Finding the Equation of a Circle

By definition, a **circle** is the set of all points *P* (x, y) whose distance from a center *C* (h, k) is the distance *r*. Thus *P* is considered a point on the circle if and only if the distance from *P* to *C* equals *r*. The general equation of a circle is $r^2 = (x-h)^2 + (y-k)^2$ where ***r* is the radius** and **the point (h, k) is the center** of the circle.

Example 1: Find the equation of a circle with radius=3 and a center (2, -5).

Step 1 **Begin with the general equation of a circle:**

$$r^2 = (x-h)^2 + (y-k)^2$$

Step 2 **Plug in the values in place of their corresponding variables:**

From the above example, *r* = 3, *h* = 2 ***and*** *k* = -5**.**
Therefore: $(3)^2 = (x-(2))^2 + (y-(-5))^2$

Step 3 **Simplify the equation:**

Simplify $(3)^2 = (x-(2))^2 + (y-(-5))^2$ **. . . *to get*** $9 = (x-2)^2 + (y+5)^2$

Example 2: Find the equation of a circle that has points *P* (1, 8) and *Q* (5, -6) as the endpoints of a diameter.

Step 1 **Use the midpoint formula,** $m = \frac{(x_1 + x_2)}{2}, \frac{(y_1 + y_2)}{2}$**, to find the center of the circle:**

a. Plug values into midpoint formula $m = \frac{(1+5)}{2}, \frac{(8+(-6))}{2}$

b. Simplify $m = \frac{6}{2}, \frac{2}{2}$ **. . . *to get*** $m = (3,1)$

The center between *P* and *Q* is *x* = 3 and *y* = 1, which is also the center coordinates, (*h*, *k*), of the circle.

Step 2 **Use the general equation of a circle, one of the points given and the coordinates for the center to find the value of *r*:**

a. General equation of a circle is $r = \sqrt{(x-h)^2 + (y-k)^2}$

b. Plug in values P (1, 8) and C (3, 1) ➔ $r = \sqrt{(1-3)^2 + (8-1)^2}$

c. Simplify ➔ $r = \sqrt{(-2)^2 + (7)^2}$ ➔ $r = \sqrt{4+49}$ ➔ $r = \sqrt{53}$

Thus, the **radius** of the circle is $\sqrt{53}$.

Step 3 **Plug in values into the general equation of a circle:**

a. General equation of a circle is $r^2 = (x-h)^2 + (y-k)^2$

b. Plug in values ➔ $(\sqrt{53})^2 = (x-3)^2 + (y-1)^2$

c. Simplify to get ➔ $53 = (x-3)^2 + (y-1)^2$

Example 3 Find the radius and center of a circle with equation $x^2 + y^2 + 2x - 6y + 7 = 0$.

Step 1 **Group all *x*-terms and *y*-terms together, and move all constants to right-hand side of the equal sign:**

a. The original equation is $x^2 + y^2 + 2x - 6y + 7 = 0$.

b. Group all x-terms and y-terms together ➔ $(x^2 + 2x) + (y^2 - 6y) + 7 = 0$

c. Move all constants to right-hand side of the equal sign ➔ $(x^2 + 2x) + (y^2 - 6y) = -7$

Step 2 **Complete the square for each group by adding the square of half the coefficient of the *x* and *y* variable to each respective group and adding the same amount to the right-hand side of the equal sign:**

a. The coefficient of x = 2 and of y = -6 ➔ $(x^2 + 2x + \left(\frac{2}{2}\right)^2) + (y^2 - 6y + \left(\frac{-6}{2}\right)^2) = -7 + \left(\frac{2}{2}\right)^2 + \left(\frac{-6}{2}\right)^2$

b. Simplify ➔ $(x^2 + 2x + (1)^2) + (y^2 - 6y + (-3)^2) = -7 + (1)^2 + (-3)^2$ ➔ $(x^2 + 2x + 1) + (y^2 - 6y + 9) = 3$

c. Factor the resulting perfect square trinomials and write them as squares of a binomial ➔ $(x+1)^2 + (y-3)^2 = 3$

Step 3 **Use the resulting equation to obtain the radius and coordinates for the center:**

From the equation, $(x+1)^2 + (y-3)^2 = 3$, the **center** is (-1, 3), and the **radius** is $\sqrt{3}$.

<u>Practice Exercises</u>

Find the equation of a circle.

1. Center (-1, -4); radius 8
2. Endpoints of a diameter are P (-1, 3) and Q (7, -5)

Given the equation of a circle, find the center and radius for each.

3. $x^2 + y^2 - 2x - 2y = 2$
4. $x^2 + y^2 + 6y + 2 = 0$

Answers:

1. $(x+1)^2 + (y+4)^2 = 64$ **2.** $(x-3)^2 + (y+1)^2 = 32$ **3.** Center (1, 1); radius 2 **4.** Center (0, -3); radius $\sqrt{7}$