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Revised Spring 2015

MAT 1375 Final Exam Review Sheet

#1 Solve the inequality and express the solution as an interval.

a) $x^2 - 2x - 3 \geq 0$ b) $6x - 2x^2 > 0$ c) $x^2 + x > 0$ d) $\frac{x+3}{x-4} \leq 0$

#2 Solve the inequalities algebraically and write the solution as an interval.

a) $|3x-2| < 4$ b) $|5-2x| \geq 9$

#3 Given each function: Find the x- and y-intercepts, the domain, the vertical and horizontal asymptotes, and then sketch a complete graph of the function.

a) $f(x) = \frac{1-x}{x+2}$ b) $f(x) = \frac{x+2}{3-x}$ c) $f(x) = \frac{2x-4}{1-x}$

#4 Find the difference quotient $\frac{f(x+h)-f(x)}{h}$ (assume $h \neq 0$) for:

a) $f(x) = x^2 + 5x + 2$ b) $f(x) = 2x^2 - 3x$ c) $f(x) = -2x^2 + 4x + 1$

#5 For each polynomial, find all roots of the function algebraically in simplest radical form. Then sketch the complete graph, indicating the viewing window, the x-intercepts and the relative maximum and minimum.

a) $f(x) = x^3 + 2x^2 - 3x - 6$ b) $f(x) = 2x^3 - 7x^2 - 3x + 8$

#6 Find the magnitude and the direction angle for:

a) $v = \langle 2, -2\sqrt{3} \rangle$ b) $v = \langle -3, -3 \rangle$ c) $v = \langle -5\sqrt{3}, 5 \rangle$

#7 Find the quotient or product and write the result in standard complex form:

a) $\frac{6(\cos 225^\circ + i \sin 225^\circ)}{3(\cos 135^\circ + i \sin 135^\circ)}$ b) $2(\cos 120^\circ + i \sin 120^\circ) \cdot 4(\cos 90^\circ + i \sin 90^\circ)$

#8 Let $u = \ln(x)$ and $v = \ln(y)$, where $x, y > 0$. Write the following expression in terms of u and v.

a) $\ln\left(x^8 \cdot \sqrt[3]{y^4}\right)$ b) $\ln\left(\frac{\sqrt{x^5}}{y^3}\right)$ c) $\ln\left(\sqrt{x^3 \cdot \sqrt[4]{y}}\right)$

#9 Find the domain, asymptotes, and x-intercepts of the function, and then sketch its graph.

a) $f(x) = \log(x-4)$ b) $f(x) = \log(5-x)$ c) $f(x) = -\ln(x+2)$

#10 State the amplitude, period and phase shift, and then sketch one complete cycle of the graph. Label all maximum, minimum and x-intercepts.

a) $y = 3\cos(2x + \pi)$ b) $y = 2\sin(4x - \pi)$ c) $y = -4\sin\left(x - \frac{\pi}{2}\right)$

#11 Find all exact solutions in radians.

a) $2\sin^2 x = \sin x$ b) $2\cos^2 x - \cos x - 1 = 0$ c) $\tan^2 x - \tan x = 0$

#12 In 1998, the population of a colony is 10,000, and is decreasing exponentially at 1.5% per year.
a) What will the population be after 5 years? b) In what year will there be half of the population left?

#13 In 2003, the population of a city is 80,000 people, and is growing at a rate of 4% per year.
a) What will the population be in 2015? b) In what year will the population be triple?

#14 Find the inverse for the following function.

a) $y = 3 - 4x$ b) $y = \frac{4}{x-3}$ c) $y = \frac{2}{8x+5}$ d) $y = \frac{x-1}{x+2}$

#15
a) Find the sum of the first 70 terms of the arithmetic sequence: 22, 19, 16, 13, …
b) Find the sum of the first 95 terms of the arithmetic sequence: -17, -12, -7, -2, …
c) Find the sum of the first 777 terms of the arithmetic sequence: 3, 9, 15, 21, …

#16 Find the exact sum of the infinite geometric sequence.

a) $\frac{1}{2}, -\frac{1}{4}, \frac{1}{8}, \ldots$ b) 32, -16, 8, -4, … c) $3, 2, \frac{4}{3}, \frac{8}{9}, \ldots$ d) -54, -18, -6, -2, …

#17 Use the Binomial Theorem to write the following terms in simplest form.

a) Find the first four terms of the expansion of $\left(2x - \frac{y}{2}\right)^8$.

