Verification of Work-Energy Theorem

Lab 16

Learning Objectives

This laboratory activity has two objectives: determination of the coefficient of kinetic friction and verification of the Work-Energy Theorem.

Required Equipment

- Wooden block
- Triple beam balance
- Ruler
- Wooden inclined plane
- Science Workshop Interface box
- Two photogate accessories
- Set of masses

Learning Outcomes

You will be able to:

- *Qualitatively understand the Work-Energy Theorem.*
- *Apply the Work-Energy Theorem to situation involving potential and kinetic energy when an object slides on an inclined plane and there is a force of friction.*
- *Make measurements to determine the amount of work done on an object by a force of friction.*
- *Calculate potential and kinetic energies when an object slides on an inclined plane.*
- *Compare the change in the total mechanical energy of a sliding block to the work done by the frictional force and, as a result, verify the Work-Energy Theorem.*

Theoretical Background

The concept most central to all of science is energy. The concept of energy appears throughout every area of physics and is the cornerstone of the fundamental law of nature, - namely the conservation of energy. The study of various forms of energy and their transformation from one form into another has led to one of the greatest generalizations in physics: in an isolated system energy can be transformed from one form into another, but the total amount of energy of the system remains a constant. In other words, energy cannot be created or destroyed, it may be transformed from one type to another, but the total amount of energy of the isolated system never changes. This principle, or law of conservation of energy not only has its origins, but is also bounded with the fundamental property of the space-time continuity, - the homogeneity of time. Homogeneity of time means that mechanical properties for an isolated system do not change with time.

There are two kinds of mechanical energy. The energy that an object possesses by virtue of its position and interaction with another body is defined as its potential energy. The energy that the object possesses by virtue of its motion is its kinetic energy. The sum of these two kinds of energy is called the total mechanical energy, E. The law of conservation of mechanical energy states that the total mechanical energy of an isolated system is constant. This principle only applies when ***conservative forces*** like gravitational forces or spring forces are present. When external ***nonconservative forces*** are present instead of the law of conservation of mechanical energy, we have ***the Work-Energy Theorem***, which is a powerful tool for studying a wide variety of systems.

Consider an object of mass m that is acted on by a net force F along the motion as it moves along the x-axis. By Newton's second law this force F is related to the rate of change of speed of the object. Thus the work done on the object by this net force as the object moves from initial position x_1 to a final position x_2 is given as

$$W = \int_{x_1}^{x_2} F dx \, . \qquad (16.1)$$

From Newton's second law,

$$F = m \frac{dv}{dt} \, . \qquad (16.2)$$

If we consider the speed as a function of the displacement x measured along x-axis, we can apply the chain rule for derivatives and equation (16.2) becomes

$$F = m\frac{dv}{dt} = m\frac{dv}{dx}\frac{dx}{dt} = mv\frac{dv}{dx}, \qquad (16.3)$$

where we used the fact that $v = dx/dt$.
Substituting equation (16.3) into equation (16.1) yields

$$W = \int_{x_1}^{x_2} mv\frac{dv}{dx}dx = \int_{v_1}^{v_2} mvdv. \qquad (16.4)$$

Note that when we change the variable from x to v we must express the limits on the integral in terms of the new variable v. Because the mass m is a constant in the Newtonian mechanics, we are able to move it outside the integral in equation (16.4). The evaluation of the integral gives

$$W = \int_{v_1}^{v_2} mvdv = m\int_{v_1}^{v_2} vdv = \frac{mv_2^2}{2} - \frac{mv_1^2}{2} \qquad (16.5)$$

or

$$W = \frac{mv_2^2}{2} - \frac{mv_1^2}{2}. \qquad (16.6)$$

The change in the kinetic energy of the object is equal to the work done on that object. This result is known as ***the work-kinetic energy theorem***. Its follows from the definition of the work and from Newton's second law.

