

MAT 1375 Final Exam Review Sheet

#1 Solve the inequality and express the solution as an interval.

a) $x^2 - 2x - 3 \geq 0$

b) $6x - 2x^2 > 0$

c) $x^2 + x > 0$

d) $\frac{x+3}{x-4} \leq 0$

#2 Solve the inequalities algebraically and write the solution as an interval.

a) $|3x-2| < 4$

b) $|5-2x| \geq 9$

#3 Given each function: Find the x- and y-intercepts, the domain, the vertical and horizontal asymptotes, and then sketch a complete graph of the function.

a) $f(x) = \frac{1-x}{x+2}$

b) $f(x) = \frac{x+2}{3-x}$

c) $f(x) = \frac{2x-4}{1-x}$

#4 Find the difference quotient $\frac{f(x+h)-f(x)}{h}$ (assume $h \neq 0$) for:

a) $f(x) = x^2 + 5x + 2$

b) $f(x) = 2x^2 - 3x$

c) $f(x) = -2x^2 + 4x + 1$

#5 For each polynomial, find all roots of the function algebraically in simplest radical form. Then sketch the complete graph, indicating the viewing window, the x-intercepts and the relative maximum and minimum.

a) $f(x) = x^3 + 2x^2 - 3x - 6$

b) $f(x) = 2x^3 - 7x^2 - 3x + 8$

#6 Find the magnitude and the direction angle for:

a) $v = \langle 2, -2\sqrt{3} \rangle$

b) $v = \langle -3, -3 \rangle$

c) $v = \langle -5\sqrt{3}, 5 \rangle$

#7 Find the quotient or product and write the result in standard complex form:

a) $\frac{6(\cos 225^\circ + i \sin 225^\circ)}{3(\cos 135^\circ + i \sin 135^\circ)}$

b) $2(\cos 120^\circ + i \sin 120^\circ) \cdot 4(\cos 90^\circ + i \sin 90^\circ)$

#8 Let $u = \ln(x)$ and $v = \ln(y)$, where $x, y > 0$. Write the following expression in terms of u and v .

a) $\ln(x^8 \cdot \sqrt[3]{y^4})$

b) $\ln\left(\frac{\sqrt{x^5}}{y^3}\right)$

c) $\ln(\sqrt{x^3 \cdot \sqrt[4]{y}})$

#9 Find the domain, asymptotes, and x-intercepts of the function, and then sketch its graph.

a) $f(x) = \log(x-4)$

b) $f(x) = \log(5-x)$

c) $f(x) = -\ln(x+2)$

#10 State the amplitude, period and phase shift, and then sketch one complete cycle of the graph. Label all maximum, minimum and x-intercepts.

a) $y = 3\cos(2x + \pi)$

b) $y = 2\sin(4x - \pi)$

c) $y = -4\sin\left(x - \frac{\pi}{2}\right)$

#11 Find all exact solutions in radians.

a) $2\sin^2 x = \sin x$

b) $2\cos^2 x - \cos x - 1 = 0$

c) $\tan^2 x - \tan x = 0$

#12 In 1998, the population of a colony is 10,000, and is decreasing exponentially at 1.5% per year.

- a) What will the population be after 5 years? b) In what year will there be half of the population left?

#13 In 2003, the population of a city is 80,000 people, and is growing at a rate of 4% per year.

- a) What will the population be in 2015? b) In what year will the population be triple?

#14 Find the inverse for the following function.

- a) $y = 3 - 4x$ b) $y = \frac{4}{x-3}$ c) $y = \frac{2}{8x+5}$ d) $y = \frac{x-1}{x+2}$

#15

- a) Find the sum of the first 70 terms of the arithmetic sequence: 22, 19, 16, 13, ...
b) Find the sum of the first 95 terms of the arithmetic sequence: -17, -12, -7, -2, ...
c) Find the sum of the first 777 terms of the arithmetic sequence: 3, 9, 15, 21, ...

#16 Find the exact sum of the infinite geometric sequence.

- a) $\frac{1}{2}, -\frac{1}{4}, \frac{1}{8}, \dots$ b) 32, -16, 8, -4, ... c) 3, 2, $\frac{4}{3}, \frac{8}{9}, \dots$ d) -54, -18, -6, -2, ...

#17 Use the Binomial Theorem to write the following terms in simplest form.

