NYC College of Technology **Revised Spring 2018**
Mathematics Department **Professor Rozenblyum**

MAT 1275 Final Exam Review Sheet

 # 1 Solve for x and simplify the answer:

a) $6x^2 - 2x - 3 = 0$ b) $-2x^2 + 4x - 5 = 0$ c) $2x^2 + 2x = -3$

 # 2 Find the vertex of the quadratic function. Graph the function and label the vertex and the x- and y-intercepts with numbers or coordinates. Do not round the numbers:

a) $y = x^2 - 2x - 3$ b) $y = x^2 + 2x$ c) $y = -x^2 + 4x - 2$

 # 3 Simplify the complex fraction: a) $\dfrac{\frac{2}{y^2} + \frac{1}{y}}{\frac{4}{y^2} - \frac{1}{y}}$ b) $\dfrac{\frac{2}{x} + \frac{1}{y}}{\frac{3}{y} - \frac{4}{x}}$ c) $\dfrac{3 - \frac{1}{2x}}{2 + \frac{1}{x^2}}$

 # 4 Write the equation of circle, given below, in standard form. Identify the center and radius of the circle and graph it. Label four points on the graph with coordinates:

a) $x^2 + y^2 - 6x + 4y - 4 = 0$ b) $x^2 + y^2 + 8x - 2y - 8 = 0$

c) $x^2 + y^2 + 10x + 6y + 18 = 0$

 # 5 Perform the indicated operations and express the answer in the form $a + bi$:

a) $(-2 + 3i)(5 - 7i)$ b) $(4 - 9i)(3 - 2i)$ c) $\dfrac{3 - 2i}{1 - i}$ d) $\dfrac{2 - 7i}{3 + 4i}$

 # 6 Solve the system of equations:

a) $\begin{aligned} 3x + 2y - z &= 4 \\ 2x - y + 2z &= 10 \\ x + 3y - 4z &= -7 \end{aligned}$ b) $\begin{aligned} -2x + 5y + z &= 8 \\ x - 2y - 3z &= -13 \\ x + 3y - z &= 5 \end{aligned}$ c) $\begin{aligned} 2x - 3y + z &= -9 \\ 3x + 5y + 2z &= 16 \\ -4x + 2y - 3z &= 4 \end{aligned}$

 # 7 Solve the system of equations:

a) $\begin{aligned} 3x + y &= 4 \\ x^2 - 3y &= -32 \end{aligned}$ b) $\begin{aligned} x^2 + y^2 &= 5 \\ x - y^2 &= -3 \end{aligned}$ c) $\begin{aligned} 4x - y &= 2 \\ x^2 + 2y &= 5 \end{aligned}$ d) $\begin{aligned} x^2 - y^2 &= 3 \\ 2x + y^2 &= 5 \end{aligned}$

 # 8 For the information given, state the value of the five remaining trig functions of θ:

a) $\sin\theta = \frac{2}{5}$ and $\cos\theta < 0$ b) $\tan\theta = 2$ and $\sin\theta < 0$

c) $\cos\theta = \frac{2}{3}$ and $\tan\theta < 0$ d) $\cos\theta = \frac{3}{7}$ and $\sin\theta < 0$

 # 9 Given ΔABC, answer the following (round each answer to the nearest tenth):

a) If $a = 12$, $b = 8$ and $c = 5$, find $\angle C$

b) If $\angle A = 50^\circ$, $\angle B = 75^\circ$, and $a = 20$, find side b

c) If $b = 9$, $c = 6$ and $\angle A = 67^\circ$, find side a

d) If $\angle C = 37^\circ$, $\angle B = 79^\circ$, and $b = 13$, find side a

 # 10 Prove the identities:

a) $\cos x + \sin x \cdot \tan x = \sec x$ b) $\csc x - \sin x = \cot x \cdot \cos x$

c) $\frac{1}{\cos x} - \frac{1}{\sec x} = \tan x \cdot \sin x$ d) $\csc x \cdot \cot x \cdot \sec x = 1 + \cot^2 x$

e) $\sin x \cdot \tan x \cdot \cos x = 1 - \cos^2 x$ f) $\sec x \cdot \csc x = \tan x + \cot x$

 # 11 Find the **exact** solutions to the trig equations for $x \in [0, 2\pi)$:

a) $4\sin x + 2 = 0$ b) $6\cos x - 3 = 0$ c) $2\tan x - 2 = 0$ d) $6\tan x = -2\sqrt{3}$

 # 12 Solve for x and round the answer to the nearest tenth:

a) $3^x = 38$ b) $2^x = 19$ c) $4^x = 7$

 # 13 Evaluate the logarithm **without** using a calculator:

a) $\log_2 \sqrt[3]{2}$ b) $\log_3 \frac{1}{9}$ c) $\log_4 \left(16\sqrt[3]{4}\right)$ d) $\log_2 \left(8\sqrt[6]{2}\right)$

Answers

 # 1 a) $\dfrac{1 \pm \sqrt{19}}{6}$ b) $\dfrac{2 \pm i\sqrt{6}}{2}$ c) $\dfrac{-1 \pm i\sqrt{5}}{2}$

 # 2 a) Vertex: $(1, -4)$, x-intercepts: $(3, 0)$ and $(-1, 0)$, y-intercept: $(0, -3)$

b) Vertex: $(-1, -1)$, x-intercepts: $(0, 0)$ and $(-2, 0)$, y-intercept: $(0, 0)$

c) Vertex: $(2, 2)$, x-intercepts: $\left(2 - \sqrt{2}, 0\right)$ and $\left(2 + \sqrt{2}, 0\right)$, y-intercept: $(0, -2)$