In the work-kinetic energy theorem it is convenient to subdivide the work into two categories: W_{con}, the work done by the conservative forces, and W_{nonc}, the work done by the nonconservative forces. The work-kinetic energy theorem then becomes

$$W_{con} + W_{nonc} = \frac{mv_2^2}{2} - \frac{mv_1^2}{2}. \qquad (16.7)$$

By definition, a conservative force is one whose work is independent of the path taken as an object moves from one point to another. Examples of the conservative force are the gravitational force and the force of a stretched or compressed spring. For the conservative force we can define a potential energy. It is well known that the work done by the conservative force between initial position 1 and final position 2 is equal to the difference in potential energy evaluated at positions 1 and 2. Thus, the work of the conservative forces between initial and final positions may be written as

$$W_{con} = U_1 - U_2, \qquad (16.8)$$

where U_1 and U_2 are potential energies of the object at the initial and final position, respectively. Therefore, we may rewrite the work-kinetic energy theorem as

$$W_{nonc} + U_1 - U_2 = \frac{mv_2^2}{2} - \frac{mv_1^2}{2}. \qquad (16.9)$$

From equation (16.9)

$$W_{nonc} = \left(\frac{mv_2^2}{2} + U_2\right) - \left(\frac{mv_1^2}{2} + U_1\right). \qquad (16.10)$$

The sum of the kinetic and potential energies, $\frac{mv^2}{2} + U$ is the total mechanical energy. Thus equation (16.10) shows that the work done by the nonconservative forces is equal to the change in the mechanical energy. This result is known as ***the Work-Energy Theorem***.

An example of a nonconservative force is kinetic friction. In this experimental activity we study the relation between the work done by the nonconservative kinetic frictional force on an object and the change in the mechanical energy of the object, when it moves on an inclined plane. The actual configuration of the experiment is illustrated in Fig. 16.1. A block of mass m

slides down on a plane inclined at an angle of θ with the horizontal. At the height y_1 the block has the velocity v_1 and its total mechanical energy at the position **A** is $E_1 = \frac{mv_1^2}{2} + mgy_1$. At the height y_2 the block's velocity is v_2 and its total mechanical energy at the position **B** is $E_2 = \frac{mv_2^2}{2} + mgy_2$. The work done by the kinetic frictional force between **A** and **B** is equal to the change in the total mechanical energy of the block between these points. That is

$$W_f = \left(\frac{mv_2^2}{2} + mgy_2\right) - \left(\frac{mv_1^2}{2} + mgy_1\right), \tag{16.11}$$

where W_f is the work done by the kinetic friction. If the distance between points **A** and **B** is d that work is

$$W_f = F_k d\ . \tag{16.12}$$

In equation (16.12) F_k is the force of kinetic friction. The kinetic frictional force is

$$F_f = \mu_k N = \mu_k mg\cos\theta\ , \tag{16.13}$$

where $N = mg\cos\theta$ is the normal force and the coefficient of kinetic friction μ_k depends on the nature of the surfaces in contact. Combining equations (16.12) and (16.13), we obtain

$$W_f = \mu_k mgd\cos\theta\ . \tag{16.14}$$

Thus, to determine the work done by kinetic friction we need to know the coefficient of kinetic friction μ_k. One way of determining the coefficient of kinetic friction is to observe the block moving with a uniform speed on the inclined plane. The arrangement is illustrated in Fig. 16.2. The free body diagram in Fig. 16.2 shows all forces acting on the block. The force of gravity mg acts down, a normal force N acts

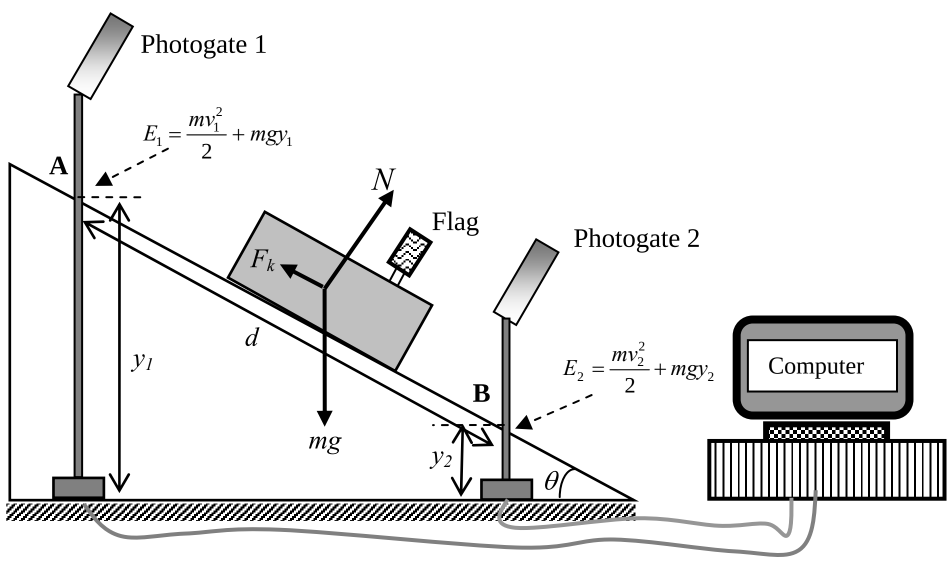

Fig. 16.1. Arrangement of an incline plane and photogates for measuring the kinetic and potential energy of a sliding block.

