates, and indicate or explain how this gives the solution set of the inequality.



2. (5 points) Write down **and simplify** the following for $f(x) = 2x^2 + x + 14$:

(a) $f(x+h) =$

Solution: $f(x+h) = 2(x+h)^2 + (x+h) + 14 = 2(x^2 + 2xh + h^2) + x + h + 14 = 2x^2 + 4xh + 2h^2 + x + h + 14$

(b) $f(x+h) - f(x) =$

Solution: $f(x+h) - f(x) = (2x^2 + 4xh + 2h^2 + x + h + 14) - (2x^2 + x + 14) = 4xh + 2h^2 + h$

(c) $\dfrac{f(x+h) - f(x)}{h} =$

Solution: $\dfrac{f(x+h) - f(x)}{h} = \dfrac{4xh + 2h^2 + h}{h} = 4x + 2h + 1$

3. (10 points) Evaluate the following logarithms:

   (a) $\log_3(81) =$

   Solution: $\log_3(81) = 4$ (since $3^4 = 81$)

   (b) $\ln(e^3) =$

   Solution: $\ln(e^3) = 3$

   (c) $\log_4(0.25) =$

   Solution: $\log_4(0.25) = -1$ (since $4^{-1} = \frac{1}{4} = 0.25$)

4. (10 points) Use the properties of logarithmic functions to write the following expressions in terms of $\ln(x)$ and $\ln(y)$:

   (a) $\ln\left(\sqrt{\frac{y}{x}}\right) =$

   Solution: $\ln\left(\sqrt{\frac{y}{x}}\right) = \ln\left(\frac{\sqrt{y}}{\sqrt{x}}\right) = \ln(\sqrt{y}) - \ln(\sqrt{x}) = \frac{1}{2}\ln(y) - \frac{1}{2}\ln(x)$

   or $\ln\left(\left(\frac{y}{x}\right)^{1/2}\right) = \frac{1}{2}(\ln(y) - \ln(x)) = \frac{1}{2}\ln(y) - \frac{1}{2}\ln(x)$

   (b) $\ln\left(x^3\sqrt{y^7}\right) =$

   Solution: $\ln\left(x^3\sqrt{y^7}\right) = \ln(x^3) + \ln(y^{7/2}) = 3\ln(x) + \frac{7}{2}\ln(y)$

5. (5 points) Solve the following exponential equation for x:

   $$3^x = 10$$

   First solve for x algebraically, expressing the value of x in terms of logarithms. Then use a calculator to give a decimal approximation for x (to 2 decimal places).

   (Hint: start by taking the logarithm of both sides of the equation, and then use the properties of logarithms to simplify.)

   Solution: $\ln 3^x = \ln 10 \Longrightarrow x\ln 3 = \ln 10 \Longrightarrow x = \frac{\ln 10}{\ln 3} \approx 2.10$

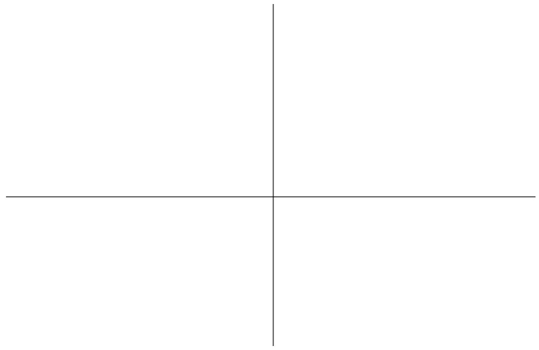
   or $\log 3^x = \log 10 \Longrightarrow x\log 3 = 1 \Longrightarrow x = \frac{1}{\log 3} \approx 2.10$

6. (5 points) Consider the functions $f(x) = b^x$ and $g(x) = \log_b x$ for some fixed constant $b > 1$.

(a) What is the relationship between these two functions (in particular, how is $g(x)$ defined in terms of $f(x)$?

Solution: The logarithmic function $g(x) = \log_b x$ is defined as the inverse of the exponential function $f(x) = b^x$

(b) Sketch rough graphs of $y = f(x)$ and $y = g(x)$ on the coordinate axes below. Label the x- and y-intercepts on each graph with their coordinates.



7. (10 points) Consider the function $f(x) = \log(2x - 3)$.

(Note: For (a)-(c), you can use the graph of $f(x)$ to check your answers but you must **show how you get the solutions algebraically** for full credit.)

(a) What is the domain of $f(x)$?

Solution: The domain consists of all x such that $2x - 3 > 0$. Thus the domain is $x > 1.5$, or $(1.5, \infty)$.

(b) What is the $x-$intercept of $f(x)$?

Solution: To find the x-intercept, solve $f(x) = 0$:

$$\log(2x - 3) = 0 \iff 2x - 3 = 10^0 = 1 \iff x = 2$$

So the only x-intercept is the point $(2, 0)$.

(c) What is the vertical asymptote of $f(x)$?

Solution: The vertical asymptote occurs when $2x - 3 = 0$, i.e., the vertical line $x = 1.5$.

(d) Sketch the graph of $f(x)$. Label the x–intercept and the vertical asymptote.