## TRIGONOMETRIC IDENTITIES

Recall that:

| $\sin x$   | $\cos x$   | 1          | 1          |
|------------|------------|------------|------------|
| $\tan x =$ | $\cot x =$ | $\sec x =$ | $\csc x =$ |
| $\cos x$   | $\sin x$   | $\cos x$   | $\sin x$   |

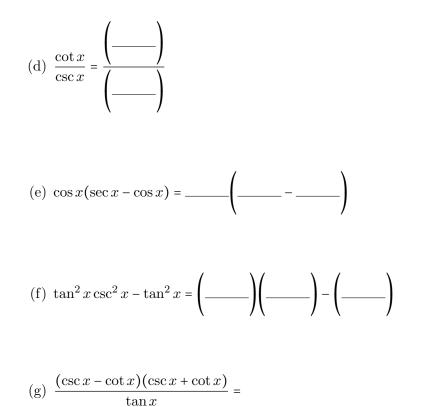
(1) **Rewrite** the following trigonometric functions in terms of  $\cos x$  and  $\sin x$  only. Do not perform any algebraic simplification yet.

| <b>Example:</b> The function $\cot x(\sec x + \sin x)$ can be rewritten as | $\left(\frac{1}{+\sin x}\right)$ . |
|--|------------------------------------|
|  | $\left( \cos x \right)$            |

(b) 
$$1 + \tan^2 x = 1 + \left( \underbrace{-----}_{\text{and } \tan^2 x} = (\tan x)(\tan x) = (\tan x)^2$$
  
and  $\tan^2 x \neq \tan(x^2)$ ]

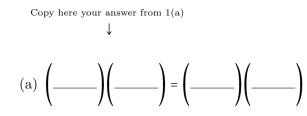
[Note: don't forget to add the

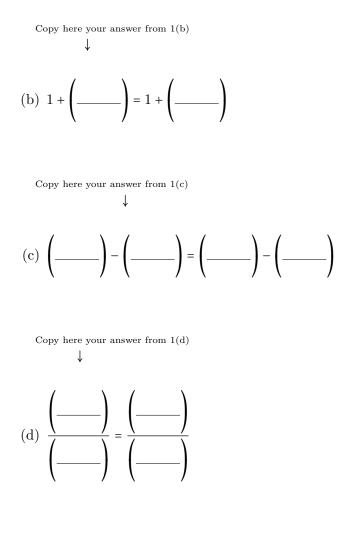
(c) 
$$\csc^2 x - \cot^2 x = \left( \underbrace{----} \right) - \left( \underbrace{-----} \right)$$
  
"squares"]

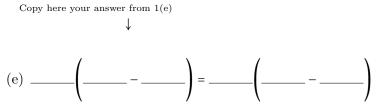


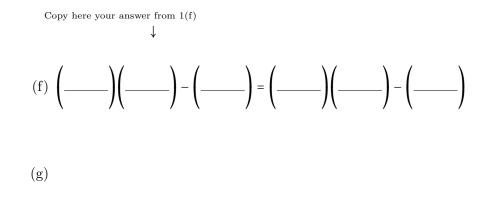
- $\tan x$
- (2) For each function in question (1), substitute each  $\cos x$  by a and each  $\sin x$  by b. Do not perform any algebraic simplification yet.

| Example (cont'd): The function |  | $\left(\frac{1}{\cos x} + \sin x\right)$ | can be rewritten as $\frac{a}{b}$ | $\left(\frac{1}{a}+\frac{b}{a}\right).$ |  |
|--------------------------------|--|--|-----------------------------------|---|--|
|--------------------------------|--|--|-----------------------------------|---|--|



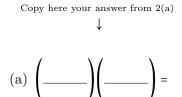


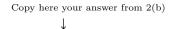


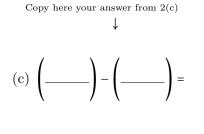


(3) **Simplify** each expression in question (2) algebraically.