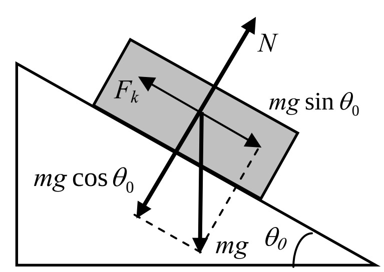

Fig. 16.2. Block moves down incline with uniform speed.

perpendicularly to the plane and the kinetic frictional force F_k always opposes the sliding motion. If the block slides down with constant speed the resultant force of all those forces equals zero. Thus, the component of the force of gravity $mg\cos\theta_0$, which is perpendicular to the plane is equal to the normal force N:

$$N = mg\cos\theta_0\ . \tag{16.15}$$

The kinetic frictional force opposes the component of the force of gravity parallel to the inclined plane that acts to move the block down the plane. As long as the block moves with the uniform speed the kinetic frictional force must be equal to the component of the force of gravity parallel to the plane. Therefore, the kinetic frictional force is equal to:

$$F_k = \mu_k N = mg\sin\theta_0 . \qquad (16.16)$$

In equations (16.15) and (16.16) θ_0 is the angle at which the block slides at a constant speed on the plane. From equations (16.15) and (16.16) the coefficient of kinetic friction can be determine as

$$\mu_k = \frac{mg\sin\theta_0}{N} = \frac{mg\sin\theta_0}{mg\cos\theta_0} = \tan\theta_0 . \qquad (16.17)$$

Thus equation (16.17) can be used to determine coefficient of kinetic friction μ_k by measuring the angle θ_0 at which the block slides down on the inclined plane with a constant speed.

Procedure

1. Measure and record the mass of the block and the width of the flag, Δx.
2. Place the block on the surface of an adjustable inclined plane and incline the surface until the block moves down the plane at constant speed after being given a slight tap. Measure and record the angle θ_0 at which the block slides down on the inclined plane with a constant speed.
3. Repeat step 2 placing masses 100, 200 and 400 grams successively on top of the wooden block. Record the angle θ_0 in each case.
4. Set the angle θ of the adjustable inclined plane greater than θ_0. Record the angle θ. Set up two photogates as shown in Fig. 16.1.
5. Measure the vertical distances y_1 and y_2 between the base of the inclined plane and the points of the inclined plane, where the photogate accessories are located. Record y_1 and y_2. Measure the distance *d* between the photogates along the inclined plane. Record *d.*
6. Connect the Photogate accessories stereo phone plugs into Digital Channel 1 and Digital Channel 2 on the Science Workshop Interface.
7. Set up the sensors in the software.
   - Open *DataStudio* Window. Click “Create Experiment”.
   - Find the “Photogate” in the Sensors list in the Experiment Setup window. Double-click the “Photogate”, and as a result the “Photogate” icon appears below Digital Channel 1 of the Interface box 750. Double-click the “Photogate” again, and as a result the “Photogate” icon appears below Digital Channel 2 of the interface.
   - In the Experimental Setup Window click “Timers”, and as a result the “Timer Setup” window appears. According to equation (16.11), to measure the kinetic energy of a sliding block you need to know the velocities v_1 and v_2, when the block passes Photogate 1 and Photogate 2, respectively. To measure the velocities v_1 and v_2, we will use the same techniques as in Experiment 5 and determine the velocities as

     $$v_1 = \frac{\Delta x}{\Delta t_1}, \qquad \text{and} \qquad v_2 = \frac{\Delta x}{\Delta t_2} . \qquad (16.18)$$

