MAT 1275 Final Exam Scoring Guide

1. a) Solve for x and simplify the answer [5 pts]: $-x^2 + 6x + 2 = 0$

If using the completing the square method:

[1 pt] for completing the square,
[1 pt] for factoring the perfect square,
[1 pt] for applying the square root,
[1 pt] for remembering +/-,
[1 pt] for final answer.

OR If using the quadratic formula:

[1 pt] for stating the quadratic formula,
[1 pt] for substituting into the q.f.,
[1 pt] for initial simplification using arithmetic,
[1 pt] for simplication of the radical,
[1 pt] for simplication of the fraction

(Students are not *required* to explicitly state the q.f. Students may skip to directly substituting values for a, b and c without penalty).

b) Find the vertex of the quadratic function: $y = -x^2 + 6x + 2$. Graph the function and label the vertex and the x- and y-intercepts with numbers or coordinates, using the provided graph paper [5 pts].

[2 pts] for finding the vertex ([1 pt] per coordinate),
[3 pts] for graphing with:
- [1 pt] for vertex and x and y intercept placements,
- [1 pt] for vertex and x and y intercept labels,
- [1 pt] for sketching smooth curve which gets steeper away from vertex.

2. a) Simplify the complex fraction [5 pts]:

$$\frac{\frac{5}{x} - \frac{3}{x^2}}{\frac{7}{x^2} + \frac{4}{x}}$$

For the combine-invert-reduce method:

[2 pts] for combining the numerator and the denominator,
[1 pt] for applying invert and multiply,
[2 pts] for reducing the result.

OR For the LCD method:

[1 pt] for identifying the LCD,
[4 pts] for reducing each individual fraction.

b) Find the quotient $\frac{3+5i}{4-i}$ and express it in $a+bi$ form [5 pts].

[1 pt] for identifying the complex conjugate,
[2 pts] for performing the multiplication in the numerator and denominator,
[1 pt] for performing the addition in the numerator and denominator,
[1 pt] for writing the result in $a+bi$ form.

3. Write the equation of the circle; given below, in standard form. Identify the center and radius of the circle and graph the circle using the provided graph paper. Label four points with coordinates on the circle on your graph [10 pts]:

$$x^2 + y^2 - 14x + 8y + 64 = 0.$$

[1 pt] for shifting the constant term,
[2 pts] for completing both squares,
[1 pt] for factoring both perfect squares,
[1 pt] for identifying the center,
[1 pt] for identifying the radius,
[2 pts] for marking four points on the circle,
[2 pts] for labeling the four points with their coordinates.

4. Solve the linear system of equations [10 pts]:

$$-x + 2y - z = -1$$

$$2x - y - 2z = -6$$

$$2x - y - z = -1$$

[3 pts] for eliminating any variable in any fashion,

[3 pts] for eliminating the same variable in a second fashion,

[2 pts] for using the first two results to eliminate a second variable and finding first value,

[1 pt] for finding a second value,

[1 pt] for finding the final value.

5. Solve the system of equations [10 pts]:

$$3x + y = 4$$

$$x^2 - 2y = -16$$

[2 pts] for isolating a variable in the linear equation,

[2 pts] for correct substitution,

[1 pts] for writing in standard form ($ax^2 + bx + c = 0$),

[2 pts] for finding both solutions to the quadratic equation,

[2 pts] for finding corresponding values of the second variable,

[1 pt] for linking each x value with its associated y value, e.g. by providing 2 pairs of coordinates.

6. For the information given, state the values of the 6 trigonometric functions of θ [10 pts]:

$$\tan\theta = -\frac{3}{4} \text{and} \cos\theta < 0$$

[1 pt] for creating the triangle and labelling with the given information,

[2 pts] for using Pythagoras to find third side of right triangle,

[2 pts] for proper identification of quadrant for θ,

[5 pts] for exact values for each trig ratio.

7. This problem will have any one of the following three options:

Version asking to find an $\angle$: In $\triangle\ ABC$, side $a = 19$, side $b = 36$ and $c = 30$. Find side $\angle C$ of $\triangle\ ABC$ to the nearest tenth of a degree [10 pts].

[3 pts] for selecting the correct Law of Cosines form and stating it correctly,

[3 pts] for substituting the correct values,

[2 pts] for manipulating the equation to solve for $\cos(angle)$,

[2 pts] for applying the inverse trig function.

Version asking to find a side: In $\triangle\ ABC$, $\angle A = 82\,\text{deg}$, $\angle C = 33\,\text{deg}$ and side $a = 18$. Find side c of $\triangle\ ABC$ to the nearest tenth [10 pts].

[3 pts] for selecting the correct Law of Sines and stating it correctly,

[3 pts] for substituting the correct values,

[2 pts] for manipulating the equation to solve for the unknown side,

[2 pts] for computing the correct final result.

Version asking to find a side: In $\triangle\ ABC$, side $a = 22$, side $b = 39$ and $\angle C = 58\,\text{deg}$. Find side c of $\triangle\ ABC$ to the nearest tenth [10 pts].

[3 pts] for selecting the correct Law of Cosines form and stating it correctly,

[3 pts] for substituting the correct values,

[4 pts] for computing the correct final result.

8. Prove the identity [10 pts]:

$$\frac{\cos(x)}{1-\sin(x)} = \sec(x) + \tan(x)$$

roughly [2 pts] per step towards literal equality

9. Find all exact solutions to the trigonometric equations for $x \in [0, 2\pi)$:

   a) $\tan(x) = -\sqrt{(}3)$ [5 pts]

   b) $2\cos(x) = \sqrt{(}2)$[5 pts]

   Parts a) and b) can be assigned points by:

   [1 pt] for solving for trig(x),
   [1 pt] for identifying quadrants for x (implicitly or explicitly),
   [1 pt] for identifying the reference angle (implicitly or explicitly),
   [2 pt] for correct solutions.

   (Students are not *required* to explicitly state the quadrants and reference angles. The breakdowns are for the purpose of assigning partial credit in the case that students do not find the correct angles via the unit circle.)

10. a) Evaluate the logarithm without a calculator: $\log_5(25\sqrt[4]{5})$ [5 pts].

   [4 pts] for identifying the correct exponent for separate pieces,
   [1 pt] for adding these two exponents together.

   OR

   [1 pt] for expressing as a logarithmic equation,
   [3 pts] for rewriting as an equivalent exponential equation,
   [1 pt] for identifying the correct exponent (solution).

   b) Solve for x and round the answer to the nearest tenth [5 pts]: $17^x = 373$.

   [2 pts] for writing the log-form with given base,
   [2 pts] for applying change of base formula,
   [1 pt] for the correct decimal approximation.

   OR

   [1 pt] for expressing as a logarithmic equation ("taking the log of both sides"),
   [1 pt] for rewriting using properties of logarithms to bring exponent (x) down,
   [2 pts] for solving for x,
   [1 pt] for the correct decimal approximation.