# A Probabilistic Population Model with Cyclic Birth Rates

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# Background

- A model is used to simulate population changes in a group of organisms living in a single region
- Random processes are inherent to actual living populations, but they can be extremely complex, so difficult to describe mathematically
- To simplify: approximate possible events on an individual scale, and then simulate them for a larger group (implementing mean field theory)

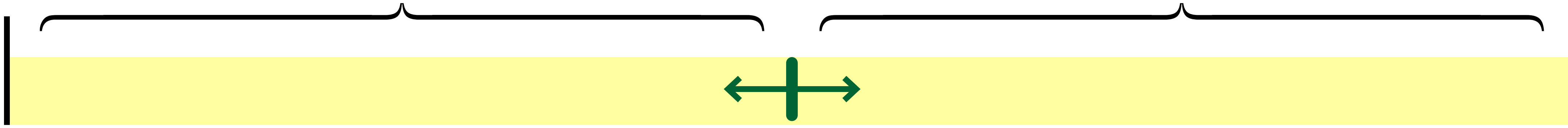
  An extension of a mean field Bolker-Pacala Model

- Each event is modeled by an exponential random variable whose parameters we control
- This makes more efficient simulation of a large and complex population possible

# Model Parameters

| | | | |
|---|---|---|---|
| $b_i$ | birth rate after $i^{th}$ event | $j$ | initial population |
| $\mu$ | death rate (natural) | $t$ | population threshold |
| $a$ | death rate (competition) | $L$ | # of locations in region |
| $r$ | resource growth rate | $p$ | probability of change in b |

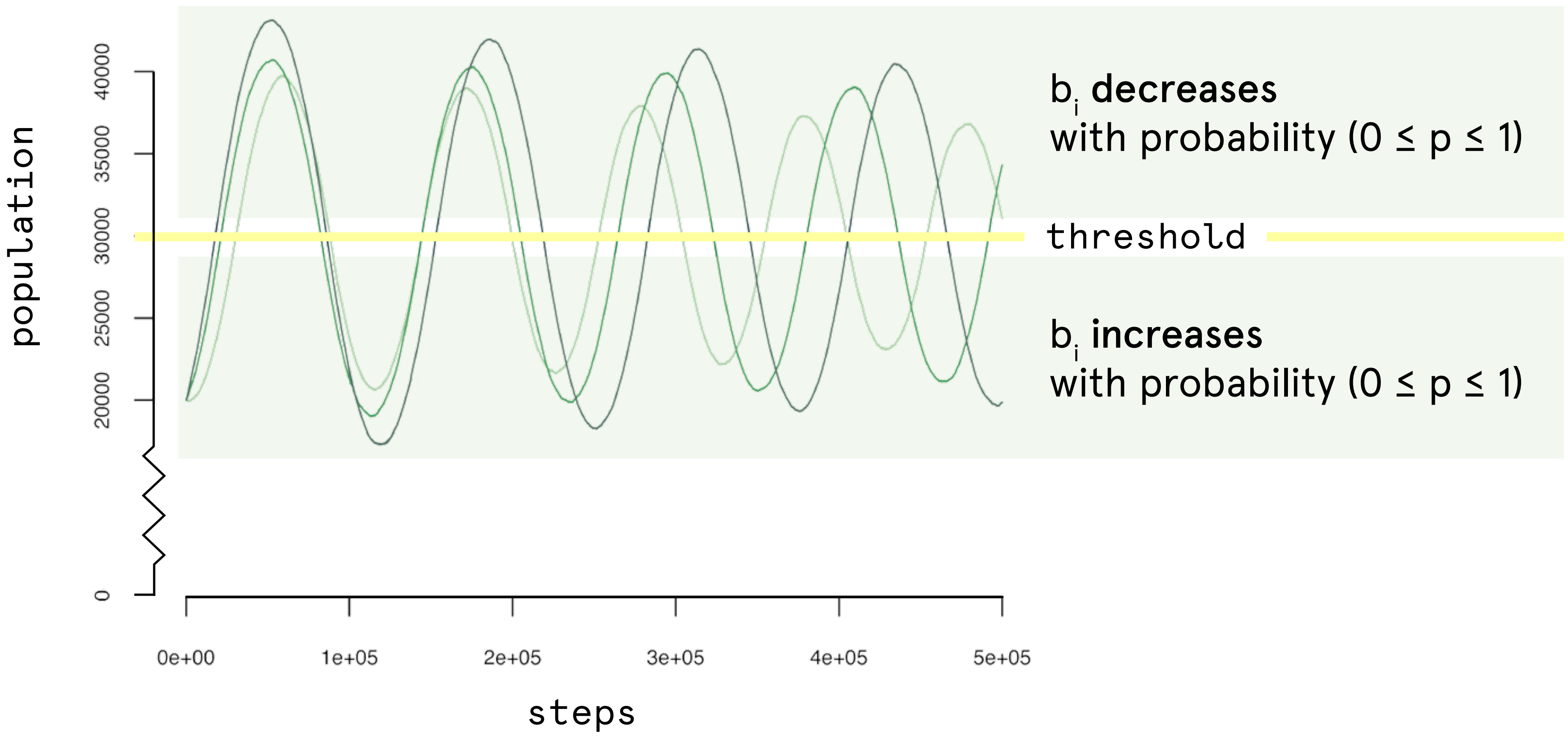
Probability of population increase + Probability of population decrease = 1



