MAT 1275/D506 Fall 2016
Professor Bonanome

NAME: Solutions

1. Simplify, and give your answer in simplest radical form. [5 points each]

(a) $(\sqrt{2x} - 3\sqrt{6})(5\sqrt{3x} + 10\sqrt{6})$

$$= 5\sqrt{6}x^2 - 15\sqrt{18}x + 10\sqrt{12}x - 30\sqrt{6}^2$$

$$= 5x\sqrt{6} - 45\sqrt{2}x + 20\sqrt{3}x - 180$$

(b) $\dfrac{4x\sqrt{2}}{3\sqrt{10}} \cdot \dfrac{\sqrt{10}}{\sqrt{10}} = \dfrac{4x\sqrt{20}}{30} = \dfrac{8x\sqrt{5}}{30} = \boxed{\dfrac{4x\sqrt{5}}{15}}$

(c) $\dfrac{2}{\sqrt{3}+\sqrt{7}} \cdot \dfrac{(\sqrt{3}-\sqrt{7})}{(\sqrt{3}-\sqrt{7})} = \dfrac{2\sqrt{3}-2\sqrt{7}}{3-7} = \dfrac{2\sqrt{3}-2\sqrt{7}}{-4}$

$$= \boxed{\dfrac{\sqrt{3}-\sqrt{7}}{-2}} \text{ or } -\dfrac{(\sqrt{3}-\sqrt{7})}{2} = \boxed{\dfrac{\sqrt{7}-\sqrt{3}}{2}}$$

Check

$$\sqrt{2\cdot3+19} \overset{?}{=} 3+2$$
$$\sqrt{6+19} \overset{?}{=} 5$$
$$\sqrt{25} \overset{?}{=} 5 \checkmark$$
$$5=5 \checkmark$$

$$\sqrt{(-5)\cdot2+19} \overset{?}{=} -5+2$$
$$\sqrt{-10+19} \overset{?}{=} -3$$
$$\sqrt{9} \overset{?}{=} -3$$
$$3 \neq -3$$

2. Solve and check [15 points]: $\sqrt{2x+19} = x+2$

$$\left(\sqrt{2x+19}\right)^2 = (x+2)^2$$
$$2x+19 = x^2+4x+4$$
$$-2x-19 \quad\quad -2x-19$$
$$x^2+2x-15 = 0$$
$$(x-3)(x+5) = 0$$

$$x-3=0 \quad\Big|\quad x+5=0$$
$$x=3 \quad\Big|\quad x=\cancel{-5} \text{ reject}$$

3. Simplify, write your answer in $a+bi$ form. [10 points]

$$5\sqrt{-49}\sqrt{-50}$$
$$= 5\cdot7i\cdot5i\sqrt{2}$$
$$= 175i^2\sqrt{2}$$
$$= \boxed{-175\sqrt{2} + 0i}$$

4. Simplify, write your answer in $a+bi$ form. [5 points each]

(a) $(3-7i) - (5+2i)$

$$(3-5) + (-7-2)i$$
$$= \boxed{-2-9i}$$

(b) $(4-i)(7+5i)$

$$= 28 + 20i - 7i - 5i^2$$

$$= 28 + 13i + 5$$

$$\boxed{= 33 + 13i}$$

(c) $\dfrac{6+i}{2-3i} \quad \dfrac{(2+3i)}{(2+3i)}$

$$= \dfrac{12 + 18i + 2i + 3i^2}{4 + 6i - 6i - 9i^2}$$

$$\boxed{= \dfrac{9 + 20i}{13}}$$

$$\boxed{= \dfrac{9}{13} + \dfrac{20}{13}i}$$

5. Solve by factoring [15 points]: $x^2 - 14 = 5x$

$$x^2 - 5x - 14 = 0$$

$$(x+2)(x-7) = 0$$

$$x + 2 = 0 \qquad x - 7 = 0$$

$$\boxed{x = -2} \qquad \boxed{x = 7}$$

Check!

$(-2)^2 - 14 \overset{?}{=} 5(-2)$

$4 - 14 \overset{?}{=} -10$

$-10 = -10$ ✓

$(7)^2 - 14 \overset{?}{=} 5 \cdot 7$

$49 - 14 \overset{?}{=} 35$

$35 = 35$ ✓

* Begin

6. Solve by completing the square [15 points]: $3x^2 + 6 = 24x$

imagine This

$$\frac{3x^2}{3} - \frac{24x}{3} = -\frac{6}{3}$$

$$x^2 - 8x = -2$$

$$(x^2 - 8x + 16) = -2 + 16$$

$$(x - 4)^2 = 14$$

$$x - 4 = \pm\sqrt{14}$$

$$x - 4 = +\sqrt{14} \qquad x - 4 = -\sqrt{14}$$

$$\boxed{x = 4 + \sqrt{14}} \qquad \boxed{x = 4 - \sqrt{14}}$$

	x	-4
x	x^2	$-4x$
-4	$-4x$	$+16$

7. Solve using the quadratic formula [15 points]: $2x(x+4) = -5$

$$2x^2 + 8x + 5 = 0 \qquad a = 2 \quad b = 8 \quad c = 5$$

$$x = \frac{-8 \pm \sqrt{8^2 - 4(2)(5)}}{2(2)} = \frac{-8 \pm \sqrt{64 - 40}}{4} = \frac{-8 \pm \sqrt{24}}{4}$$

$$= \frac{-8 \pm 2\sqrt{6}}{4} = \frac{-4 \pm \sqrt{6}}{2}$$

$$\boxed{= \frac{-4 \pm \sqrt{6}}{2}}$$

8. Extra Credit [15 points] The width of a rectangle is 5 inches less than 3 times the length. The area is 2 square inches. Find the length and the width of the rectangle.

$$\boxed{A = 2 \text{ in}^2} \qquad \ell$$

$w = 3\ell - 5$

$(3\ell - 5)\ell = 2$

$3\ell^2 - 5\ell - 2 = 0$

$a = 3 \quad b = -5 \quad c = -2$

$$x = \frac{5 \pm \sqrt{49}}{6} \rightarrow x = 2$$

$$\searrow x = -\frac{3}{6} = \frac{1}{3} \text{ reject}$$

$$\boxed{\ell = 2 \text{ inches}}$$

$$\boxed{w = 3\ell - 5 = 3 \cdot 2 - 5 = 1 \text{ inch}}$$