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A RESOURCE MANUAL FOR COLLEGE ALGEBRA

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Contents

Dedicated to all the determined students
who learn, no matter what, and the brave
teachers who endeavor to teach the students
in front of them.

Introduction

Write a little introduction here

Rational Expressions

Adding and Subracting Rational Expressions

Each item in the lists below describes either a video (marked by the symbol ▷) or an exercise (marked by the symbol ⋆. After the title of the video, the length of the video is given in parentheses.

Total video time:

Total number of problems:

1. ▷ Adding and Subtracting rational expressions with like denominators (3:45)

   Example 1
   $$\frac{6}{2x^2-7}+\frac{-3x-8}{2x^2-7}=\frac{-2-3x}{2x^2-7}$$
   Example 2
   $$\frac{9x^2+3}{14x^2-9}-\frac{-3x^2+5}{14x^2-9}=\frac{12x^2-2}{14x^2-9}$$
2. ⋆ Practice adding and subtracting rational expressions with like denominators

   $$\frac{k^3}{k+5}+\frac{7k^3+3}{k+3}=\frac{8k^3+3}{k+3}$$
3. ▷ Intro to adding rational expressions with unlike denominators (2:43)

   $$\frac{a}{b}+\frac{c}{d}$$

   Note: this video uses proper notation, such as $\frac{a}{b}\cdot\frac{d}{d}$.
4. ▷ Adding rational expressions: unlike denominators (5:11)

   $$\frac{5x}{2x-3}+\frac{-4x^2}{3x+1}=\frac{-8x^3+27x^2+5x}{(2x-3)(3x+1)}$$

5. ▷ Subtracting rational expressions: unlike denominators (4:47)

$$\frac{-5x}{8x+7} - \frac{6x^3}{3x+1} = \frac{-48x^4 - 42x^3 - 15x^2 - 5x}{(8x+7)(3x+1)}$$

6. ⋆ Practice adding rational expressions with unlike denominators

$$\frac{2k}{k-6} + \frac{k^3}{k+9}$$

7. ▷ Finding the least common multiple of two integers (4:15)

   Example 1 Find the *lcm* of 36 and 12.

   Example 2 Find the *lcm* of 12 and 18 (uses the prime factorization method).

8. ▷ Finding the least common multiple with repeating factors (2:34)

   Find the $lcm(30, 25)$

9. ⋆ Find the $lcm(7, 5)$

10. ▷ Finding the least common multiple of polynomials (6:51) Finds the least common multiple of $3z^3 - 6z^2 - 9z$ and $7z^4 + 21x^3 + 14z^2$.

11. ⋆ Add the rational expressions

$$\frac{4}{(8-k)(k+5)} + \frac{3k^2}{(6k+1)(k+5)}$$

12. ▷ Subtracting rational expressions (4:36)

$$\frac{a-2}{a+2} - \frac{a-3}{a^2+4a+4}$$

13. ⋆ Combine the rational expressions.

$$\frac{2k}{3k+15} + \frac{k^2}{6k^2+60k+150}$$

Videos and exercises for Adding and Subtracting Rational Expressions

1. ▷ Adding and Subtracting rational expressions with like denominators (3:45) https://www.khanacademy.org/math/algebra2/rational-expressions-equations-and-functions/adding-and-subtracting-rational-expressions/v/adding-and-subtracting-rational-expressions-with-like-denominators

2. ⋆ Practice adding and subtracting rational expressions with like denominators

   https://www.khanacademy.org/math/algebra2/rational-expressions-equations-and-functions/adding-and-subtracting-rational-expressions/e/adding_and_subtracting_rational_expressions

3. ▷ Adding rational expressions with unlike denominators (2:43)
https://www.khanacademy.org/math/algebra2/rational-expressions-equations-and-functions/adding-and-subtracting-rational-expressions/v/algebraic-expression-adding-fractions

4. ⋆ Practice adding rational expressions with unlike denominators
https://www.khanacademy.org/math/algebra2/rational-expressions-equations-and-functions/adding-and-subtracting-rational-expressions/e/adding_and_subtracting_rational_expressions_2

5. ▷ Adding rational expressions: unlike denominators (5:11) https://www.khanacademy.org/math/algebra2/rational-expressions-equations-and-functions/adding-and-subtracting-rational-expressions/v/adding-rational-expression-w-unlike-denominators

6. ▷ Subtracting rational expressions: unlike denominators (4:47)
https://www.khanacademy.org/math/algebra2/rational-expressions-equations-and-functions/adding-and-subtracting-rational-expressions/v/subtracting-rational-expressions-w-unlike-denominators

7. ⋆ Practice adding rational expressions with unlike denominators
https://www.khanacademy.org/math/algebra2/rational-expressions-equations-and-functions/adding-and-subtracting-rational-expressions/e/adding_and_subtracting_rational_expressions_2

8. ▷ Finding the least common multiple of two integers (4:15) https://www.khanacademy.org/math/algebra2/rational-expressions-equations-and-functions/adding-and-subtracting-rational-expressions/v/least-common-multiple-exercise

9. ▷ Finding the least common multiple with repeating factors (2:34)
https://www.khanacademy.org/math/algebra2/rational-expressions-equations-and-functions/adding-and-subtracting-rational-expressions/v/least-common-multiple-exercise

10. ⋆ Find the least common multiple of two integers https://www.khanacademy.org/math/algebra2/rational-expressions-equations-and-functions/adding-and-subtracting-rational-expressions/e/least_common_multiple

11. ▷ Finding the least common multiple of polynomials (6:51)
https://www.khanacademy.org/math/algebra2/rational-expressions-equations-and-functions/adding-and-subtracting-rational-expressions/v/least-common-multiples-of-polynomials

12. ⋆ Add two rational expressions https://www.khanacademy.org/math/algebra2/rational-expressions-equations-and-functions/adding-and-subtracting-rational-expressions/e/adding_and_subtracting_rational_expressions_3

13. ▷ Subtracting rational expressions (4:36) https://www.khanacademy.org/math/algebra2/rational-expressions-equations-and-functions/adding-and-subtracting-rational-expressions/v/adding-and-subtracting-rational-expressions-3

14. ⋆ Combine the rational expressions. https://www.khanacademy.org/math/algebra2/rational-expressions-equations-and-functions/adding-and-subtracting-rational-expressions/e/adding_and_subtracting_rational_expressions_5

Simplifying Rational Expressions (For remediation)

This sequence of videos uses many different variables and works out the problems in detail (yea!), though perhaps doesn't include enough $\pm$ signs.