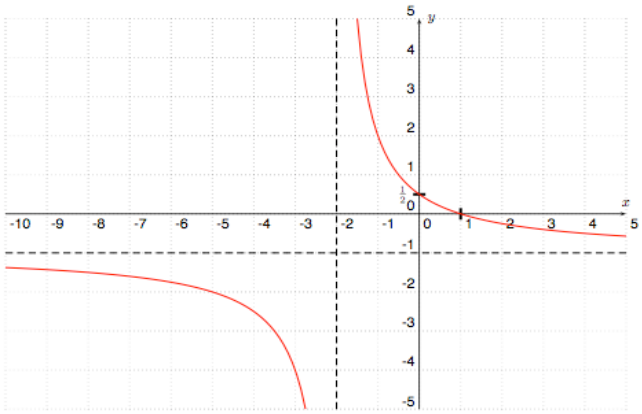
b) Find the 7^{th} term in the expansion of $\left(2x - \frac{y}{2}\right)^8$.

Answers:

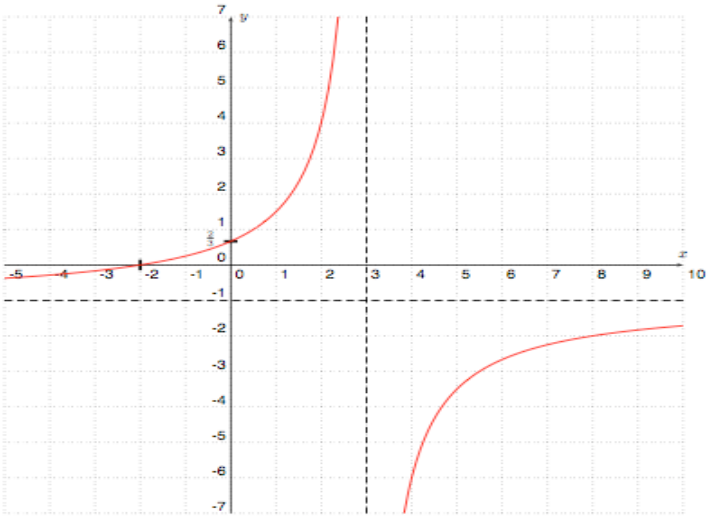
#1 a) $(-\infty,-1]\cup[3,\infty)$ b) $(0,3)$ c) $(-\infty,-1)\cup(0,\infty)$ d) $[-3,4)$

#2 a) $\left(-\frac{2}{3},2\right)$ b) $(-\infty,-2]\cup[7,\infty)$

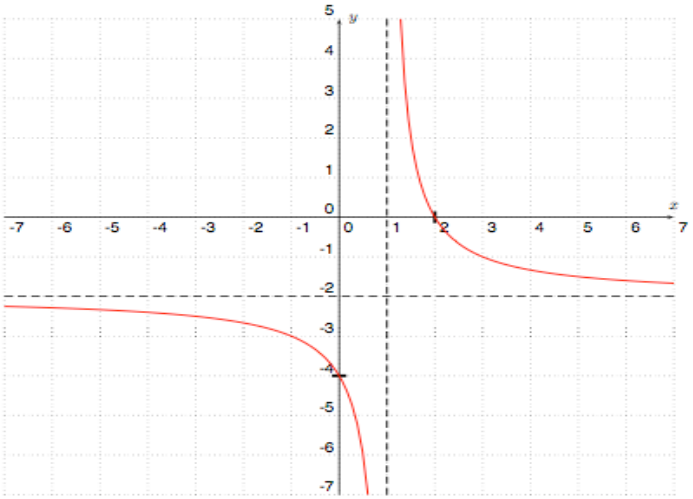
#3 a) domain: $D=(-\infty,-2)\cup(-2,\infty)$
vertical asymptote: $x=-2$
horizontal asymptote: $y=-1$
x-int: $(1,0)$, y-int: $\left(0,\frac{1}{2}\right)$



b) domain: $D=(-\infty,3)\cup(3,\infty)$
vertical asymptote: $x=3$
horizontal asymptote: $y=-1$
x-int: $(-2,0)$, y-int: $\left(0,\frac{2}{3}\right)$



c) domain: $D=(-\infty,1)\cup(1,\infty)$
vertical asymptote: $x=1$
horizontal asymptote: $y=-2$
x-int: $(2,0)$, y-int: $(0,-4)$



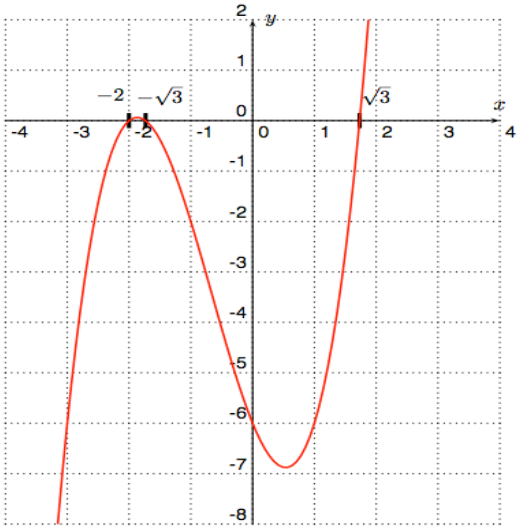
#4 a) $2x+h+5$ b) $4x+2h-3$ c) $-4x-2h+4$