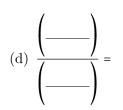
**Example (cont'd):** The expression  $\frac{a}{b}\left(\frac{1}{a}+b\right)$  can be rewritten as  $\frac{a}{b}\left(\frac{1}{a}+\frac{ab}{a}\right) = \frac{a}{b}\left(\frac{1+ab}{a}\right) = \frac{a(1+ab)}{ba} = \frac{1+ab}{b}.$ Notice that the simplification led to a single fraction.

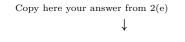


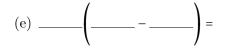


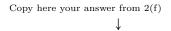


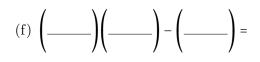
Copy here your answer from 2(d)  $\downarrow$ 











(g)

Now we can finally start proving trigonometric identities. Basically, we will put together the three procedures we have just practiced: **rewrite**, **substitute**, and **simplify**.

**Step 1: Rewrite** the identity in terms of  $\cos x$  and  $\sin x$  only. **Step 2: Substitute** each  $\cos x$  by a and each  $\sin x$  by b. **Step 3: Simplify** each side **separately**. You are done when the LHS is equal to the RHS.

**Remark:** sometimes, in order to show that the two sides are equal, we have to use the fundamental identity:  $\cos^2 x + \sin^2 x = 1.$ 

which can be rewritten as

which can be rewritten as  $(\cos x)^2 + (\sin x)^2 = 1$ , or  $a^2 + b^2 = 1$ . So, whenever you see  $a^2 + b^2$ , **remember** to replace it by 1. Things will be simpler!

**Example:** Show that  $\cos x + \sin x \cdot \tan x = \sec x$ . Solution:  $\cos x + \sin x \cdot \tan x = \sec x$  $\cos x + \sin x \cdot \frac{\sin x}{\cos x} = \frac{1}{\cos x}$ Step 1: rewrite  $a+b\cdot\frac{b}{a} = \frac{1}{a}$ Step 2: substitute  $a + \frac{b^2}{a} = \frac{1}{a}$ Step 3: simplify turn the LHF into a single fraction  $\frac{a \cdot a}{a} + \frac{b^2}{a} = \frac{1}{a}$ Don't move the terms from one side to the other  $\frac{a^2}{a} + \frac{b^2}{a} = \frac{1}{a}$  $\frac{a^2+b^2}{a} = \frac{1}{a}$ **Remember:**  $a^2 + b^2 = 1$  $\frac{1}{a} = \frac{1}{a}$ Done! ©  $\checkmark$ 

 $\mathbf{6}$ 

(1) Show that:  
(a) 
$$\sin \theta \cot \theta = \cos \theta$$
  
(b)  $\sec^2 \theta \cot^2 \theta = \csc^2 \theta$   
(c)  $\cos \theta (\sec \theta - \cos \theta) = \sin^2 \theta$   
(d)  $\sin \theta (\csc \theta - \sin \theta) = \cos^2 \theta$   
(e)  $\tan \theta (\csc \theta + \cot \theta) = \sec \theta + 1$   
(f)  $\tan^2 \theta \csc^2 \theta - \tan^2 \theta = 1$   
(g)  $\frac{\sin \theta \cos \theta + \sin \theta}{\cos \theta + \cos^2 \theta} = \tan \theta$   
(h)  $\frac{1 + \sin \theta}{\cos \theta + \cos \theta \sin \theta} = \sec \theta$   
(i)  $\frac{(\sin \theta + \cos \theta)^2}{\cos \theta} = \sec \theta + 2\sin \theta$   
(j)  $(1 + \sin \theta)(1 - \sin \theta) = \cos^2 \theta$   
(k)  $\frac{\cos^2 \theta}{\sin \theta} + \sin \theta = \csc \theta$   
(l)  $\frac{\tan \theta}{\csc \theta} - \frac{\sin \theta}{\cos \theta} = \frac{\sin \theta - 1}{\cot \theta}$   
(m)  $\cos^2 \tan^2 x = 1 - \cos^2 x$   
(n)  $\tan x + \cot x = \sec x \csc x$   
(o)  $\frac{\cos x}{\tan x} = \csc x - \sin x$ 

(p) 
$$\frac{\cos \theta}{1 - \sin \theta} = \sec \theta + \tan \theta$$