

Question:	1	2	3	Total
Points:	4	2	4	10
Score:				

1. (4 points) Recall DeMorgan's Law for conjunction:

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

Show this logical equivalence is true by two different methods:

- a. Verbally explain (in 3-4 sentences) why the LHS is true if and only if the RHS is true:

Solution: Since the compound proposition on the LHS is a negation, it is true if and only if the proposition inside the parentheses, $p \wedge q$, is false. But such conjunction is false if and only if either of the conjuncts are false, i.e., if and only if $\neg p$ or $\neg q$ is true, i.e., $\neg p \vee \neg q$ is true.

- b. Using truth tables: show that $\neg(p \wedge q)$ and $\neg p \vee \neg q$ have identical truth tables.

Solution:

p	q	$p \wedge q$	$\neg(p \wedge q)$	$\neg p$	$\neg q$	$\neg p \vee \neg q$
T	T	T	F	F	F	F
T	F	F	T	F	T	T
F	T	F	T	T	F	T
F	F	F	T	T	T	T

2. (2 points) Write down the RHS of DeMorgan's Law for disjunction:

Solution: $\neg(p \vee q) \equiv \neg p \wedge \neg q$

3. (4 points) Consider the predicates $H(x) = "x \text{ did the homework.}"$ and $P(x) = "x \text{ passed the course}"$; let the domain consist of all students in our class.

- a. Translate the following statement of predicate logic into an English sentence:

$$\exists x(\neg H(x) \wedge P(x))$$

Solution: "There is a student in our class who did not do the homework but who did pass the course."

- b. Express the following sentence as a statement of predicate logic using quantifiers, logical connectives and the predicates $H(x)$ and $P(x)$:

"Every student in the class who passed the course also did the homework."

Solution: $\forall x(P(x) \rightarrow H(x))$