Exam #1 Sheet

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow$

$p)$ $p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$

$p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$

$\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$

TABLE 6 **Logical Equivalences.**

Equivalence	*Name*
$p \wedge \mathbf{T} \equiv$ p $p \vee \mathbf{F}$ $\equiv p$	Identity laws
$p \vee \mathbf{T} \equiv \mathbf{T}$ $p \wedge \mathbf{F} \equiv \mathbf{F}$	Domination laws
$p \vee p \equiv$ p $p \wedge p$ $\equiv p$	Idempotent laws
$\neg(\neg p) \equiv p$	Double negation law
$p \vee q \equiv q \vee$ p $p \wedge q \equiv q$ $\wedge p$	Commutative laws
$(p \vee q) \vee r \equiv p \vee (q \vee$ $r)$ $(p \wedge q) \wedge r \equiv p \wedge (q$ $\wedge r)$	Associative laws
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee$ $r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge$ $r)$	Distributive laws
$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$	De Morgan's laws
$p \vee (p \wedge q) \equiv$ p $p \wedge (p \vee q)$ $\equiv p$	Absorption laws
$p \vee \neg p \equiv \mathbf{T}$ $p \wedge \neg p \equiv \mathbf{F}$	Negation laws

$p \rightarrow q \equiv \neg p \vee q$

$p \rightarrow q \equiv \neg q \rightarrow \neg p$

$p \vee q \equiv \neg p \rightarrow q$

$p \wedge q \equiv \neg(p \rightarrow \neg q)$

$\neg(p \rightarrow q) \equiv p \wedge \neg q$

$(p \rightarrow q) \wedge (p \rightarrow r) \equiv p \rightarrow (q \wedge r)$

$(p \rightarrow r) \wedge (q \rightarrow r) \equiv (p \vee q) \rightarrow r$

$(p \rightarrow q) \vee (p \rightarrow r) \equiv p \rightarrow (q \vee r)$

Modus Ponens:

$$\begin{array}{l} p \to q \\ p \\ \hline \therefore q \end{array}$$

Corresponding Tautology: $(p \wedge (p \to q)) \to q$

Modus Tollens:

$$\begin{array}{l} p \to q \\ \neg q \\ \hline \therefore \neg p \end{array}$$

Corresponding Tautology: $(\neg p \wedge (p \to q)) \to \neg q$

Hypothetical Syllogism:

$$\begin{array}{l} p \to q \\ q \to r \\ \hline \therefore p \to r \end{array}$$

Corresponding Tautology: $((p \to q) \wedge (q \to r)) \to (p \to r)$

Disjunctive Syllogism:

$$\begin{array}{l} p \vee q \\ \neg p \\ \hline \therefore q \end{array}$$

Corresponding Tautology: $(\neg p \wedge (p \vee q)) \to q$

Addition:

$$\begin{array}{l} p \\ \hline \therefore p \vee q \end{array}$$

Corresponding Tautology: $p \to (p \vee q)$

Simplification:

$$\begin{array}{l} p \wedge q \\ \hline \therefore q \end{array}$$

Corresponding Tautology: $(p \wedge q) \to p$

Conjunction:

$$\begin{array}{l} p \\ q \\ \hline \therefore p \wedge q \end{array}$$

Corresponding Tautology: $((p) \wedge (q)) \to (p \wedge q)$

Resolution:

$$\begin{array}{l} \neg p \vee r \\ p \vee q \\ \hline \therefore q \vee r \end{array}$$

Corresponding Tautology: $((\neg p \vee r) \wedge (p \vee q)) \to (q \vee r)$

Universal Instantiation (UI):

$$\begin{array}{l} \forall x P(x) \\ \hline \therefore P(c) \end{array}$$

Universal Generalization (UG):

$$\begin{array}{l} P(c) \text{ for an arbitrary } c \\ \hline \therefore \forall x P(x) \end{array}$$

Existential Instantiation (EI):

$$\begin{array}{l} \exists x P(x) \\ \hline \therefore P(c) \text{ for some element } c \end{array}$$

Existential Generalization (EG):

$$\begin{array}{l} P(c) \text{ for some element } c \\ \hline \therefore \exists x P(x) \end{array}$$