     To measure the initial velocity v_1 of the block you need to measure the time Δt_1 the block’s flag requires to pass through the Photogate 1, when the Photogate 1 is first blocked by the flag and when it is unblocked as the back end of the flag moves away from the Photogate 1. In the timing sequences choices click **Ch. 1** and select “Blocked”. Click once again and select “Unblocked”.
   - Now in the “Timer Setup” window click **+New** and Timer 2 appears. To measure the final velocity v_2 of the block you need to measure the time Δt_2 the block’s flag requires to pass through the Photogate 2, when the Photogate 2 is first blocked by the flag and when it is unblocked as the back end of the flag moves away from Photogate 2. In the timing sequences choices click **Ch. 2** and select “Blocked”. Click once again and select “Unblocked”.
   - To record your measurements of time, double-click the Table icon in the Display window, and as a result the “Choose a Data Source” window appears. Choose “Timer 1” and click OK. (You can also click-and-drag the Table icon from the Display window to the Data window for Timer 1). Repeat this procedure for Timer 2. For your measurements you only need “Elapsed time”. Click on the “Clock” icon in each table menu, and as a result you will record just “Elapsed time”. Resize the tables to fit your screen.
8. Put the block at the starting point on the inclined plane. The block must be released from the same point for each trip. Click the “**Start**” button and release the block from the starting point. Timing begins when the beams of the photogates

are blocked and ends when the beams are unblocked. The time intervals Δt_1 and Δt_2 that the flag of the width Δx takes to pass between the photogates will be immediately displayed. Catch the block, put it at the same starting point and release again. Repeat this step a few times.

9. Click the "Σ" button and take the mean value of Δt_1 and Δt_2. Record these mean values of Δt_1 and Δt_2 in Data Table. 16.2.
10. Repeat steps 8 and 9 placing masses 100 and 200 grams successively on top of the wooden block.
11. Increase the angle θ of the inclined plane and repeat steps 8 through 10.

Computations and Data Analysis

1. Compute the mean value of the angle θ_0 at which the block slides down the inclined plane with constant speed. Use this mean value of θ_0 and calculate the coefficient of kinetic friction as $\mu_k = \tan\theta_0$.
2. Use equation (16.14) and compute the work done by the kinetic friction for each trial.
3. Use equations (16.18) and calculate velocities $v_1 = \frac{\Delta x}{\Delta t_1}$ and $v_2 = \frac{\Delta x}{\Delta t_2}$ for each trial based on the parameter Δx, the width of the flag you measured and time intervals Δt_1 and Δt_2 the flag took to pass through each of the photogates. Record the values of v_1 and v_2 in Data Table 16.2.
4. Compute the mechanical energy of the block at the positions **A** ($E_1 = \frac{mv_1^2}{2} + mgy_1$) and **B** ($E_2 = \frac{mv_2^2}{2} + mgy_2$) for each trial.
5. Compute the change in the mechanical energy, $\Delta E = E_2 - E_1$ for each trial.
6. Compare the change in the mechanical energy, $\Delta E = E_2 - E_1$ with work done by the frictional force for each trial by calculating the percent difference. Make conclusions about the work-energy theorem.

Questions

1. When the block moves on an inclined plane, what kind of energy transformations take place?
2. Distinguish between the static and kinetic friction. Explain why it is necessary for the block to move at constant velocity to determine the coefficient of kinetic friction.
3. Step 8 of the procedure states that the block must be released from the same point for each trip. Why?
4. Does your data confirm the expected relationship between the work done by the kinetic friction and the change in the mechanical energy of the block? State clearly what is expected and what your data indicates.
5. A 2 kg block slides down on an inclined plane and reaches the bottom with speed 4 m/s. How much work does the force of friction do if the block starts from rest at a height of 1.5 m.
6. A ball of mass 0.2 kg is thrown straight upward with an initial speed of 15 m/s. When the ball returns to the same level, its speed is 13 m/s. How much work does air resistance do on the ball during its flight?
7. A car is moving horizontally with the speed of 20 m/s when it breaks to a stop. Where does all the kinetic energy of the moving car go?

Student’s name ______________________________*Section*__________ *Date* __________

Mass of the block _______ kg.   Width of the flag ______ m

Data Table 16.1. **Determination of a coefficient of kinetic friction**

Mass, *kg*	Angle θ_0, *degree*	Mean value of angle θ_0, *degree*	Coefficient of kinetic friction $\mu_k = \tan\theta_0$

Data Table 16.2. **Data and Results of Calculations**

Angle θ	Mass, *kg*	Time, *s*		Velocity, *m/s*		Distance, *m*			Energy, *J*		Change in energy $\Delta E = E_2 - E_1$, *J*	Work W_f, *J*	% difference
		Δt_1	Δt_2	v_1	v_2	y_1	y_2	d	$E_1 = \frac{mv_1^2}{2} + mgy_1$	$E_2 = \frac{mv_2^2}{2} + mgy_2$			