$$\frac{b_i j}{b_i j + \mu j + \frac{aj^2}{L}} + \frac{\mu j + \frac{aj^2}{L}}{b_i j + \mu j + \frac{aj^2}{L}} = 1$$

# Cyclic Birth Rate

- Births become more likely when the population is below a certain **threshold**, and less likely when the population is above that threshold

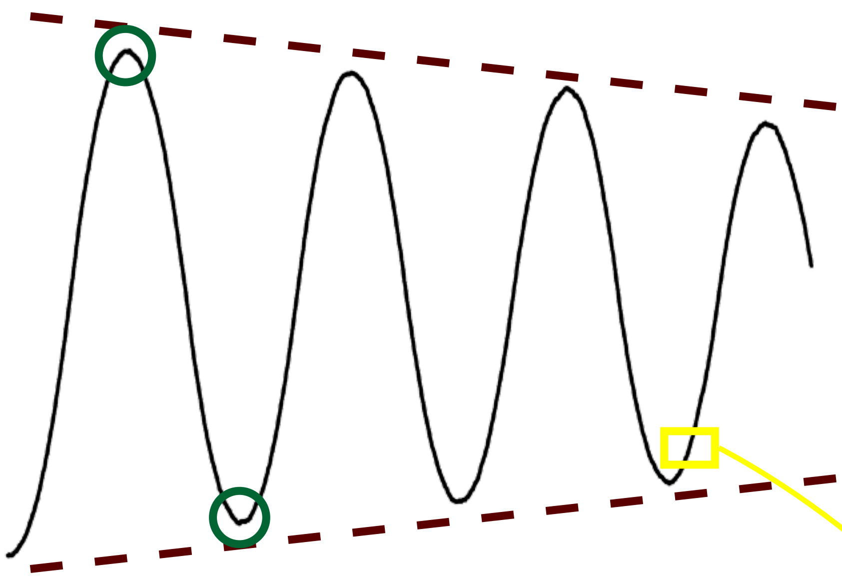



- These birth rate cycles result in **oscillations**, whose period and amplitude are affected by the input parameters
- Two methods for implementing: **input**-based vs. **density**-based

# Key Features of Population Oscillations

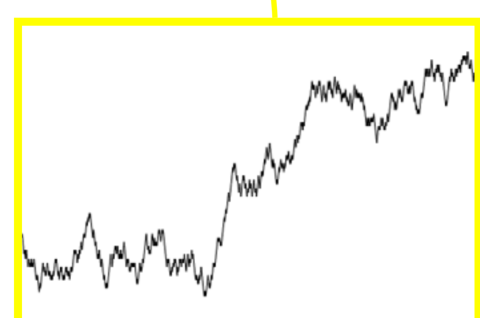
## Max (peak) and min values

Overall relatively consistent for any given starting population (despite randomness inherent to model)