#5 a) roots: $x=-2,\ x=\sqrt{3},\ x=-\sqrt{3}$

window: X-min: -4, X-max: 4,
Y-min: -8, Y-max: 2

max: $(x,y)\approx(-1.87,0.06)$

min: $(x,y)\approx(0.54,-6.88)$

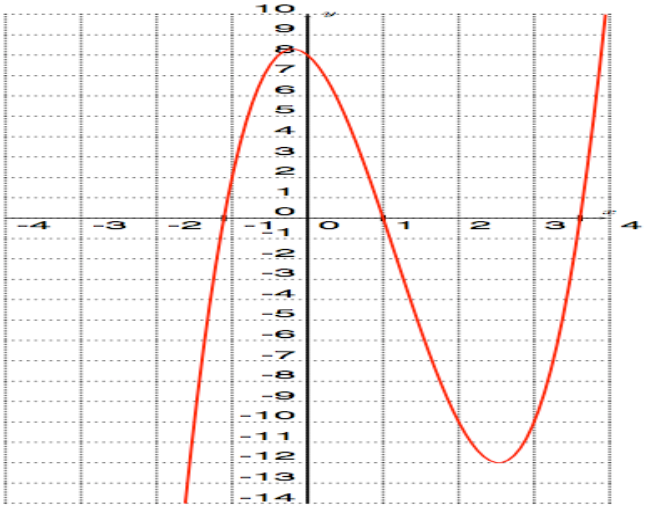


b) roots: $x=1,\ x=\dfrac{5+\sqrt{89}}{4},\ x=\dfrac{5-\sqrt{89}}{4}$

window: X-min: -4, X-max: 4,
Y-min: -14, Y-max: 10

max: $(x,y)\approx(-0.20,8.30)$

min: $(x,y)\approx(2.53,-12.01)$



#6 a) $\|v\|=4$, Direction Angle: $\theta=300^\circ$

b) $\|v\|=3\sqrt{2}$, Direction Angle: $\theta=225^\circ$

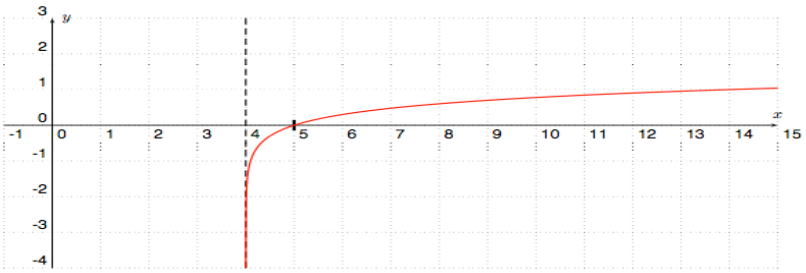
c) $\|v\|=10$, Direction Angle: $\theta=150^\circ$

#7 a) $2(\cos 90^\circ + i\sin 90^\circ)=2i$

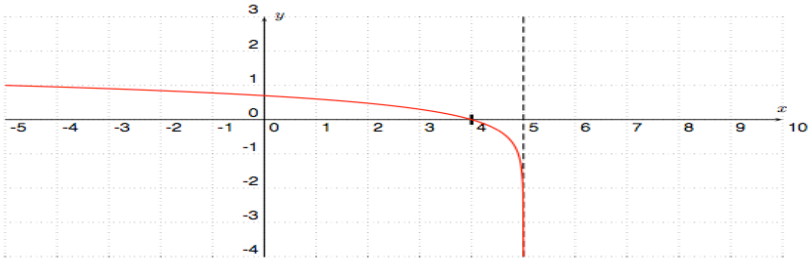
b) $8(\cos 210^\circ + i\sin 210^\circ)=-4\sqrt{3}-4i$

