**MAT2440/D641** **Exam #1**
**Instructor: Suman Ganguli** **Wednesday, October 3** **Name:** ____________________

| Question: | 1 | 2 | 3 | 4 | Total |
|---|---|---|---|---|---|
| Points: | 10 | 15 | 10 | 15 | 50 |
| Score: | | | | | |

1. Recall that a compound proposition is *satisfiable* if there is an assignment of truth values to its variables that makes the proposition true.

   Determine whether the following compound propositions are satisfiable by constructing their truth tables. Circle **is** or **is not** after constructing the truth table. If the proposition **is** satisfiable, circle the line(s) of the truth table that show the proposition is satisfiable.

   a. (5 points) **Solution:**

   $(p \longleftrightarrow q) \wedge \neg p$ **is** satisfiable, as demonstrated by the last line of the truth table:

| $p$ | $q$ | $p \longleftrightarrow q$ | $\neg p$ | $(p \longleftrightarrow q) \wedge \neg p$ |
|---|---|---|---|---|
| T | T | T | F | F |
| T | F | F | F | F |
| F | T | F | T | F |
| F | F | T | T | T |

   b. (5 points) **Solution:**

   $p \wedge (p \longrightarrow q) \wedge \neg q$ **is not** satisfiable:

| $p$ | $q$ | $p \longrightarrow q$ | $p \wedge (p \longrightarrow q)$ | $\neg q$ | $p \wedge (p \longrightarrow q) \wedge \neg q$ |
|---|---|---|---|---|---|
| T | T | T | T | F | F |
| T | F | F | F | T | F |
| F | T | T | F | F | F |
| F | F | T | F | T | F |

2. Consider the conditional statement $[(p \vee q) \wedge \neg p] \longrightarrow q$.

a. (6 points) Construct the truth table for $[(p \vee q) \wedge \neg p] \longrightarrow q$:

**Solution:**

| $p$ | $q$ | $p \vee q$ | $\neg p$ | $(p \vee q) \wedge \neg p$ | $[(p \vee q) \wedge \neg p] \longrightarrow q$ |
|---|---|---|---|---|---|
| T | T | T | F | F | T |
| T | F | T | F | F | T |
| F | T | T | T | T | T |
| F | F | F | T | F | T |

b. (3 points) What feature of its truth table establishes that $[(p \vee q) \wedge \neg p] \longrightarrow q$ is a tautology? (In other words, what is the definition of a tautology?)

**Solution:** A tautology is any compound propositions that is true for all combinations of truth values of the propositional variables that occur in it, i.e., its column in the truth table is all $T$ (as seen in the rightmost column above).

c. (3 points) Recall that each rule of inference we discussed is based on a certain tautology. The rule of inference associated with $[(p \vee q) \wedge \neg p] \longrightarrow q$ is called disjunctive syllogism:

$$\begin{array}{l} p \vee q \\ \neg p \\ \hline \therefore q \end{array}$$

Briefly describe (in one or two complete sentences) the relationship between the tautology $[(p \vee q) \wedge \neg p] \longrightarrow q$ and this rule of inference. Here are some terms you should use in your answer:

- the propositions above the horizontal line in a rule of inference, in this case $p \vee q$ and $\neg p$, are called the *hypotheses* of the rule of inference
- the proposition below the horizontal line, $q$, is called the *conclusion* of the rule of inference
- in the conditional statement $[(p \vee q) \wedge \neg p] \longrightarrow q$, the part before the arrow, $(p \vee q) \wedge \neg p$, is called the *antecedent*, and $q$ is called the *consequence*. )

**Solution:** The antecedent of the tautology is a conjunction of the premises of the rule of inference, and its consequence is the conclusion of the rule of inference.

d. (3 points) Consider the following premises: "I will choose soup or salad" and "I will not choose soup." What are the atomic propositions $p$ and $q$ so that we can apply the rule of inference above to these premises (and thus reach the conclusion $q$ from these premises)?