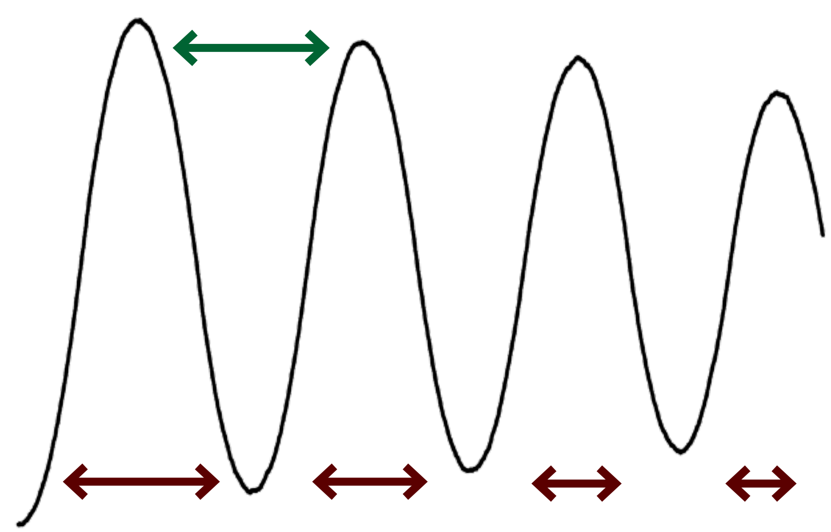
## Dampening effect

Stronger as competition rates increase



## Distance between peaks

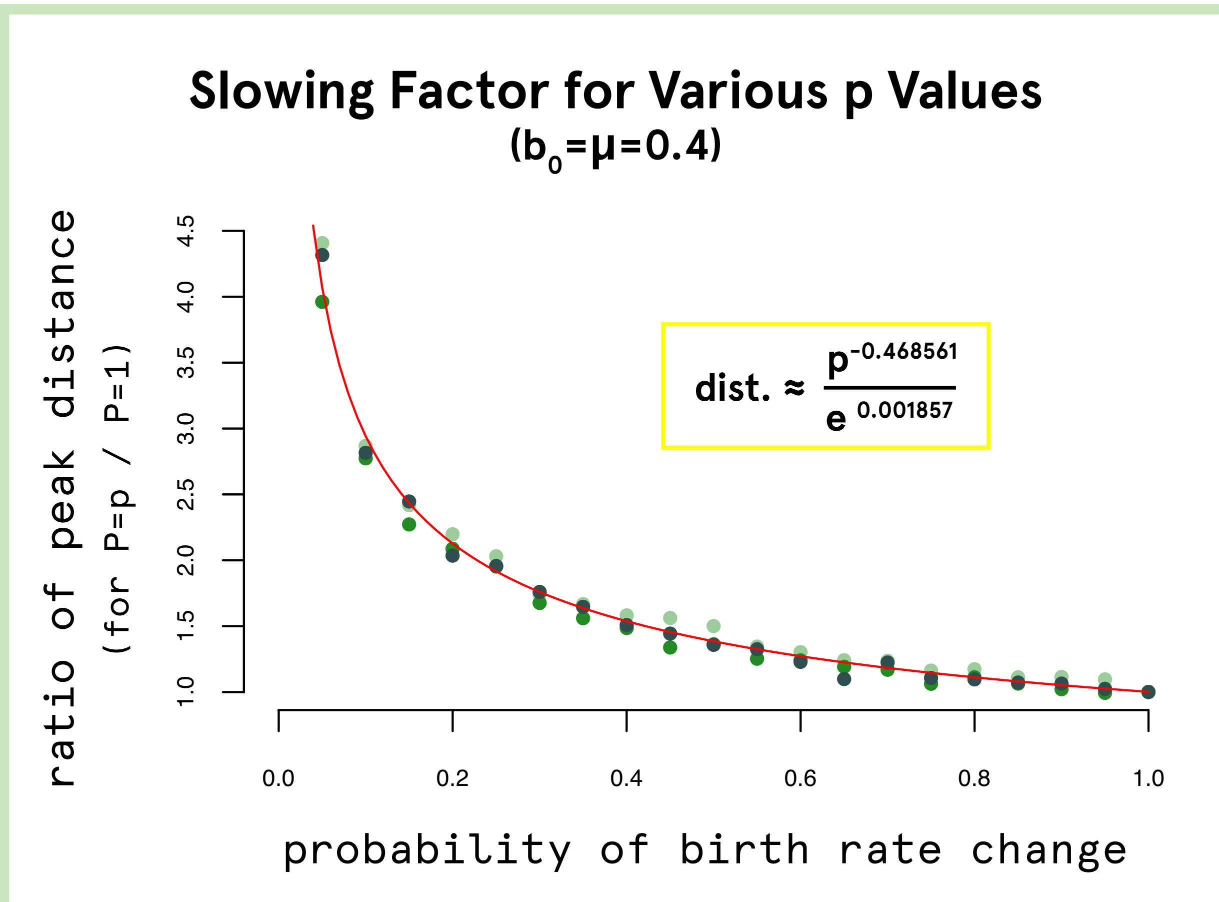
(Related to the "slowing factor")
Varies according to p, the probability that the birth rate will change