1. ▷ Introduction to simplification of rational expressions (15:22)

   These videos include a proper treatment of excluded values. (Hooray!)

   https://www.khanacademy.org/math/algebra2/rational-expressions-equations-and-functions/simplify-rational-expressions/v/simplifying-rational-expressions-introduction

   Example 1
   $$\frac{3}{6}$$
   Example 2
   $$\frac{8}{24}$$
   Example 3
   $$\frac{9x+3}{12x+4}$$
   Example 4
   $$\frac{x^2-9}{5x+15}$$
   Example 5
   $$\frac{x^2+6x+5}{x^2-x-2}$$
   Example 6
   $$\frac{3x^2+3x-18}{2x^2+5x-3}$$
   (factoring by grouping)

2. ▷ Simplify rational expressions with common monomial factors (0:55) https://www.khanacademy.org/math/algebra2/rational-expressions-equations-and-functions/simplify-rational-expressions/v/simplifying-rational-expressions-w-common-monomial-factors

$$\frac{28b^6}{7b} = 4b^5$$

3. ⋆ Simplify the rational expression with common monomial factors. Which values of n make the expression undefined?

   https://www.khanacademy.org/math/algebra2/rational-expressions-equations-and-functions/simplify-rational-expressions/e/simplifying_rational_expressions_1

$$\frac{3n^4 + 6n^3}{6n^4 + 9n^3}$$

4. ▷ Simplifying rational expressions with common binomial factors. (4:11)

   https://www.khanacademy.org/math/algebra2/rational-expressions-equations-and-functions/simplify-rational-expressions/v/simplifying-rational-expressions-1

   Given a rectangle with width $w = z^2 - 9$ and length $l = z^2 + 6a - 27$, find the ratio of the width to the length of the rectangle.

5. ▷ Simpifying Rational Expressions: opposite (sign) common binomial factors $x - a$ and $a - x$.

   https://www.khanacademy.org/math/algebra2/rational-expressions-equations-and-functions/simplify-rational-expressions/v/simplifying-rational-expressions-3

$$\frac{x^2 - 36}{6 - x} = -(x + 6)$$

   What is the domain of the simplified expression?

6. ⋆ Simplifying rational expressions with common binomial factors

   https://www.khanacademy.org/math/algebra2/rational-expressions-equations-and-functions/simplify-rational-expressions/e/simplifying_rational_expressions_2

$$\frac{2z^2 - 288}{z^2 - 9z - 36}$$

7. ▷ Simplifying Rational Expressions: Advanced Factorization (Grouping) (6:41)

   https://www.khanacademy.org/math/algebra2/rational-expressions-equations-and-functions/simplify-rational-expressions/v/simplifying-rational-expressions-2

$$\frac{2x^2 + 13x + 20}{2x^2 + 17x + 30} = \frac{(x + 4)(2x + 5)}{(x + 6)(2x + 5)}$$

   What is the domain of the resulting expression?

8. ⋆ Simplify the rational expression

$$\frac{5k^2 - 26k + 5}{25k^3 - k}$$

https://www.khanacademy.org/math/algebra2/rational-expressions-equations-and-functions/simplify-rational-expressions/e/simplifying_rational_expressions_3

Multiplying and Dividing Rational Expressions (for remediation)

In this section Sal encourages them to pause the videos, to think about restrictions needed for expressions to really be equivalent.

1. ▷ Monomials (5:55) This video uses function notation $f(x)$ and analyzes the expressions as functions.

   Example 1

   $$\frac{6x^3}{5} \cdot \frac{2}{3x}$$

   Example 2

   $$\frac{2x^4}{7} \div \frac{5x^4}{4}$$

2. ⋆ (basic) Mulitply the following monomials and determined if the result is defined when $b = 0$.

   $$\frac{-5b^3}{6} \cdot \frac{3b^2}{-10}$$

3. ▷ Multiply rational expressions (4:51)

   $$\frac{a^2 - 4}{a^2 - 1} \cdot \frac{a - 2}{a - 1,}$$

   noting that $a \neq -2, -1$.

4. ▷ Dividing rational expressions (4:09)

   $$\frac{2p + 6}{p + 5} \div \frac{10}{4p + 20} = \frac{5(p + 3)}{4}$$

   where $p \neq -5$.

5. ⋆

   $$\frac{4z^2 + 24z}{3z^2 - 9z - 12} \cdot \frac{z^2 - 4z - 5}{z - 4}$$

6. ▷ Multiply and expression as a simplified rational expression. State the domain. (3:37)

   $$\frac{3x^2y}{2ab} \frac{14a^2b}{18xy^2} = \frac{7ax}{6y}.$$

   Note that $a, b, x, y \neq 0$.

7. Divide the rational expressions and simplify.

$$\frac{10n^2+23n-5}{4n^2+6n} \div \frac{25n^2-10n+1}{7n+3}$$

Nested (or complex) Fractions

Note that there is only one video and one exercise on this topic, so more supplementary information will likely be necessary.

1. ▷ Nested Fractions (5:45)

   Example 1

$$\frac{a}{b} \cdot \frac{c}{d}$$

   Example 2

$$\frac{a}{b} \div \frac{c}{d}$$

   Example 3

$$\frac{\frac{1}{a}-\frac{1}{b}}{c} \div \frac{1}{d} = \frac{b-a}{ab} \cdot \frac{1}{c} \cdot \frac{d}{1} = \frac{db-da}{abc}$$

2. ⋆ Simplify the nested fraction: (which expression is equivalent to)

$$\frac{3+\frac{3}{a}}{\frac{3}{a}}$$

1. ▷ Nested Fractions (5:45) https://www.khanacademy.org/math/algebra2/rational-expressions-equations-and-functions/nested-fractions/v/algebraic-expressions-with-fraction-division

2. ⋆ Nested fraction practice https://www.khanacademy.org/math/algebra2/rational-expressions-equations-and-functions/nested-fractions/e/nested-fractions

Solving Rational Equations

1. ▷ Equations with one rational expression (3:11)

$$\frac{14x+4}{-3x-2} = 8.$$

   Solution: $x = -\frac{-10}{19}$

2. ⋆ Solve for r:

$$\frac{4r-6}{r-7} = \frac{1}{2}$$

3. ▷ Solving equations with one rational expression (advanced)
   Solve:
   $$x^2 - \frac{x^2 - 4}{x - 2} = 4$$
   with $x \neq 2$.

