

Exam 1.

< solutions and answers >

Class MAT 1275 CO

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Date: 9/18/2019, Time: 8:00am~9:40am at Room: N719, Last name:

First name:

Show your work on the blue book and answer it. You do not get any point without showing work.

Factor out.

$$1. \quad 8x^2 - 2y^2 = 2(4x^2 - y^2) = 2((2x)^2 - y^2) = 2(2x + y)(2x - y) \checkmark$$

by using $a^2 - b^2 = (a+b)(a-b)$

$$2. \quad x^3 - 2x^2 - 4x + 8 = x^2(x-2) - 4(x-2) = (x-2)(x^2 - 4)$$

$$= (x-2)(x^2 - 2^2) = (x-2)(x+2)(x-2)$$

$$= (x-2)^2(x+2) \checkmark$$

3. Solve a quadratic equation.

$$x(-1)(-x^2 + 2x + 3) = 0 \quad x(-1)$$

$$x^2 - 2x - 3 = 0 \quad \rightarrow \quad -3 = \begin{matrix} 1 \\ -3 \\ \hline -2 \end{matrix}$$

$$(x+1)(x-3) = 0$$

$$\therefore x = -1, \quad x = 3 \checkmark$$

4. Solve a quadratic equation by using a completing square form.

$$x^2 + 6x + 5 = 0$$

$$x^2 + 6x + 5 = 0 \xrightarrow{+\frac{6}{2}=3} (x+3)^2 - 3^2 + 5 = 0$$

$$(x+3)^2 - 9 + 5 = 0, \quad (x+3)^2 - 4 = 0, \quad (x+3)^2 = 4$$

$$x+3 = \pm\sqrt{4}, \quad x+3 = \pm 2, \quad x = -3 \pm 2$$

$$\text{so, } x = -3 + 2 \checkmark \text{ and } x = -3 - 2 \checkmark$$

$$= -1 \checkmark \qquad \qquad \qquad = -5 \checkmark$$

5. Solve a quadratic equation.

$$\frac{4(x+3)^2}{4} = \frac{-8}{4}, \quad (x+3)^2 = -2$$

$$\sqrt{-1} = i$$

$$(x+3) = \pm\sqrt{-2}, \quad x+3 = \pm\sqrt{(-1)(2)}$$

$$x+3 = \pm\sqrt{-1}\sqrt{2}, \quad x+3 = \pm i\sqrt{2}$$

$$\therefore x = -3 \pm i\sqrt{2} \checkmark$$

6. Simplify.

$$\frac{4s}{s-2} + \frac{8(-1)}{(2-s)(-1)} = \frac{4s}{s-2} + \frac{-8}{-2+s}$$

$$= \frac{4s}{s-2} - \frac{8}{s-2}$$

$$= \frac{4s - 8}{s-2}$$

$$= \frac{4(s-2)}{s-2}$$

$$= 4 \checkmark$$

7. Simplify.

$$\text{L.C.M}(t, t-1)$$

$$= t(t-1)$$

$$\left(\frac{\frac{3}{t} - \frac{2}{t-1}}{3 - \frac{6}{t-1}} \right) t(t-1)$$

$$= \frac{3(t-1) - 2t}{3t(t-1) - 6t}$$

$$= \frac{3t - 3 - 2t}{3t^2 - 3t - 6t}$$

$$= \frac{t - 3}{3t^2 - 9t}$$

$$= \frac{\cancel{t} - 3}{3t(\cancel{t} - 3)}$$

$$= \frac{1}{3t} \checkmark$$

8. Solve the system of equations.

$$\begin{cases} 3x + y = 8 & \text{--- ①} \\ 2y - x = -12 & \text{--- ②} \end{cases}$$

$$(-2) \times \text{①} \rightarrow \begin{cases} -6x - 2y = -18 & \text{--- ①} \\ -x + 2y = -12 & \text{--- ②} \end{cases}$$

$$\frac{-7x}{-7} = \frac{-28}{-7} \rightarrow$$

$$\div (-7) \quad \boxed{x = 4} \checkmark$$

by ①,

$$3x + y = 8 \text{ --- ①}$$

$$\downarrow \downarrow \quad \downarrow \downarrow$$

$$3(4) + y = 8$$

$$12 + y = 8$$

$$y = 8 - 12$$

$$\boxed{y = -4} \checkmark$$

$$\therefore \{ (4, -4) \} \checkmark$$

Write the answers in the form $a + bi$.

9. Simplify.

$$\begin{aligned} & (3 - 6i) - (2 - 4i) \\ &= 3 - 6i - 2 + 4i \\ &= 3 - 2 - 6i + 4i \\ &= 1 - 2i \checkmark \end{aligned}$$

10. Simplify.

$$\frac{1-2i}{1+i}$$

conjugate $\leftrightarrow (1-i)$

$$\frac{(1-2i)(1-i)}{(1+i)(1-i)} = \frac{1-i-2i+2i^2}{1^2-i^2}$$

$$= \frac{1+2i^2-i-2i}{1-i^2} = \frac{1+2(-1)-3i}{1-(-1)}$$

$$= \frac{1-2-3i}{2} = \frac{-1-3i}{2} = -\frac{1}{2} - \frac{3}{2}i \checkmark$$

$= a + bi \text{ form}$

$$i^2 = -1$$

$$\begin{aligned} (a+b)(a-b) \\ = a^2 - b^2 \end{aligned}$$