

Let $f(x) = x^2 - 2x - 3$.

1. (2 points) Find the roots of $f(x)$.

(Hint: either factor $f(x)$ or use the quadratic formula.)

Solution: To find the roots, we solve the equation $f(x) = 0$:

By factoring: $f(x) = x^2 - 2x - 3 = (x - 3)(x + 1)$, so $f(x) = 0$ for $x = 3$ and $x = -1$.

By the quadratic formula:

$$x = \frac{2 \pm \sqrt{4 - 4(-3)}}{2} = \frac{2 \pm \sqrt{16}}{2} = \frac{2 \pm 4}{2}$$

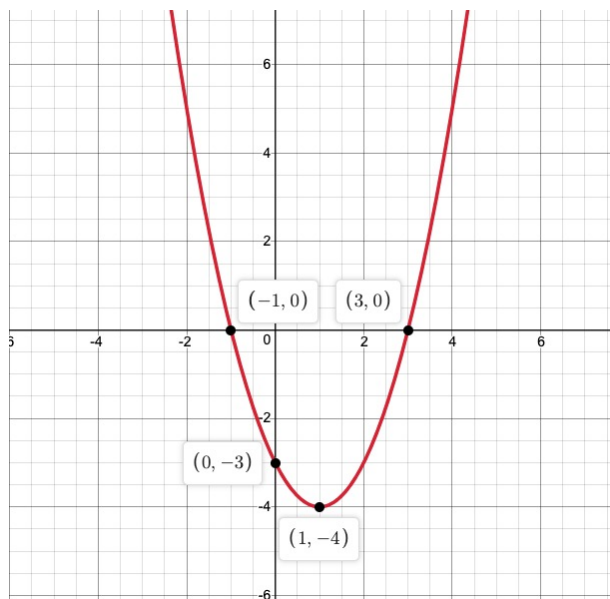
and so again we see that the roots of $f(x)$ are $x = 3$ and $x = -1$.

2. (5 points) Sketch the graph of $f(x)$, labelling the x -intercepts, the y -intercept, and the vertex with their coordinates:

Solution: From #1, we know the x -intercepts occur at $(-1, 0)$ and $(3, 0)$.

Since $f(0) = 0^2 - 2(0) - 3 = -3$, the y -intercept occurs at $(0, -3)$.

From the vertex formula, the x -coordinate of the vertex is at $x = -\frac{b}{2a} = -\frac{-2}{2} = 1$ and so the y -coordinate of the vertex is $f(1) = 1^2 - 2(1) - 3 = 1 - 2 - 3 = -4$. Thus, the vertex of the parabola occurs at $(1, -4)$.



3. (3 points) Use the graph to solve the following inequality; express the solution in interval notation:

$$x^2 - 2x - 3 \geq 0$$

Solution: From the graph (or really just from the fact that we know the graph of $y = x^2 - 2x - 3$ is an parabola opening upwards, with x -intercepts at $x = 3$ and $x = -1$), we see that the solution set of $x^2 - 2x - 3 \geq 0$ is

$$(-\infty, -1] \cup [3, \infty)$$