   This produces the equation $x^2 - x - 6 = 0$, so $x = 3$ or $x = -2$. Then he correctly verifies — doesn't assume the conclusion! (Says: $f(-2) = 4$ after defining $f(x)$ to be the given rational expression.)

4. ⋆ Solve
   $$\frac{-3k - 38}{k^2 - 16} = -1$$

5. ▷ Equations with two rational expressions (4:16)

   Solve and find excluded values ($p \neq 1, -3$).
   $$\frac{4}{p - 1} = \frac{5}{p + 3}$$

   Solution: $p = 17$, verifies solution.

6. ▷ Equations with two rational expressions (by finding the least common multiple) (4:07)
   $$\frac{5}{2x} - \frac{4}{3x} = \frac{7}{18}$$
   (note that $x \neq 0$. Solution: $x = 3$

7. ▷ Rational equations with extraneous solutions (3:02)

   Solve:
   $$\frac{x^2}{x + 2} = \frac{4}{x + 2}$$
   . The values $x = -2$ and $x = 2$ satisfy the resulting equation, but $x = -2$ is extraneous.

8. ⋆ Equations with two rational expressions. Solve:
   $$\frac{k + 5}{k^2 - 5k + 6} = \frac{k - 9}{k - 2}$$

Videos and exercises for Solving Rational Equations

1. ▷ Equations with one rational expression (3:11)

   Solution: $x = -\frac{-10}{19}$

2. ⋆ Solve for r:
   $$\frac{4r - 6}{r - 7} = \frac{1}{2}$$

3. ▷ Solving equations with one rational expression (advanced)
   Solve:
   $$x^2 - \frac{x^2 - 4}{x - 2} = 4$$
   with $x \neq 2$.

   This produces the equation $x^2 - x - 6 = 0$, so $x = 3$ or $x = -2$. Then he correctly verifies — doesn't assume the conclusion! (Says: $f(-2) = 4$ after defining $f(x)$ to be the given rational expression.)

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   (note that $x \neq 0$. Solution: $x = 3$

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   $$\frac{x^2}{x + 2} = \frac{4}{x + 2}$$
   . The values $x = -2$ and $x = 2$ satisfy the resulting equation, but $x = -2$ is extraneous.

8. ⋆ Equations with two rational expressions. Solve:
   $$\frac{k + 5}{k^2 - 5k + 6} = \frac{k - 9}{k - 2}$$

Additonal Related Topics

Below are some topics that fall outside the scope of the existing City Tech MAT 1275 syllabus, but on which there exist Khan Academy videos that are likely useful to many students.

- Direct and Inverse variation
- End behavior of rational functions

- Discontinuities of rational functions
- Graphs of rational functions
- Modeling with rational functions

Expressions with Rational Exponents and Radicals

To begin, we will present a sequence of videos from an introductory algebra course. There are further videos available on Khan Academy. As this material is covered in MAT 1175 (or elsewhere in remedial or high school courses), we expect that this material will be covered more quickly and that these videos might be used to help struggling students.

Introduction to Rational Exponents and Radicals

1. ▷ Introduction to rational exponents (exponents are unit fractions such as $\frac{1}{2}$ and $\frac{1}{3}$ (4:59)
   - $4^3 = 64$
   - $4^{-3} = \frac{1}{64}$
   - $4^{\frac{1}{2}} = \sqrt{4} = 2$
   - $8^{\frac{1}{3}} = \sqrt[3]{8}2$ as $2^3 = 8$
   - $32^{\frac{1}{5}} = 2$ as $2^5 = 32$

   And a few similar problems.
2. ▷ Radicals and Exponents (8:45)
   - $\sqrt{9} = \sqrt[2]{9} = 3$
   - $\sqrt[3]{27} = 3$
   - $\sqrt[4]{16} = 2$
   - $\sqrt[5]{96} = \sqrt[5]{2^5 \cdot 3} = 2 \cdot \sqrt[5]{3}$
   - $\sqrt[6]{64 \cdot x^8} = 2x\sqrt[6]{x^2} = 2x \cdot \sqrt[3]{x} = 2x^{\frac{4}{3}}$
3. ⋆ $(-128)^{\frac{1}{7}}$
4. ▷ Evaluating rational exponents which are negative unit fractions (3:02)
   - $9^{-\frac{1}{2}} = \frac{1}{3}$

- $-27^{-\frac{1}{3}} = -\frac{1}{3}$

5. ▷ Evaluating rational expressions (5:53)
   - $(64)^{\frac{2}{3}} = (4^3)^{\frac{2}{3}} = 4^2 = 16$
   - $(\frac{8}{27})^{-2}3 = \frac{9}{4}$
6. ▷ Evaluating rational expressions with rational bases (3:24) $(\frac{25}{9})^{\frac{1}{2}} = \frac{5}{3}$ and $(\frac{81}{256})^{-\frac{1}{4}} = \frac{4}{3}$
7. ▷ Converting from radical to fractional exponents (4:01) If $3^a = \sqrt[3]{\frac{3}{2}}$, then $a = \frac{2}{5}$.
8. ⋆ Rational exponents $(-125)^{\frac{2}{3}}$.