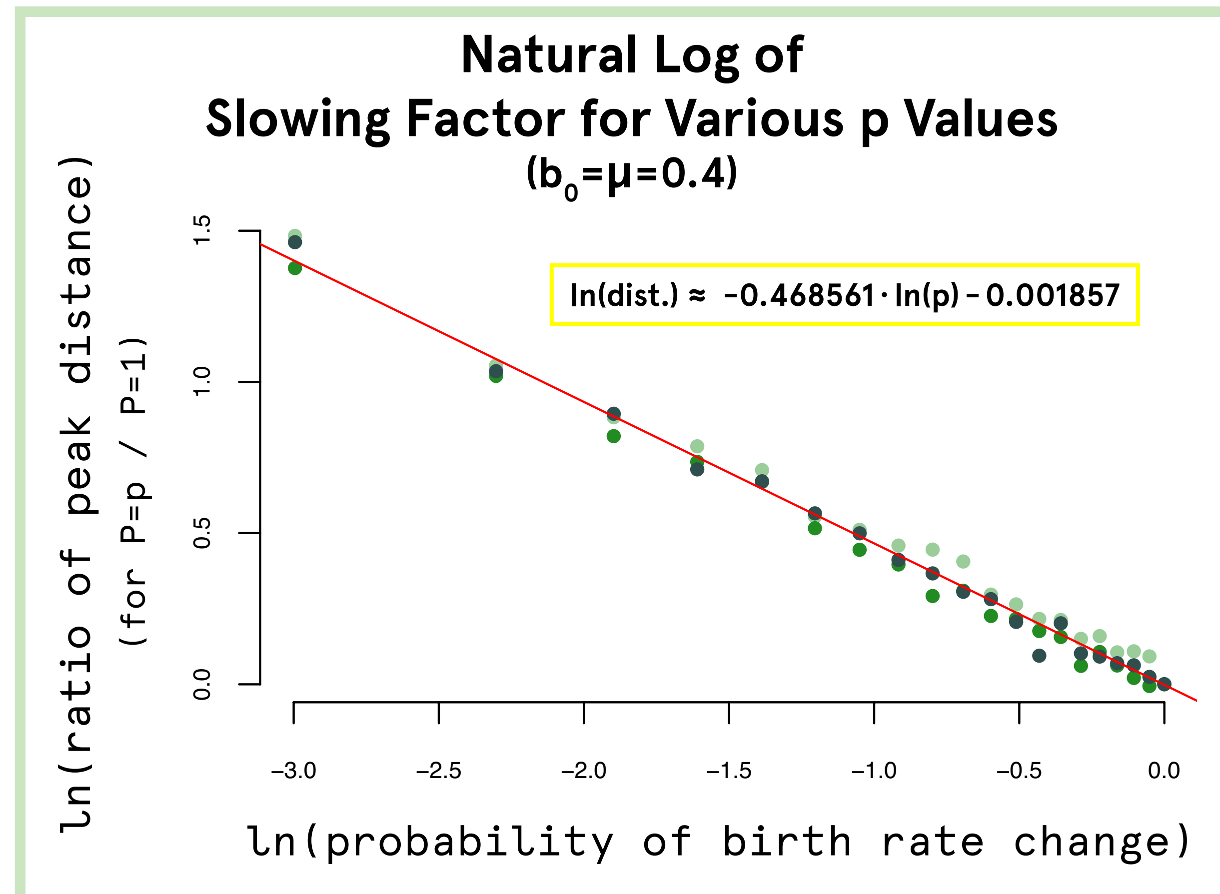


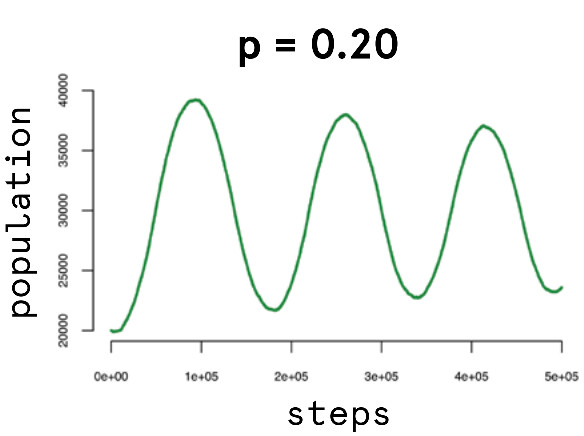
## Condensing effect

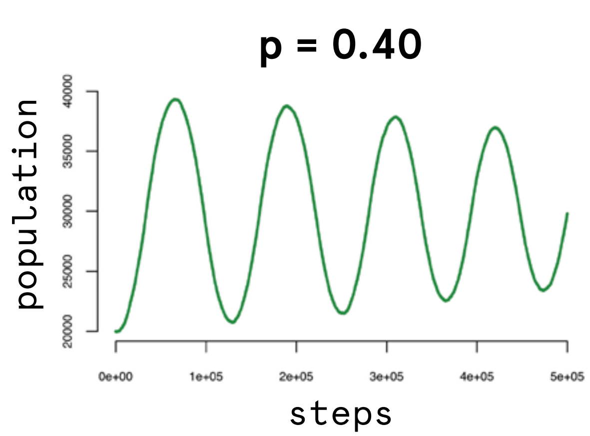
