MAT2440/D641 **Quiz #3**
Instructor: Suman Ganguli **Wednesday, October 24** **Name:** ____________

Question:	1	2	Total
Points:	5	5	10
Score:			

Recall the formal definitions of even and odd integers:

Definition: An integer n is *even* if $n = 2k$ for some integer k. An integer n is *odd* if $n = 2k + 1$ for some integer k.

1. (5 points) Prove that the product of two odd integers is also odd.
   (Hint: use a direct proof to prove "If m and n are both odd integers, then $m * n$ is also an odd integer.")

   Solution:
   (For a direct proof, we assume the antecedent–in that case that m and n are odd integers–and proceed to prove the consequence–that then $m * n$ is also odd.)

   Take any two odd integers m and n. By definition of being odd, $m = 2k + 1$ and $n = 2j + 1$ for integers j, k. (Note that since m and n may be different odd integers, we must assume two different representations of those odd integers, $2k + 1$ and $2j + 1$, respectively.)

   Then $m * n = (2k+1)(2j+1) = 4jk + 2k + 2j + 1 = 2(2jk + k + j) + 1$, where $2jk + k + j$ is an integer. Thus, $m * n$ fulfills the definition of being odd.

2. (5 points) Complete the proof of the statement: "If m and n are integers such that $m * n$ is even, then either m is even or n is even."

   Proof: We will provide a proof by contraposition. The statement is a conditional of the form $p \longrightarrow q$, where the propositions involved are

   $p =$ "$m * n$ is even"

   $q =$ "m is even or n is even"

   In order to provide a proof by contraposition, we need to prove the contrapositive $\neg q \to \neg p$. Write out the proof of the contrapositive below.

   (Hint: Use DeMorgan's Law to formulate $\neg q$, and then complete the proof below by reasoning from $\neg q$ to show $\neg p$, using the fact given in #1 above.)

   Solution: Applying DeMorgan's Law to formulate $\neg q$, we have:

   $$\neg q \equiv \neg(m \text{ is even or } n \text{ is even}) \equiv \neg(m \text{ is even}) \text{ and } \neg(n \text{ is even}) \equiv (m \text{ is odd}) \text{ and } (n \text{ is odd})$$

   Thus, for a proof by contraposition, we initially assume that it is not the case that either m is even or n is even, which we have shown is equivalent to assuming m is odd *and* n is odd.

   But by #1 above, if we have two such odd integers m and n, then $m * n$ is odd. This establishes the theorem (since "$m * n$ is odd" is equivalent to $\neg p$).