Factoring – Summary of the Method

I. ALWAYS start with the Greatest Common Factor (GCF):

example: $12x^2y^4 + 15xy^3 \rightarrow$ rewrite the terms in exponential form

$2^2 \bullet 3^1 \bullet x^2 \bullet y^4 + 3^1 \bullet 5^1 \bullet x^1 \bullet y^3$ the smallest exponent occuring on each common factor is part of the GCF so the GCF is $3^1 \bullet x^1 \bullet y^3$. Therefore, the answer is $3xy^3(4xy+5)$

II. Once the GCF is factored out (if there is one), whatever is leftover may be factored further. The amount of terms contained in the leftovers gives you an idea which method to use next.

A. **4 OR MORE TERMS**, then use **GROUPING** to factor.

example: $8x^2 + 20ax + 6x + 15a$ since there are no common factors with all 4 terms, group the terms in pairs and see if there are common factors. $(8x^2 + 20ax) + (6x + 15a)$ The first two terms have a 4x in common and the last two terms have a 3 in common thus: $4x(2x + 5a) + 3(2x + 5a)$. Now you have 2 terms instead of 4. They now have the $(2x + 5a)$ in common. So once you factor it out, you get $4x + 3$. So your answer is $(2x+5a)(4x+3)$.

B. **3 TERMS**, then it is a **TRINOMIAL**. To factor a trinomial, use one of the following techniques:

1.) Trial and error

example: $2x^2 - 7x - 4$ list the factors of 2 and 4 and through trial and error using reverse foil find out which combination of factors will give you a -7x as the middle term. Since there is only one way to factor 2, the first slots in each pair of parenthesis is going to be 2x and 1x

(2x)(1x). There are two ways to factor 4 → 1, 4 or 2, 2.

(2x 2)(1x 2) (2x 4)(1x 1) (2x 1)(1x 4)

2x 4x 1x By subtracting we are able to get -7x if we use 1x

4x 2x 8x and -8x therefore the answer is (2x+1)(1x-4)

adding or subtracting won't give -7x.

2.) AC-Method

example: $2x^2 - 7x - 4$ multiply the first and last numbers and then list the factors of the number to see which pair of factors adding or subtracting will give you -7. Multiplying 2 and 4 we get 8. The pairs of factors of 8 are 1 & 8 and 2 & 4. The only way to get -7 is to use the factors 1 & 8, particularly if you use the 1 and -8 to add to -7. Therefore, rewrite the original problem as $2x^2 + 1x - 8x - 4$. Now use grouping to factor x(2x+1) -4(2x+1) The answer is (2x+1)(x-4).

C. **2 TERMS**, then it is a **BINOMIAL**. To factor a binomial, use one of the special factoring techniques.

1.) The difference of two squares $F^2 - L^2 = (F - L)(F + L)$

example: $25x^2 - 81 \rightarrow$ rewrite it to fit the formula $(5x)^2 - (9)^2$ so our F=5x and L=9 thus it factors as (5x-9)(5x+9)

2.) The difference of two cubes $F^3 - L^3 = (F-L)(F^2 + FL + L^2)$

example: $27x^3 - 8 \rightarrow$ rewrite the formula $(3x)^3 - (2)^3$ so our F = 3x and L = 2 thus it factors as $(3x-2)\left[(3x)^2 + (3x)(2) + (2)^2\right]$ which can be simplified as $(3x-2)(9x^2 + 6x + 4)$

3.) The sum of two cubes $F^3 + L^3 = (F+L)(F^2 - FL + L^2)$

example: $x^3 + 1000y^3 \rightarrow$ rewrite it to fit the formula $(x)^3 + (10y)^3$ so our F =x and L =10y thus it factors as $(x+10y)[(x)^2 - (x)(10y) + (10y)^2]$ which can be simplified as $(x+10y)(x^2 - 10xy + 100y^2)$