Stronger as birth rate exceeds death rate

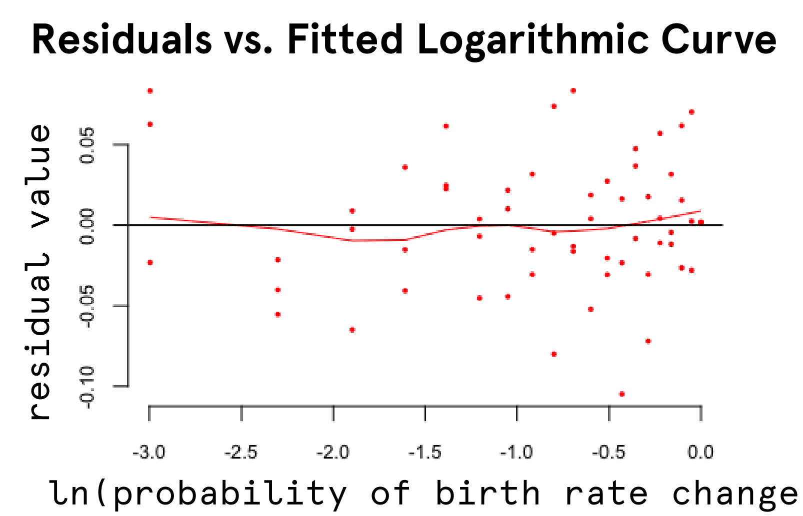
# Slowing Factor Caused by p (for initial birth rate = death rate)