- a) Find the first four terms of the expansion of $\left(2x - \frac{y}{2}\right)^8$.
b) Find the 7th term in the expansion of $\left(2x - \frac{y}{2}\right)^8$.
-
-

Answers:

#1 a) $(-\infty, -1] \cup [3, \infty)$

b) $(0, 3)$

c) $(-\infty, -1) \cup (0, \infty)$

d) $[-3, 4)$

#2 a) $\left(-\frac{2}{3}, 2\right)$

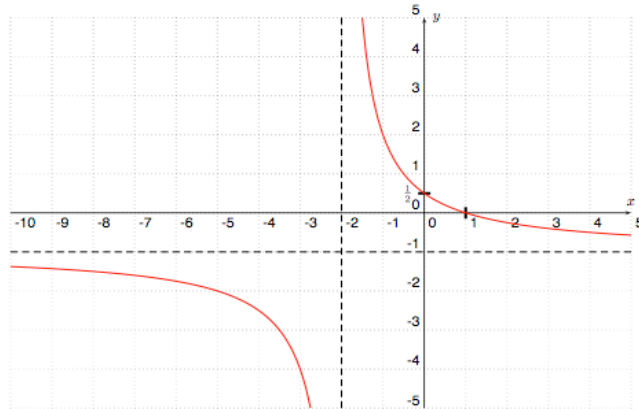
b) $(-\infty, -2] \cup [7, \infty)$

#3 a) domain: $D = (-\infty, -2) \cup (-2, \infty)$

vertical asymptote: $x = -2$

horizontal asymptote: $y = -1$

x-int: $(1, 0)$, y-int: $\left(0, \frac{1}{2}\right)$

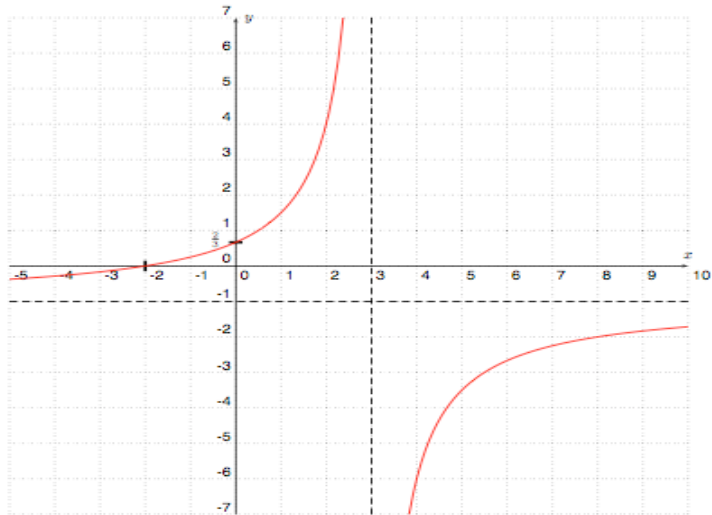


b) domain: $D = (-\infty, 3) \cup (3, \infty)$

vertical asymptote: $x = 3$