#8 a) $8u+\dfrac{4}{3}v$ b) $\dfrac{5}{2}u-3v$ c) $\dfrac{3}{2}u+\dfrac{1}{8}v$

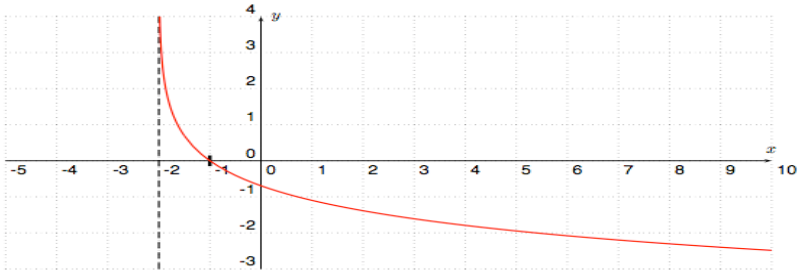
#9 a) domain: $D=(4,\infty)$
vertical asymptote: $x=4$
x-int: $(5,0)$



b) domain: $D=(-\infty,5)$
vertical asymptote: $x=5$
x-int: $(4,0)$



c) domain: $D=(-2,\infty)$
vertical asymptote: $x=-2$
x-int: $(-1,0)$

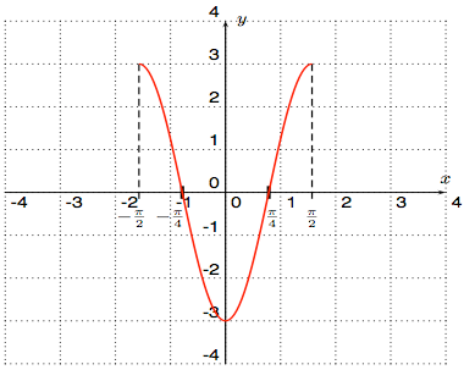


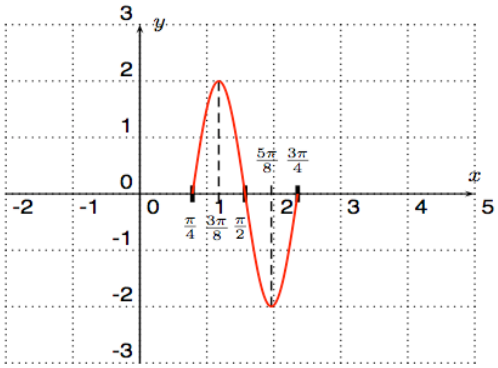
#10 a) amplitude: 3, period: π, phase shift: $-\frac{\pi}{2}$, max: $(\pm\frac{\pi}{2},3)$, min: $(0,-3)$, x-int: $(\pm\frac{\pi}{4},0)$

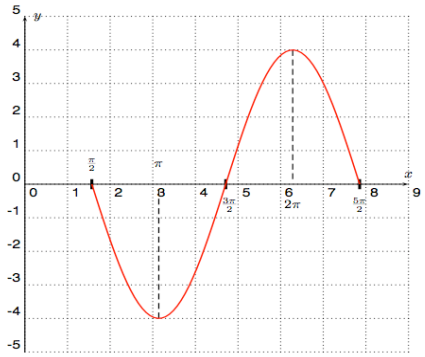
b) amplitude: 2, period: $\frac{\pi}{2}$, phase shift: $\frac{\pi}{4}$, max: $(\frac{3\pi}{8},2)$, min: $(\frac{5\pi}{8},-2)$, x-int: $(\frac{\pi}{4},0)$, $(\frac{\pi}{2},0)$, $(\frac{3\pi}{4},0)$

c) amplitude: 4, period: 2π, phase shift: $\frac{\pi}{2}$, max: $(2\pi,4)$, min: $(\pi,-4)$, x-int: $(\frac{\pi}{2},0)$, $(\frac{3\pi}{2},0)$, $(\frac{5\pi}{2},0)$

Graphs: a) b) c)







#11 a) $x = n\pi$ or $x = (-1)^n\frac{\pi}{6} + n\pi$, where $n = 0,\pm1,\pm2,\ldots$

b) $x = \pm\frac{2\pi}{3} + 2n\pi$ or $x = 2n\pi$, where $n = 0,\pm1,\pm2,\ldots$

c) $x = n\pi$ or $x = \frac{\pi}{4} + n\pi$, where $n = 0,\pm1,\pm2,\ldots$

#12 a) The population will be approximately 9,272 people.

b) Sometime in the year 2043 half of the population will be left.

#13 a) The population will be approximately 128,083 people. b) It will be triple in the year 2031.

#14 a) $f^{-1}(x) = \frac{3-x}{4}$ b) $f^{-1}(x) = \frac{4}{x} + 3$ c) $f^{-1}(x) = \frac{1}{4x} - \frac{5}{8}$ d) $f^{-1}(x) = \frac{2x+1}{1-x}$

#15 a) -5705 b) 20710 c) 1811187

#16 a) $\frac{1}{3}$ b) $\frac{64}{3}$ c) 9 d) -81

#17 a) $256x^8 - 512x^7y + 448x^6y^2 - 224x^5y^3$ b) $\frac{7}{4}x^2y^6$