Videos and Exercises for Introduction to Rational Exponents and Radicals

1. ▷ Introduction to rational exponents (exponents are unit fractions such as $\frac{1}{2}$ and $\frac{1}{3}$ (4:59) https://www.khanacademy.org/math/algebra/rational-exponents-and-radicals/introduction-to-rational-exponents-and-radicals/v/basic-fractional-exponents
2. ▷ Radicals and Exponents (8:45) https://www.khanacademy.org/math/algebra/rational-exponents-and-radicals/introduction-to-rational-exponents-and-radicals/v/radical-expressions-with-higher-roots
3. ⋆ $(-128)^{\frac{1}{7}}$ https://www.khanacademy.org/math/algebra/rational-exponents-and-radicals/introduction-to-rational-exponents-and-radicals/e/understanding-fractional-exponents
4. ▷ Evaluating rational exponents which are negative unit fractions (3:02) https://www.khanacademy.org/math/algebra/rational-exponents-and-radicals/introduction-to-rational-exponents-and-radicals/v/negative-fractional-exponent-examples
5. ▷ Evaluating rational expressions (5:53) https://www.khanacademy.org/math/algebra/rational-exponents-and-radicals/introduction-to-rational-exponents-and-radicals/v/fractional-exponents-with-numerators-other-than-1
6. ▷ Evaluating rational expressions with rational bases (3:24) https://www.khanacademy.org/math/algebra/rational-exponents-and-radicals/introduction-to-rational-exponents-and-radicals/v/negative-fractional-exponent-examples-2
7. ▷ Converting from radical to fractional exponents (4:01) https://www.khanacademy.org/math/algebra/rational-exponents-and-radicals/introduction-to-rational-exponents-and-radicals/v/solving-for-a-fractional-exponent

8. ⋆ Rational exponents $(-125)^{\frac{2}{3}}$. https://www.khanacademy.org/math/algebra/rational-exponents-and-radicals/introduction-to-rational-exponents-and-radicals/e/exponents_3

Roots and Radicals, Simplifying Radical Expressions

Review material: Simplifying numerical radical expressions

1. ▷ Simplifying square roots (3:08) $5\sqrt{117} = 5\sqrt{3^2 \cdot 13} = 15\sqrt{13}$ and $3\sqrt{26}$ cannot be simplified.
2. ▷ Simplifying square roots of fractions (4:40) $\sqrt{\frac{1}{200}} = \frac{1}{10\sqrt{2}} = \frac{\sqrt{2}}{20}$
3. ⋆ Simplifying square roots $\sqrt{\frac{1}{44}}$
4. ▷ Simplifying sums of radicals (4:41) [1]
   $\sqrt{2x^2} + 4\sqrt{8} + 3\sqrt{2x^2} + \sqrt{8} = (4|x| + 10)\sqrt{2}$
5. ▷ Simplifying differences of radicals (5:45)
   $4\sqrt[4]{81x^5} - 2\sqrt[4]{81x^5} - \sqrt{x^3} = 6|x|\sqrt[4]{x} - |x|\sqrt{x}$.
   [2]
6. ⋆ Simplifying square root expressions (with no variables) $\sqrt{\frac{6^2}{95}}$
7. ▷ Introduction to rationalizing the denominator (10:17)

   Example 1 $\frac{1}{2}$

   Example 2 $\frac{7}{15}$

   Example 3 $\frac{12}{2-\sqrt{5}}$ [3]

   Example 4 $\frac{5y}{2\sqrt{y}-5} = \frac{10}{4y-25}$

[1] He uses the correct $|x| = \sqrt{x^2}$ rather than the (false) $\sqrt{x^2} = x$. Hooray!

[2] In this video he remarks that "If $x > 0$, there's no need for the $|x|$."

[3] Sal shows why multiplying by $\frac{\sqrt{5}}{\sqrt{5}}$ doesn't help and why multiplying by $\frac{2+\sqrt{5}}{2+\sqrt{5}}$ does — difference of squares!

Videos and exercises for Roots and Radicals, Simplifying Radical Expressions

1. ▷ Simplifying square roots (3:08) https://www.khanacademy.org/math/algebra/rational-exponents-and-radicals/simplify-radical-expressions/v/simplifying-square-roots-1
2. ▷ Simplifying square roots of fractions (4:40) https://www.khanacademy.org/math/algebra/rational-exponents-and-radicals/simplify-radical-expressions/v/rewriting-square-root-of-fraction

3. ⋆ Simplifying square roots https://www.khanacademy.org/math/algebra/rational-exponents-and-radicals/simplify-radical-expressions/e/simplifying_radicals

4. ▷ Simplifying sums of radicals (4:41) https://www.khanacademy.org/math/algebra/rational-exponents-and-radicals/simplify-radical-expressions/v/adding-and-simplifying-radicals

5. ▷ Simplifying differences of radicals (5:45) https://www.khanacademy.org/math/algebra/rational-exponents-and-radicals/simplify-radical-expressions/v/subtracting-and-simplifying-radicals

6. ⋆ Simplifying square root expressions (with no variables) https://www.khanacademy.org/math/algebra/rational-exponents-and-radicals/simplify-radical-expressions/e/multiplying_radicals

7. ▷ Introduction to rationalizing the denominator (10:17) https://www.khanacademy.org/math/algebra/rational-exponents-and-radicals/simplify-radical-expressions/v/how-to-rationalize-a-denominator

Simplifying Rational Expressions

The material for this section is review. A whole sequence of videos is available at: https://www.khanacademy.org/math/algebra/rational-exponents-and-radicals/rational-exponents-and-the-properties-of-exponents/v/simplifying-numerical-expression-with-rational-expon

Adding and Subtracting Radicals

▷ Adding and Subtracting Radical Expressions (7:56) In this video Sal simplifies each of the following:

- $3\sqrt{8} - 6\sqrt{32}$ to $-18\sqrt{2}$
- $\sqrt{180} + 6\sqrt{405}$ to $60\sqrt{5}$
- $\sqrt{48a} + \sqrt{27a}$ to $7\sqrt{3a}$

https://www.khanacademy.org/math/algebra-basics/core-algebra-foundations/square-roots-for-college/v/more-simplifying-radical-expressions

Multiplying Radicals

▷ Multiply and Simplify the Radical Expressions (4:24) Simplifies $5\sqrt[3]{2x^2} * \sqrt[3]{4x^4}$ to $30x^2$ https://www.khanacademy.org/math/algebra/rational-exponents-and-radicals/rational-exponents-and-the-properties-of-exponents/v/multiply-and-simplify-a-radical-expression-1f

Division of Radicals and Rationalization

▷ Divide and Simplify Radical Expressions with Two Variables (3:06) Divides $\frac{\sqrt{6-x^2y}}{\sqrt{48x}}$ to get $\frac{\sqrt{5xy}}{2}$ https://www.khanacademy.org/math/algebra/rational-exponents-and-radicals/rational-exponents-and-the-properties-of-exponents/v/multiply-and-simplify-a-radical-expression-1