horizontal asymptote: $y = -1$

x-int: $(-2, 0)$, y-int: $\left(0, \frac{2}{3}\right)$

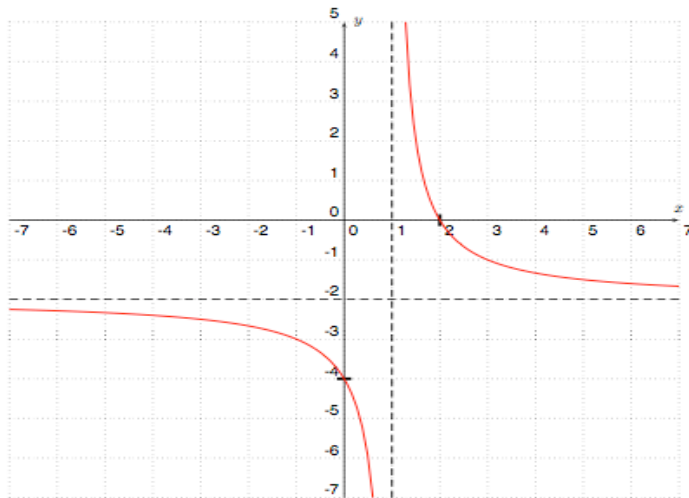


c) domain: $D = (-\infty, 1) \cup (1, \infty)$

vertical asymptote: $x = 1$

horizontal asymptote: $y = -2$

x-int: $(2, 0)$, y-int: $(0, -4)$



#4 a) $2x + h + 5$

b) $4x + 2h - 3$

c) $-4x - 2h + 4$

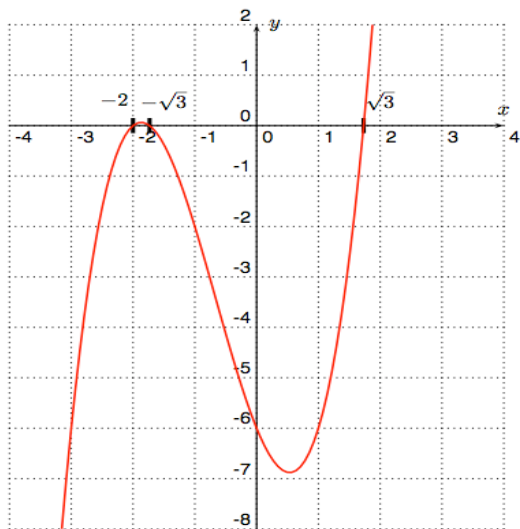
#5 a) roots: $x = -2$, $x = \sqrt{3}$, $x = -\sqrt{3}$

window: X-min: -4, X-max: 4,

Y-min: -8, Y-max: 2

max: $(x,y) \approx (-1.87, 0.06)$

min: $(x,y) \approx (0.54, -6.88)$



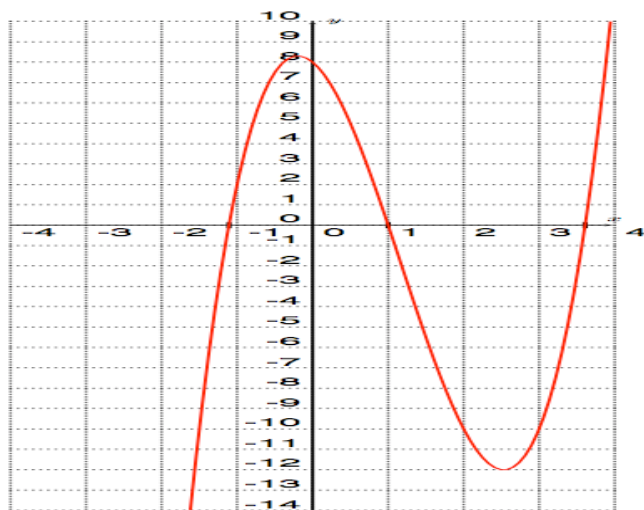
b) roots: $x = 1$, $x = \frac{5 + \sqrt{89}}{4}$, $x = \frac{5 - \sqrt{89}}{4}$