**Solution:**

$p$ : "I will choose soup"

$q$ : "I will choose salad"

3. Shown below is the truth table for the "exclusive-or" logical connective $\oplus$:

| $p$ | $q$ | $p \oplus q$ |
|---|---|---|
| T | T | F |
| T | F | T |
| F | T | T |
| F | F | F |

a. (3 points) What is the difference between the "exclusive-or" logical connective $\oplus$ and the the "inclusive-or" logical connective $\vee$? Give a brief (one or two sentence) explanation in terms of their truth tables.

**Solution:** With exclusive-or logical connective $\oplus$, a proposition $p \oplus q$ is false in the case both $p$ and $q$ are true, whereas with inclusive-or logical connective $\vee$, $p \vee q$ is true in that case; otherwise their truth tables are identical.

b. (3 points) Many restaurant menus contain a statement such as "Every entree comes with a choice of soup or salad." Do you think the correct interpretation of the "or" in such a statement is exclusive-or or inclusive-or? Justify your answer.

**Solution:** Typically such a statement should be interpreted as an instance of "exclusive-or" since the restaurant only allows you to choose either soup or salad, *but not both*.

c. (4 points) Show that $p \oplus q \equiv (p \vee q) \wedge \neg(p \wedge q)$. You can demonstrate the logical equivalence using truth tables or with a verbal argument.

**Solution:** As we noted in part (a), the difference between $p \oplus q$ and $p \vee q$ is the case where both $p$ and $q$ are true: $p \oplus q$ is false in that case, whereas $p \vee q$ is true. Thus, $p \oplus q$ is logically equivalent to $(p \vee q)$ plus the condition that not both $p$ and $q$ are truen, i.e., $(p \vee q) \wedge \neg(p \wedge q)$.

We can verify this logical equivalence with the truth table for $(p \vee q) \wedge \neg(p \wedge q)$:

| $p$ | $q$ | $p \vee q$ | $p \wedge q$ | $\neg(p \wedge q)$ | $(p \vee q) \wedge \neg(p \wedge q)$ |
|---|---|---|---|---|---|
| T | T | T | T | F | F |
| T | F | T | F | T | T |
| F | T | T | F | T | T |
| F | F | F | F | T | F |

Note that the rightmost column is identical to the truth table column for $p \oplus q$.

4. Consider the two-place predicate

$$L(x, y) = \text{“}x \text{ lives in borough } y\text{”}$$

Let the domain for $x$ consist of all CityTech students, and the domain for $y$ consist of the five boroughs of NYC, i.e., the set {Manhattan, Brooklyn, Queens, Bronx, Staten Island}.

a. (9 points) Translate the following statements of predicate logic into English sentences. For full credit, write a "natural" English translation, i.e., a sentence one would say in normal conversation. (A "literal" translation will receive partial credit.)

a. $\neg\exists x L(x, \text{Staten Island})$

**Solution:** A literal translation: "It is not the case that there is a CityTech student who lives on Staten Island." More natural translations: "There are no CityTech students who live on Staten Island" or "No CityTech students live on Staten Island."

b. $\forall x \exists y L(x, y)$

**Solution:** A literal translation: "For every CityTech student, there is some borough such that that student lives in that borough." A more natural translation: "Every CityTech student lives in some borough of NYC."

c. $\forall y \exists x L(x, y)$

**Solution:** A literal translation: "For every borough, there is some CityTech student that lives in that borough." A more natural translation: "There are CityTech students living in each borough of NYC."

d. (6 points) Express each of the following sentence as a statement of predicate logic using quantifiers, logical connectives and the predicate $L(x, y)$:

a. "All CityTech students live in Brooklyn, Queens, or Manhattan."

**Solution:** $\forall x(L(x, \text{Brooklyn}) \vee L(x, \text{Queens}) \vee L(x, \text{Manhattan}))$

b. "There are CityTech students who don't live in New York City."

**Solution:** $\exists x \forall y(\neg L(x, y))$ or equivalently

$$\exists x(\neg L(x, \text{Manhattan}) \wedge \neg L(x, \text{Brooklyn}) \wedge \neg L(x, \text{Queens}) \wedge \neg L(x, \text{Bronx}) \wedge \neg L(x, \text{Staten Island}))$$