▷ An example with multiplication and division (6:06) Simplify $\frac{(125)^{-\frac{1}{8}} * (125)^{-\frac{5}{8}}}{5^{\frac{1}{2}}}$ (Solution: 5) https://www.khanacademy.org/math/algebra/rational-exponents-and-radicals/rational-exponents-and-the-properties-of-exponents/v/simplifying-rational-exponent-expressions-advanced

▷ A further video on Rationalization. Sal rationalizes and simplifies $\frac{16+2x^2}{\sqrt{8}}$ https://www.khanacademy.org/math/algebra/rational-exponents-and-radicals/simplify-radical-expressions/v/rationalizing-denominators-of-expressions

Solving Radical Equations

Radical Equations and Functions. Focuses primarily on square roots. https://www.khanacademy.org/math/algebra2/radical-equations-and-functions

Complex Numbers

https://www.khanacademy.org/math/precalculus/imaginary-and-complex-numbers/the-complex-numbers/v/complex-number-intro

Quadratics

This material is in the Algebra I section of Khan Academy
https://www.khanacademy.org/math/algebra/quadratics

Solving Quadratic Equations by factoring

https://www.khanacademy.org/math/algebra/quadratics/solving-quadratic-equations-by-factoring/v/example-1-solving-a-quadratic-equation-by-factoring

1. ▷ Solving quadratic equations by factoring (6:21) Sal solves $s^2 - 2s - 35 = 0$ to see that $s = 7$ and $s = -5$ are solutions. https://www.khanacademy.org/math/algebra/quadratics/solving-quadratic-equations-by-factoring/v/example-1-solving-a-quadratic-equation-by-factoring
2. ⋆ Solving quadratic equations with leading coefficient 1 (i.e., of the form $x^2 + bx + c = 0$). https://www.khanacademy.org/math/algebra/quadratics/solving-quadratic-equations-by-factoring/e/solving_quadratics_by_factoring
3. ▷ Solving quadratic equations with leading coefficient not equal to one (4:35) Example solved: $6x^2 - 120x + 600 = 0$. https://www.khanacademy.org/math/algebra/quadratics/solving-quadratic-equations-by-factoring/v/solving-quadratics-by-dividing-and-factoring
4. ⋆ Solving quadratic equations with leading coefficient not equal to one, i.e., of the form $ax^2 + bx + c = 0$ where $a \neq 1$. https://www.khanacademy.org/math/algebra/quadratics/solving-quadratic-equations-by-factoring/e/solving_quadratics_by_factoring_2
5. ▷ Solving quadratics using structure (3:55) Solve the equation $(2x - 3)^2 = 4x - 3$ by making the substitution $p = 2x - 3$. https://www.khanacademy.org/math/algebra/quadratics/solving-quadratic-equations-by-factoring/v/using-structure-to-solve-quadratics

Square Root Property

https://www.khanacademy.org/math/algebra/quadratics/quadratics-square-root/v/simple-quadratic-equation

1. ▷ Solving quadratics using the square root property (2:18) Solve $2x^2 + 3 = 75$ by isolating x^2 and applying the square root property. https://www.khanacademy.org/math/algebra/quadratics/quadratics-square-root/v/simple-quadratic-equation

2. ▷ Solving quadratics by taking square roots (examples) (5:12) In this video we see how to solve the equation $(x+3)^2 - 4 = 0$ and how to find the x-intercepts of the function $f(x) = (x-2)^2 - 9$. https://www.khanacademy.org/math/algebra/quadratics/quadratics-square-root/v/solving-quadratics-by-taking-square-roots

3. ⋆ Practice solving quadratics of the form $(x+a)^2 - b = 0$ https://www.khanacademy.org/math/algebra/quadratics/quadratics-square-root/e/solving_quadratics_by_taking_the_square_root

4. ▷ Solving quadratics by taking the square root (strategy focusing on order of operations) (4:24) Solve $3(x+6)^2 = 75$ paying special attention to the order of steps imposed by order of operations (PEMDAS). https://www.khanacademy.org/math/algebra/quadratics/quadratics-square-root/v/order-of-steps-exercise-example

5. ▷ Solving quadratics by using the square root property (finding errors in steps!) (1:44) Finding and repairing errors in an attempted solution to $2(x+4)^2 = 242$. https://www.khanacademy.org/math/algebra/quadratics/quadratics-square-root/v/determining-mistakes-in-steps-example

6. ⋆ Practice problem finding the errors in an attempted solution to a quadratic equation. https://www.khanacademy.org/math/algebra/quadratics/quadratics-square-root/e/understanding-the-equation-solving-process

Completing the Square

https://www.khanacademy.org/math/algebra/quadratics/solving-quadratics-by-completing-the-square/v/solving-quadratic-equations-by-completing-the-square

1. ▷ Introduction to completing the square (14:06) How to solve equations such as $x^2 - 4x = 5$ and $10x^2 - 30x - 8 = 0$ by using the technique of completing the square.

   Be careful about answers that are irreducible radicals — be sure you give the exact, correct answer (the radical). If requested, you can also give a decimal approximation, but remember that it is only an approximation. If Sal had verified his results algebraically instead of with the calculator, he would have seen that the left hand is exactly, not approximately, zero when the solutions are substituted in for x.

https://www.khanacademy.org/math/algebra/quadratics/solving-quadratics-by-completing-the-square/v/solving-quadratic-equations-by-completing-the-square

2. ▷ Rewriting quadratics as perfect squares (3:01) Rewrite $x^2 + 16x + 9$ as $(x+8)^2 - 55$. https://www.khanacademy.org/math/algebra/quadratics/solving-quadratics-by-completing-the-square/v/rewriting-quadratics-as-perfect-squares

3. ⋆ Rewriting quadratics as perfect squares Examples such as $x^2 + 20x + 40$. https://www.khanacademy.org/math/algebra/quadratics/solving-quadratics-by-completing-the-square/e/completing_the_square_in_quadratic_expressions

4. ▷ Solving quadratics by completing the square (6:18) How to solve $x^2 - 2x - 8 = 0$ by rewriting it as $(x-1)^2 - 9 = 0$. https://www.khanacademy.org/math/algebra/quadratics/solving-quadratics-by-completing-the-square/v/solving-quadratics-by-completing-the-square

5. ▷ Solving quadratics by completing the square when the leading coefficient is not 1 (5:43) Example: $4x^2 + 40x - 300 = 0$. https://www.khanacademy.org/math/algebra/quadratics/solving-quadratics-by-completing-the-square/v/completing-the-square-to-solve-quadratic-equations

   There are more practice problems available, as well as several more videos, but these seem sufficient.

