MAT 1275/D506 Fall 2016
Professor Bonanome

NAME: $Solutions$

1. Simplify, and give your answer in simplest radical form. [5 points each]

(a) $(\sqrt{5x} - 4\sqrt{10})(2\sqrt{2x} + 6\sqrt{10})$

$= 2\sqrt{10}x + 6\sqrt{50}x - 8\sqrt{20}x - 24\sqrt{100}$

$= 2\sqrt{10x^2} + 30\sqrt{2x} - 16\sqrt{5x} - 240$

$= 2x\sqrt{10} + 30\sqrt{2x} - 16\sqrt{5x} - 240$

(b) $\dfrac{2\sqrt{3}}{5x\sqrt{6}} \cdot \dfrac{\sqrt{6}}{\sqrt{6}} = \dfrac{2\sqrt{18}}{30x} = \dfrac{6\sqrt{2}}{30x} = \boxed{\dfrac{\sqrt{2}}{5x}}$

(c) $\dfrac{3}{\sqrt{5}-\sqrt{2}} \cdot \dfrac{(\sqrt{5}+\sqrt{2})}{(\sqrt{5}+\sqrt{2})} = \dfrac{3\sqrt{5}+3\sqrt{2}}{5-2} = \dfrac{3\sqrt{5}+3\sqrt{2}}{3}$

$= \boxed{\sqrt{5}+\sqrt{2}}$

$$\sqrt{-7+23} \overset{?}{=} -7+3$$
$$\sqrt{16} \overset{?}{=} -4$$
no

$$\sqrt{2+23} \overset{?}{=} 2+3$$
$$\sqrt{25} \overset{?}{=} 5$$
$$\sqrt{5=5} ✓$$

2. Solve and check: [15 points] $\sqrt{a+23} = a+3$

$$(\sqrt{a+23})^2 = (a+3)^2$$
$$a+23 = a^2 + 6a + 9$$
$$\underline{-a-23} \qquad -a-23$$
$$a^2 + 5a - 14 = 0$$
$$(a+7)(a-2) = 0$$
$$a+7=0 \qquad a-2=0$$
$$a=\cancel{-7} \qquad \boxed{a=2}$$
(r)eject

3. Simplify, write your answer in $a+bi$ form. [10 points]

$$3\sqrt{-20}\sqrt{-16}$$

$$3\cdot 2i\sqrt{5}\cdot 4i = \boxed{-24\sqrt{5} + 0i}$$

4. Simplify, write your answer in $a + bi$ form. [5 points each]

(a) $(-2+6i) - (7-4i)$

$$= (-2-7) + ((6-(-4)))\,i$$

$$= \boxed{-9+10i}$$

(b) $(5 + 3i)(-2 + i)$

$= -10 - 6i + 5i + 3i^2$

$= \boxed{-13 - i}$

(c) $\dfrac{4 + 2i}{5 - 4i}$ $\quad \dfrac{(5 + 4i)}{(5 + 4i)}$ $\quad = \quad \dfrac{20 + 16i + 10i + 8i^2}{25 - 16i^2}$

$= \quad \dfrac{20 - 8 + 26i}{25 + 16} \quad = \quad \dfrac{12 + 26i}{41}$

$= \boxed{\dfrac{12}{41} + \dfrac{26}{41}i}$

5. Solve by factoring [15 points] (you **must factor** when solving in order to receive credit for this problem): $x^2 + 15 = 8x$

$x^2 - 8x + 15 = 0$

$(x - 3)(x - 5) = 0$

$x - 3 = 0 \qquad x - 5 = 0$

$\boxed{x = 3} \qquad \boxed{x = 5}$

Check:
$3^2 + 15 \overset{?}{=} 8 \cdot 3$

$9 + 15 \overset{?}{=} 24$

$24 = 24 \checkmark$

$5^2 + 15 \overset{?}{=} 8 \cdot 5$

$25 + 15 \overset{?}{=} 40$

$40 = 40 \checkmark$

6. Solve by completing the square [15 points] (you **must complete the square** when solving in order to receive credit for this problem): $2x^2 + 6 = 12x$

$$-6 \quad -6$$
$$-12x \quad -12x$$

$$\frac{2x^2 - 12x}{2} = \frac{-6}{2}$$

$$x^2 - 6x = -3$$

$$x^2 - 6x + 9 = -3 + 9$$

$$(x-3)^2 = 6$$

$$x - 3 = \pm\sqrt{6} \qquad x - 3 = -\sqrt{6}$$

$$\boxed{x = 3 + \sqrt{6}} \qquad \boxed{x = 3 - \sqrt{6}}$$

7. Solve using the quadratic formula [15 points] (you **must use the quadratic formula** when solving in order to receive credit for this problem): $2x(x+3) = -3$

$$2x^2 + 6x + 3 = 0$$
$$a = 2 \quad b = 6 \quad c = 3$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-6 \pm \sqrt{(6)^2 - 4(2)(3)}}{2(2)} = \frac{-6 \pm \sqrt{36 - 24}}{4} = \frac{-6 \pm \sqrt{12}}{4} = \frac{-6 \pm 2\sqrt{3}}{4}$$

$$\boxed{= \frac{-3 \pm \sqrt{3}}{2}}$$

8. **Extra Credit** [15 points] The width of a rectangle is 5 inches less than 3 times the length. The area is 2 square inches. Find the length and the width of the rectangle.

$$\boxed{A = 2 \text{ in}^2} \qquad \ell$$

$$W = 3\ell - 5$$

$$(3\ell - 5)\ell = 2$$

$$3\ell^2 - 5\ell - 2 = 0$$
$$a = 3 \quad b = -5 \quad c = -2$$

$$= \frac{-(-5) \pm \sqrt{(-5)^2 - 4(3)(-2)}}{2(3)}$$

$$= \frac{5 \pm \sqrt{25 + 24}}{6} = \frac{5 \pm \sqrt{49}}{6}$$

$$= \frac{5 \pm 7}{6} \longrightarrow \frac{5+7}{6} = \frac{12}{6} = 2$$

$$\frac{5-7}{6} = -\frac{2}{6} = -\frac{1}{3} \quad \text{reject}$$

$$\boxed{\ell = 2 \text{ inches}}$$

$$W = 3\ell - 5 = 3 \cdot 2 - 5 = \boxed{1 \text{ inch}}$$