 # 3 a) $\dfrac{2 + y}{4 - y}$ b) $\dfrac{2y + x}{3x - 4y}$ c) $\dfrac{6x^2 - x}{4x^2 + 2}$ or $\dfrac{x(6x - 1)}{2(2x^2 + 1)}$

 # 4 a) Center: $(3, -2)$, Radius: $r = \sqrt{17}$ b) Center: $(-4, 1)$, Radius: $r = 5$

c) Center: $(-5, -3)$, Radius: $r = 4$

 # 5 a) $11 + 29i$ b) $-6 - 35i$ c) $\dfrac{5}{2} + \dfrac{1}{2}i$ d) $-\dfrac{22}{25} - \dfrac{29}{25}i$

 # 6 a) $x = 3,\ y = -2,\ z = 1$ b) $x = 3,\ y = 2,\ z = 4$

c) $x = -1,\ y = 3,\ z = 2$

 # 7 a) $(-5, 19),\ (-4, 16)$ b) $(-2, 1),\ (1, 2),\ (1, -2),\ (-2, -1)$

c) $(-9, -38),\ (1, 2)$ d) $(2, 1),\ (2, -1),\ \left(-4, \sqrt{13}\right),\ \left(-4, -\sqrt{13}\right)$

 # 8 a) $\cos\theta = -\dfrac{\sqrt{21}}{5}$, $\tan\theta = -\dfrac{2}{\sqrt{21}} = -\dfrac{2\sqrt{21}}{21}$, $\sec\theta = -\dfrac{5}{\sqrt{21}} = -\dfrac{5\sqrt{21}}{21}$,

$\csc\theta = \dfrac{5}{2}$, $\cot\theta = -\dfrac{\sqrt{21}}{2}$

b) $\sin\theta = -\dfrac{2}{\sqrt{5}} = -\dfrac{2\sqrt{5}}{5}$, $\cos\theta = -\dfrac{1}{\sqrt{5}} = -\dfrac{\sqrt{5}}{5}$, $\sec\theta = -\sqrt{5}$,

$\csc\theta = -\dfrac{\sqrt{5}}{2}$, $\cot\theta = \dfrac{1}{2}$

c) $\sin\theta = -\frac{\sqrt{5}}{3}$, $\tan\theta = -\frac{\sqrt{5}}{2}$, $\sec\theta = \frac{3}{2}$,

$\csc\theta = -\frac{3}{\sqrt{5}} = -\frac{3\sqrt{5}}{5}$, $\cot\theta = -\frac{2}{\sqrt{5}} = -\frac{2\sqrt{5}}{5}$

d) $\sin\theta = -\frac{2\sqrt{10}}{7}$, $\tan\theta = -\frac{2\sqrt{10}}{3}$, $\sec\theta = \frac{7}{3}$,

$\csc\theta = -\frac{7}{2\sqrt{10}} = -\frac{7\sqrt{10}}{20}$, $\cot\theta = -\frac{3}{2\sqrt{10}} = -\frac{3\sqrt{10}}{20}$

 # 9 a) $\angle C = 17.6^{\circ}$ b) $b = 25.2$ c) $a = 8.6$ d) $a = 11.9$

 # 10 a) $\cos x + \sin x \cdot \tan x = \cos x + \sin x \cdot \frac{\sin x}{\cos x}$

$= \frac{\cos^2 x}{\cos x} + \frac{\sin^2 x}{\cos x} = \frac{\cos^2 x + \sin^2 x}{\cos x} = \frac{1}{\cos x} = \sec x$

b) $\csc x - \sin x = \frac{1}{\sin x} - \sin x = \frac{1}{\sin x} - \frac{\sin^2 x}{\sin x}$

$= \frac{1-\sin^2 x}{\sin x} = \frac{\cos^2 x}{\sin x} = \frac{\cos x}{\sin x} \cdot \cos x = \cot x \cdot \cos x$

c) $\frac{1}{\cos x} - \frac{1}{\sec x} = \frac{1}{\cos x} - \cos x = \frac{1}{\cos x} - \frac{\cos^2 x}{\cos x}$

$= \frac{1-\cos^2 x}{\cos x} = \frac{\sin^2 x}{\cos x} = \frac{\sin x}{\cos x} \cdot \sin x = \tan x \cdot \sin x$

d) $\csc x \cdot \cot x \cdot \sec x = \frac{1}{\sin x} \cdot \frac{\cos x}{\sin x} \cdot \frac{1}{\cos x} = \frac{1}{\sin^2 x} = \csc^2 x = 1 + \cot^2 x$

e) $\sin x \cdot \tan x \cdot \cos x = \sin x \cdot \frac{\sin x}{\cos x} \cdot \cos x = \sin^2 x = 1 - \cos^2 x$

f) For this identity, it is more convenient to start with the right hand side:

$\tan x + \cot x = \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = \frac{\sin^2 x + \cos^2 x}{\cos x \cdot \sin x}$

$= \frac{1}{\cos x \cdot \sin x} = \frac{1}{\cos x} \cdot \frac{1}{\sin x} = \sec x \cdot \cos x$

 # 11 a) $x = \frac{7\pi}{6}, \frac{11\pi}{6}$ b) $x = \frac{\pi}{3}, \frac{5\pi}{3}$ c) $x = \frac{\pi}{4}, \frac{5\pi}{4}$ d) $x = \frac{5\pi}{6}, \frac{11\pi}{6}$

 # 12 a) $x = 3.3$ b) $x = 4.2$ c) $x = 1.4$

 # 13 a) $\frac{1}{3}$ b) -2 c) $\frac{7}{3}$ d) $\frac{19}{6}$