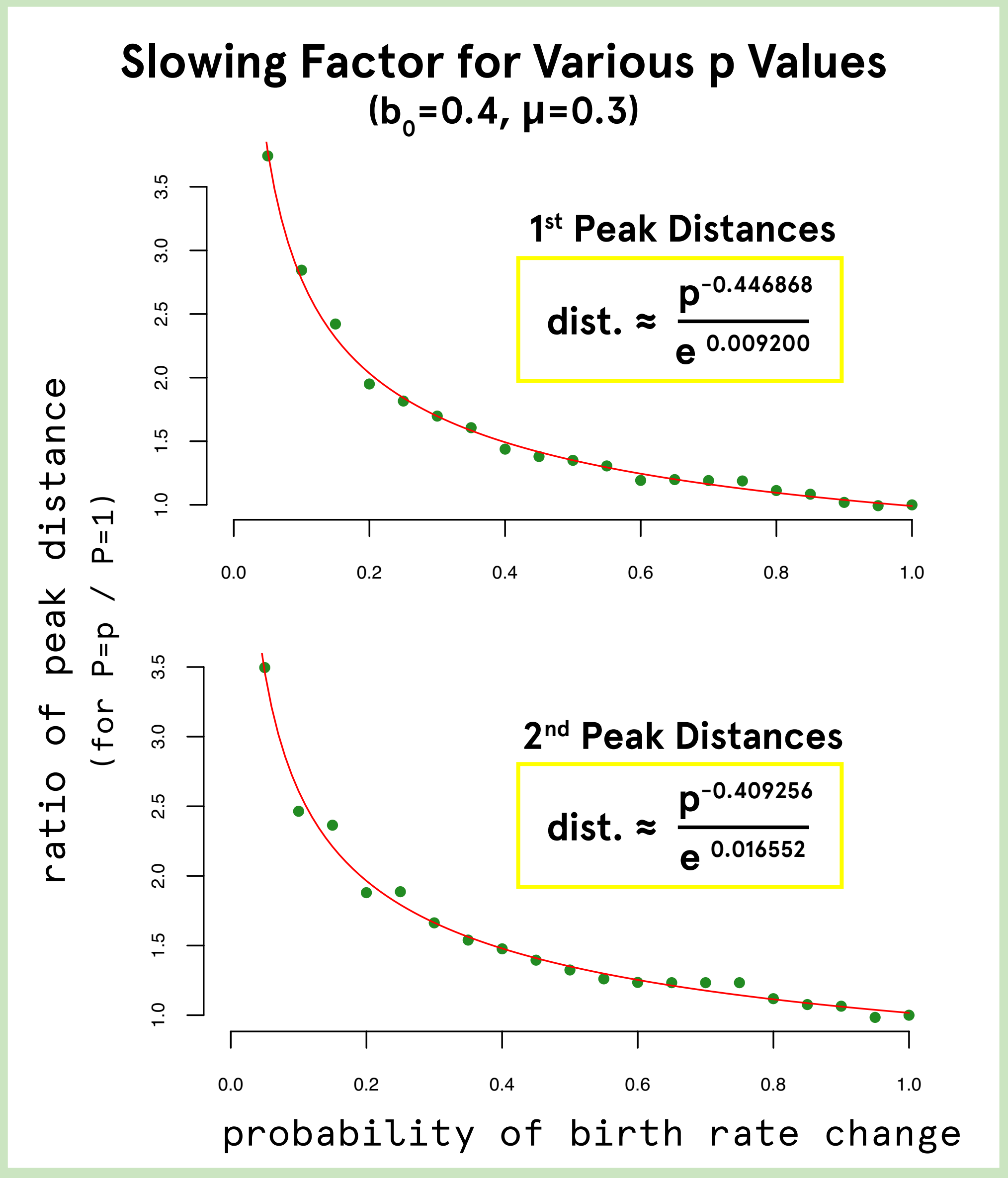



# Slowing Factor Caused by p (for initial birth rate > death rate)

- Note that condensing effect begins to have an impact when $b_i > \mu$ (peak distances are no longer constant for any given **p**)
- So, we compare peaks separately (1st-to-1st, 2nd-to-2nd)




| Input Parameters | Exponent of p for 1st Peak Distance (= slope of log line) | Exponent of p for 2nd Peak Distance (= slope of log line) |
|---|---|---|
| μ = 0.4 ($b_0$ = 0.4) | -0.449470 | -0.416982 |
| μ = 0.3 ($b_0$ = 0.4) | -0.446868 | -0.409256 |
| μ = 0.2 ($b_0$ = 0.4) | -0.492693 | -0.42614 |
| μ = 0.1 ($b_0$ = 0.4) | -0.60194 | -- |

# Conclusions and Further Study

- We are able to approximate power functions that relate the probability of birth rate change to a slowing factor in the oscillations
- Because of the stochastic processes in the simulation, these equations should be improved by conducting many more trials
- While these models do incorporate realistic events in a population, the assignments of event probabilities should be more fine-tuned to increase their meaningfulness
  - Parameters for $b_i$, u, a, and r should reflect actual ecological dynamics (*e.g.* plant species in a terrain, or human social castes in a country)
- Switching to the more realistic density-based model, where p changes in response to the threshold, may provide different solutions
- In future, these models could be extended to show interactions of multiple population groups in the same ecological system