Handout for Section 12.4

n Factorial

Let n be a positive integer. Then

$$n! = 1 \cdot 2 \cdot 3 \cdot 4 \cdots (n-2)(n-1)n.$$

0! is defined to be the number 1.

Binomial Coefficients

If r and n are integers with $0 \leq r \leq n$, then we have the following.

Either of the symbols $\binom{n}{r}$ or ${}_nC_r$ denotes the number $\dfrac{n!}{r!(n-r)!}$.

$\binom{n}{r}$ is called a **binomial coefficient.**

The Binomial Theorem

For each positive integer n,

$$(x+y)^n = x^n + \binom{n}{1}x^{n-1}y + \binom{n}{2}x^{n-2}y^2 + \binom{n}{3}x^{n-3}y^3 + \cdots + \binom{n}{n-1}xy^{n-1} + y^n$$

Properties of the Binomial Expansion

In the binomial expansion of $(x+y)^n$,

The exponent of y is always one less than the number of the term.

Furthermore, in each of the middle terms of the expansion,

The coefficient of the term containing y^r is $\binom{n}{r}$.

The sum of the x exponent and the y exponent is n.