The Quadratic Formula

https://www.khanacademy.org/math/algebra/quadratics/solving-quadratics-using-the-quadratic-formula/v/using-the-quadratic-formula

There are many more examples worked out in videos in the link above. The selection below is just a reasonable minimum.

1. ▷ The quadratic formula (16:31) The solution to three equations using the quadratic formula, and the relationship of the solutions to the graph of the function and other methods to solve the equation. The examples are: $x^2 + 4x - 21 = 0$ which is factorable and has two real solutions; $3x^2 + 6x + 10 = 0$, which has no rational roots so cannot be factored using rational numbers — indeed it has no real solutions (the graph doesn't intersect the x-axis), so the quadratic formula is the best and easiest way to solve this problem; $-3x^2 + 12x + 1 = 0$, which has two real solutions. https://www.khanacademy.org/math/algebra/quadratics/solving-quadratics-using-the-quadratic-formula/v/using-the-quadratic-formula

2. ▷ Using the quadratic formula (rearranging) (2:21) Rewrite the equation $6x^2 + 3 = 2x - 6$ in standard form and identify a, b, and c. https://www.khanacademy.org/math/algebra/quadratics/solving-quadratics-using-the-quadratic-formula/v/quadratic-equations-in-standard-form

3. ▷ Determining the number of solutions to a quadratic using the discriminant (4:58) Example used: $x^2 + 14x + 49 = 0$. https://www.khanacademy.org/math/algebra/quadratics/solving-quadratics-using-the-quadratic-formula/v/quadratic-formula-3

4. ⋆ Practice determining the number of solutions of a quadratic (including interpreting graphical information) https://www.khanacademy.org/math/algebra/quadratics/solving-quadratics-using-the-quadratic-formula/e/determine-the-number-of-solutions-of-a-quadratic-equation

5. ⋆ Practice solving quadratic equations using the quadratic formula. https://www.khanacademy.org/math/algebra/quadratics/solving-quadratics-using-the-quadratic-formula/e/quadratic_equation

6. ▷ Derivation of the quadratic formula (using completing the square to solve $ax^2 + bx + c = 0$ https://www.khanacademy.org/math/algebra/quadratics/solving-quadratics-using-the-quadratic-formula/v/proof-of-quadratic-formula

Applications of the Quadratic Formula

First a couple of word problems.

1. ▷ Dimensions of a triangle from the area (8:21) Given that the height of triangle is four units less than the length of the base, what are the dimensions of the triangle? https://www.khanacademy.org/math/algebra/quadratics/solving-quadratic-equations-by-factoring/v/example-4-solving-a-quadratic-equation-by-factoring

2. Dimensions of a box (5:53) Given that the volume of a box is 405 cubic units, and that the height if 9 units, the length is unknown, and the width is four more than the length, find the dimensions of the box in units. url

3. Projectile motion (5:52) A ball is shot into the air from a building that is 50 feet tall at an initial velocity of 20 feet per second. How long will it take to hit the ground?

   Note: this video does not discuss the equation $h(t) = at^2 + v_0t + h_0$, and probably should. https://www.khanacademy.org/math/

algebra/quadratics/solving-quadratics-using-the-quadratic-formula/v/application-problem-with-quadratic-formula

Interpreting quadratic models and a word problem with mosquitos https://www.khanacademy.org/math/algebra/quadratics/interpret-quadratic-models/v/modeling-mosquitos-with-quadratics

And a bit of practice modeling with quadratics https://www.khanacademy.org/math/algebra/quadratics/interpret-quadratic-models/e/key-features-quadratics

Graphs of Quadratic Functions

Features of parabolas https://www.khanacademy.org/math/algebra/quadratics/features-of-quadratic-functions/v/quadratic-functions-2

There are several more videos that students can watch if they are struggling with the practice problems.

1. ▷ Finding the vertex and axis of symmetry of a parabola. 7:22) Example used is $y = -x^2 + 8x + 8$. https://www.khanacademy.org/math/algebra/quadratics/features-of-quadratic-functions/v/quadratic-functions-2

2. ▷ Forms and features of quadratic functions (8:28) Here we see how factoring can be used to find the zeros (x-intercepts) and the vertex form reveals the vertex and axis of symmetry using $f(x) = x^2 - 5x + 6$. https://www.khanacademy.org/math/algebra/quadratics/features-of-quadratic-functions/v/rewriting-a-quadratic-function-to-find-roots-and-vertex

3. ⋆ Practice finding the roots, vertex, and axis of symmetry of a quadratic function. https://www.khanacademy.org/math/algebra/quadratics/features-of-quadratic-functions/e/rewriting-expressions-to-reveal-information

Graphing quadratic functions https://www.khanacademy.org/math/algebra/quadratics/graphing-quadratic-functions/v/graphing-a-quadratic-function

Shifting and Scaling Parabolas https://www.khanacademy.org/math/algebra/quadratics/transforming-quadratic-functions/v/shifting-and-scaling-parabolas

Quadratic Systems of Equations

https://www.khanacademy.org/math/algebra/quadratics/systems-of-quadratic-equations/v/non-linear-systems-of-equations-2

Geometry

Perpendicular Bisectors

▷ (1:26) Finds the perpendicular bisector of a line segment AB using a ruler and compass.

https://www.khanacademy.org/math/geometry/geometric-constructions/geo-bisectors/v/constructing-a-perpendicular-bisector-using-a-compass-and-straightedge

Distance Formula

1. ▷ Distance Fomula (9:38) Find the distance between $(3, -4)$ and $(6, 0)$ using the distance formula by deriving it from the Pythagorean Theorem. ($d = 5$)

   Then uses the formula he derived to compute the distance between $(-6, -4)$ and $(1, 7)$. In this example he uses an on-screen TI graphing calculator. (And finds $d = \sqrt{170} \approx 13.03$)

   https://www.khanacademy.org/math/basic-geo/basic-geo-pythagorean-topic/basic-geo-pythagorean-theorem/v/distance-formula

2. ⋆ Use the Pythagorean Theorem (distance formula) to find the distance between two points in the plane.

   https://www.khanacademy.org/math/geometry/hs-geo-analytic-geometry/hs-geo-distance-and-midpoints/e/distance_formula

3. ▷ Really nice application! Thiago (Brazillian soccer player) asks: How long does a keeper have to react to a penalty kick? (12:04)

   https://www.khanacademy.org/math/basic-geo/basic-geo-pythagorean-topic/basic-geo-pythagorean-theorem/v/soccer-thiago

Midpoint Formula

1. ▷ Midpoint Formula. Finds the midpoint between $(3, -4)$ and $(6, 1)$ (which is $(4.5, -1.5)$) and the midpoint between $(4, -5)$ and $(8, 2)$ (which is $(6, -1.5)$ and derives the midpoint formula from the examples.

