Arithmetic and Geometric Series

1. A sequence $\{a_n\}$ is called an **arithmetic sequence** if any two consecutive terms have a *common difference* d. The arithmetic sequence is dertmined by d and the first value a_1. This can be written recursively as:

$$a_n = a_{n-1} + d$$

for $n \geq 2$. Alternatively, the formula for the nth term of the sequence is

$$a_n = a_1 + d(n-1).$$

2. Let $\{a_n\}$ be an **arithmetic sequence**, whose nth term is given by $a_n = a_1 + d(n-1)$. Then, the sum $a_1 + a_2 + a_3 + \ldots + a_k$ is given by adding $(a_1 + a_k)$ precisely $\frac{k}{2}$ times:

$$\sum_{i=1}^{k} a_i = \frac{k}{2} \cdot (a_1 + a_l).$$

3. A sequence $\{a_n\}$ is called a **geometric sequence** if any two consecutive terms have a *common ratio* r. The geometric sequence is determined by r and the first value a_1. This can be written recursively as:

$$a_n = a_{n-1} \cdot r^{n-1}$$

for $n \geq 2$. Alternatively, we have the general formula for the nth term of the geometric sequence

$$a_n = a_1 \cdot r^{n-1}.$$

4. Let $\{a_n\}$ be an **geometric sequence**, whose nth term is given by $a_n = a_1 \cdot r^{n-1}$. Furthermore, assume that $r \neq 1$. Then, the sum $a_1 + a_2 + a_3 + \ldots + a_k$ is given by:

$$\sum_{i=1}^{k} a_i = a_1 \cdot \frac{1 - r^k}{1 - r}.$$

5. An **infinite series** is given by the

$$\sum_{i=1}^{\infty} a_i = a_1 + a_2 + a_3 + a_4 + \ldots$$

6. Let $\{a_n\}$ be an **geometric sequence**, whose nth term is given by $a_n = a_1 \cdot r^{n-1}$.Then the infinite geometric series is defined whenever $-1 < r < 1$. In this case, we have:

$$\sum_{i=1}^{k} a_i = a_1 \cdot \frac{1}{1 - r}.$$