window: X-min: -4, X-max: 4,

Y-min: -14, Y-max: 10

max: $(x,y) \approx (-0.20, 8.30)$

min: $(x,y) \approx (2.53, -12.01)$



#6 a) $\|v\| = 4$, Direction Angle: $\theta = 300^\circ$

c) $\|v\| = 10$, Direction Angle: $\theta = 150^\circ$

b) $\|v\| = 3\sqrt{2}$, Direction Angle: $\theta = 225^\circ$

#7 a) $2(\cos 90^\circ + i \sin 90^\circ) = 2i$

b) $8(\cos 210^\circ + i \sin 210^\circ) = -4\sqrt{3} - 4i$

#8 a) $8u + \frac{4}{3}v$

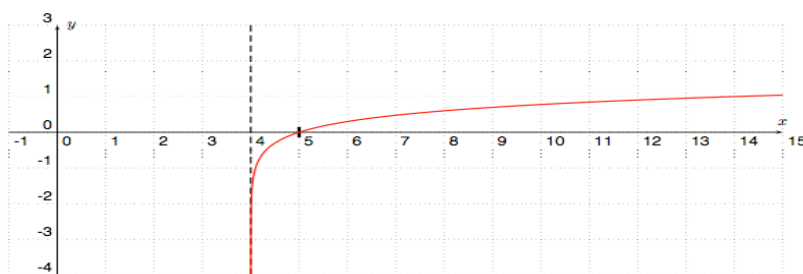
b) $\frac{5}{2}u - 3v$

c) $\frac{3}{2}u + \frac{1}{8}v$

#9 a) domain: $D = (4, \infty)$

vertical asymptote: $x = 4$

x-int: $(5, 0)$



b) domain: $D = (-\infty, 5)$

vertical asymptote: $x = 5$

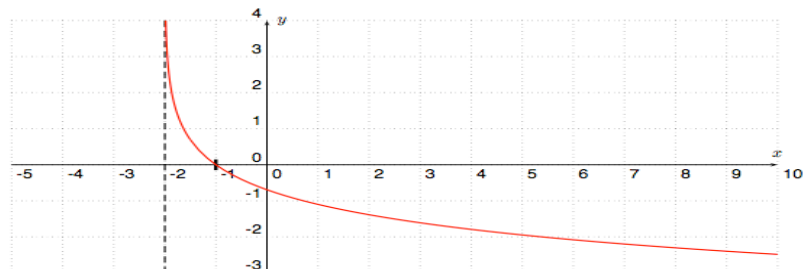
x-int: $(4, 0)$



c) domain: $D = (-2, \infty)$

vertical asymptote: $x = -2$

x-int: $(-1, 0)$

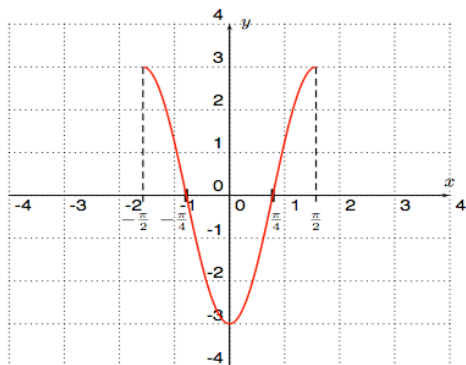


#10 a) amplitude: 3, period: π , phase shift: $-\frac{\pi}{2}$, max: $(\pm\frac{\pi}{2}, 3)$, min: $(0, -3)$, x-int: $(\pm\frac{\pi}{4}, 0)$

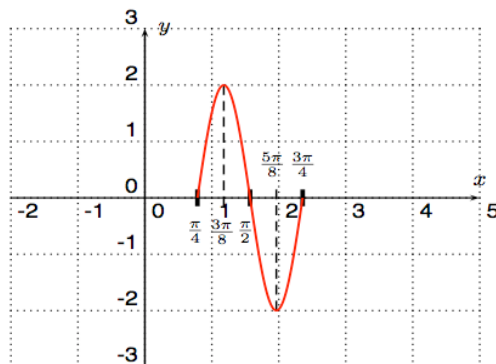
b) amplitude: 2, period: $\frac{\pi}{2}$, phase shift: $\frac{\pi}{4}$, max: $(\frac{3\pi}{8}, 2)$, min: $(\frac{5\pi}{8}, -2)$, x-int: $(\frac{\pi}{4}, 0), (\frac{\pi}{2}, 0), (\frac{3\pi}{4}, 0)$

c) amplitude: 4, period: 2π , phase shift: $\frac{\pi}{2}$, max: $(2\pi, 4)$, min: $(\pi, -4)$, x-int: $(\frac{\pi}{2}, 0), (\frac{3\pi}{2}, 0), (\frac{5\pi}{2}, 0)$

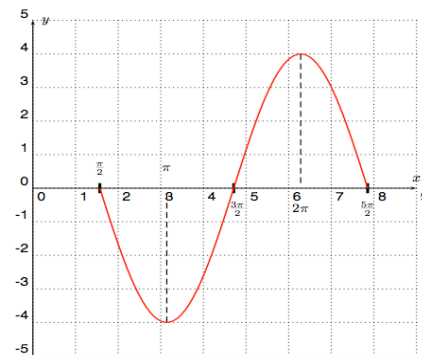
Graphs: a)



b)



c)



#11 a) $x = n\pi$ or $x = (-1)^n \frac{\pi}{6} + n\pi$, where $n = 0, \pm 1, \pm 2, \dots$

b) $x = \pm \frac{2\pi}{3} + 2n\pi$ or $x = 2n\pi$, where $n = 0, \pm 1, \pm 2, \dots$

c) $x = n\pi$ or $x = \frac{\pi}{4} + n\pi$, where $n = 0, \pm 1, \pm 2, \dots$

#12 a) The population will be approximately 9,272 people.

b) Sometime in the year 2043 half of the population will be left.

#13 a) The population will be approximately 128,083 people. b) It will be triple in the year 2031.

#14 a) $f^{-1}(x) = \frac{3-x}{4}$

b) $f^{-1}(x) = \frac{4}{x} + 3$

c) $f^{-1}(x) = \frac{1}{4x} - \frac{5}{8}$

d) $f^{-1}(x) = \frac{2x+1}{1-x}$

#15 a) -5705

b) 20710

c) 1811187

#16 a) $\frac{1}{3}$

b) $\frac{64}{3}$

c) 9

d) -81

#17 a) $256x^8 - 512x^7y + 448x^6y^2 - 224x^5y^3$

b) $\frac{7}{4}x^2y^6$