Find the average of the x values and the average of the y values.
https://www.khanacademy.org/math/geometry/analytic-geometry-topic/distance-and-midpoints/v/midpoint-formula

2. ⋆ Find the midpoint of a line segment.

   https://www.khanacademy.org/math/geometry/analytic-geometry-topic/distance-and-midpoints/e/midpoint_formula

3. Written derivation of the midpoint formula https://www.khanacademy.org/math/geometry/analytic-geometry-topic/distance-and-midpoints/a/midpoint-formula

Pythagorean Theorem Review

In case students don't know or have forgotten the Pythagorean Theorem, you can direct them here.

1. ▷ Intro to the Pythagorean Theorem I (10:45)

   https://www.khanacademy.org/math/basic-geo/basic-geo-pythagorean-topic/basic-geo-pythagorean-theorem/v/the-pythagorean-theorem

2. ▷ Intro to the Pythagorean Theorem II (13:03)

   https://www.khanacademy.org/math/basic-geo/basic-geo-pythagorean-topic/basic-geo-pythagorean-theorem/v/pythagorean-theorem

3. ▷ Pythagorean Theorem Word Problem–fishing boat (4:32)

   https://www.khanacademy.org/math/basic-geo/basic-geo-pythagorean-topic/basic-geo-pythagorean-theorem/v/pythagorean-theorem-1

4. ▷ Pythagorean Theorem Example (4:17) Finding the height of right triangle with base 9 units and hypotenuse 14 units.

   https://www.khanacademy.org/math/basic-geo/basic-geo-pythagorean-topic/basic-geo-pythagorean-theorem/v/pythagorean-theorem-2

5. ▷ Pythagorean Theorem Example (carpet) (2:59)

   https://www.khanacademy.org/math/basic-geo/basic-geo-pythagorean-topic/basic-geo-pythagorean-theorem/v/pythagorean-theorem-3

6. ⋆ Find the leg or hypotenuse of a right triangle using the Pythagorean Theorem.

   https://www.khanacademy.org/math/basic-geo/basic-geo-pythagorean-topic/basic-geo-pythagorean-theorem/e/pythagorean_theorem_1

Circles

1. ▷ Pythagorean Theorem and Radii of Circles (5:52)

   Finds missing segments of triangles within a circle using the Pythagorean Theorem.

   https://www.khanacademy.org/math/geometry/analytic-geometry-topic/equation-of-a-circle/v/pythagorean-theorem-and-radii-of-circles

2. ▷ Standard Equation of a Circle (6:18)

   Equation for the points (x, y) on a circle with radius r and center (h, k).

   https://www.khanacademy.org/math/geometry/analytic-geometry-topic/equation-of-a-circle/v/equation-for-a-circle-using-the-pythagorean-theorem

3. ▷ Features of a circle from its equation in standard form (3:50)
   Finding the center and radius of a circle from the equation in standard form.

   Given $(x+3)^2 + (y-4)^2 = 49$, find the center and radius. Solution: center is $(-3, 4)$ and radius is 7.

   https://www.khanacademy.org/math/geometry/analytic-geometry-topic/equation-of-a-circle/v/radius-and-center-for-a-circle-equation-in-standard-form

4. ⋆ Features of a circle from its equation in standard form. Find the center and radius of $x^2 + (y - 2.25)^2 = \frac{169}{196}$.

   https://www.khanacademy.org/math/geometry/analytic-geometry-topic/equation-of-a-circle/e/equation_of_a_circle_1

5. ▷ Features of a circle from its expanded equation (4:20)

   Graph the circle $x^2 + y^2 + 4x - 4y - 17 = 0$ (Puts the equation into standard form by completing the square.)

   https://www.khanacademy.org/math/geometry/analytic-geometry-topic/equation-of-a-circle/v/completing-the-square-to-write-equation-in-standard-form-of-a-circle

6. ⋆ Features of a circle from its expanded equation.

   Find the center and radius of the circle $x^2 + y^2 - 6x - 6y + 2 = 0$

   https://www.khanacademy.org/math/geometry/analytic-geometry-topic/equation-of-a-circle/e/equation_of_a_circle_2

Systems of Equations

Systems of Three Linear Equations and Three Variables

1. ▷ Intro to Systems with Three Variables (8:23)

   Describes how we approach solving systems of three variables algebraically, and how we visualize it graphically.

$$\begin{array}{rcrcrcr} x & + & y & - & 3z & = & -10 \\ x & - & y & + & 2z & = & 3 \\ 2x & + & y & - & z & = & -6 \end{array}$$

   Solution: $x = -2$, $z = 3$, and $y = 1$.

2. ▷ Solving Linear Systems with Three Variables (7:00) Uses the method of variable elimination to solve:

$$\begin{array}{rcrcrcr} x & + & 2y & - & 5z & = & -17 \\ 2x & - & 3y & + & 2z & = & -16 \\ 3x & + & y & - & z & = & 3 \end{array}$$

   Solution: $x = -1$, $y = 2$, and $z = -4$. Also verifies solutions.

3. ▷ Solving Linear Systems with 3 Variables: no solution (5:05) Determine whether this system has infinitely many or no solutions.

$$\begin{array}{rcrcrcr} 2x & - & 4y & - & z & = & 3 \\ 8x & - & 2y & + & 4z & = & 7 \\ -4x & + & y & - & 2z & = & -14 \end{array}$$

4. ▷ Three variable linear system word problem (8:15)

   This video solves a word problem about the angles of a given triangle by modeling the given information as a system of three equations and three variables.

   The second angle of a triangle is 50 degrees less than 4 times the first angle. The third angle is 40 degrees less than the first angle. Find the measures of the three angles.

Solution summary: the resulting system of equations is

$$\begin{array}{rcrcrcl} a & + & b & + & c & = & 180 \\ & & b & & & = & 4a - 50 \\ & & & & c & = & a - 40 \end{array}$$

which has $a = 45$, $c = -5$, $b = 130$. Sal then verifies that all constraints are satisfied.

1. ▷ Intro to Systems with Three Variables (8:23) https://www.khanacademy.org/math/algebra2/advanced-equations-and-inequalities/systems-with-three-variables/v/systems-of-three-variables
2. ▷ Solving Linear Systems with Three Variables (7:00) https://www.khanacademy.org/math/algebra2/advanced-equations-and-inequalities/systems-with-three-variables/v/systems-of-three-variables-2
3. ▷ Solving Linear Systems with 3 Variables: no solution (5:05) https://www.khanacademy.org/math/algebra2/advanced-equations-and-inequalities/systems-with-three-variables/v/solutions-to-three-variable-system-2
4. ▷ Three variable linear system word problem (8:15) https://www.khanacademy.org/math/algebra2/advanced-equations-and-inequalities/systems-with-three-variables/v/three-equation-application-problem

Determinants and Cramer's Rule

This section is option, and these videos probably require some supplementary discussion if you plan to use them and cover the topic of Cramer's rule and determinants.

1. ▷ Determinants of 3×3 matrices and linear Transformations. (10:00)

   This is a linear algebra video but gives more information about what a determinant is. https://www.youtube.com/watch?v=0c7dt2SQfLw
2. ▷ Determinant of a 3x3 matrix: standard method (1 of 2) (3:56) https://www.khanacademy.org/math/precalculus/precalc-matrices/determinants-and-inverses-of-large-matrices/v/finding-the-determinant-of-a-3x3-matrix-method-2
3. ▷ Determinant of a 3x3 matrix: shortcut method (2 of 2) — Cramer's Rule (2:39) https://www.khanacademy.org/math/precalculus/precalc-matrices/determinants-and-inverses-of-large-matrices/v/finding-the-determinant-of-a-3x3-matrix-method-1
4. ⋆ Find the determinant of a 3 matrix. https://www.khanacademy.org/math/precalculus/precalc-matrices/determinants-and-inverses-of-large-matrices/e/matrix_determinant_3x3

Systems Involving Nonlinear Equations

1. Nonlinear systems of equations I (5:44)

   Solve the system of equations by graphing. Check your solution algebraically.

$$\begin{aligned} y &= x^2 + 6 \\ y &= -2x - 2 \end{aligned}$$

   Solutions: $(-2, 2)$ and $(4, -10)$

2. Systems of nonlinear equations II (3:33)

   Find the solution(s) of

$$\begin{aligned} y &= \frac{1}{2}x \\ 2x^2 - y^2 &= 7 \end{aligned}$$

   Solutions: $x = 2$ or $x = -2$ giving points $(2, 1)$ and $(-2, -1)$, both of which satisfy both constraint equations.

3. Systems of nonlinear equations III (4:13) Find the solution(s) of the system of equations

$$\begin{aligned} y &= x + 1 \\ x^2 + y^2 &= 25 \end{aligned}$$

   Solutions: $(-4, -3)$ and $(4, 3)$

Videos: systems of nonlinear equations

1. Nonlinear systems of equations I (5:44) https://www.youtube.com/watch?v=hjigR_rHKDI
2. Systems of nonlinear equations II (3:33) https://www.youtube.com/watch?v=GQf1vjfxuo8
3. Systems of nonlinear equations III (4:13) https://www.youtube.com/watch?v=swFohliPgmQ

Trigonometry

Here is a link to the Khan Academy Trigonometry overview page
https://www.khanacademy.org/math/algebra2/trig-functions#intro-to-radians-alg2

Trigonometry of Right Triangles

Here is a review of material from basic geometry. https://www.khanacademy.org/math/geometry/right-triangles-topic/intro-to-the-trig-ratios-geo/v/basic-trigonometry-ii

Solving Right Triangles (for a side)

https://www.khanacademy.org/math/geometry/right-triangles-topic/trig-solve-for-a-side-geo/a/unknown-side-in-right-triangle-w-trig

Solving Right Triangles (for an angle)

https://www.khanacademy.org/math/geometry/right-triangles-topic/trig-solve-for-an-angle-geo/a/inverse-trig-functions-intro

Applications of Static Geometry

Modeling with right triangles https://www.khanacademy.org/math/geometry/right-triangles-topic/modeling-with-right-triangles-geo/v/angle-to-aim-to-get-alien

Similar Triangles https://www.khanacademy.org/math/geometry/right-triangles-topic/trig-ratios-similarity-geo/v/similarity-to-define-sine-cosine-and-tangent

Angle Measure: Introduction to Radians

1. ▷ Introduction to radians (10:51)

   This video explains the definition and motivation for radians and the relationship between radians and degrees, including the identities by which we can convert between radians and degrees.

2. ▷ Radians and degrees (7:12)

   Sal discusses the general approach to converting between radians and degrees and vice versa. Describes identities, then converts 30 degrees to radians — including nice explanation of unit conversion.

   Also converts 45 degrees to radians and $-\frac{\pi}{2}$ radians to degrees.

3. ▷ Converting degrees to radians (7:01)

   This video shows how to convert the degree measures 150Âř and -45Âř to radians.

4. ▷ Converting radians to degrees (3:19) Converts π and $-\frac{\pi}{3}$ radians to degrees.

5. ⋆ Converting from radians to degrees and visa versa Assorted example problems

6. ▷ Radians and Quandrants (3:51) This video show how to find the quandrant in which a ray formed by rotation of θ radians lies.

   Examples: $\frac{3\pi}{5}$, $\frac{2\pi}{7}$, 3 radians.

Videos and exercises: Angle measures

1. ▷ Introduction to radians (10:51) https://www.khanacademy.org/math/algebra2/trig-functions/intro-to-radians-alg2/v/introduction-to-radians

2. ▷ Radians and degrees (7:12) https://www.khanacademy.org/math/algebra2/trig-functions/intro-to-radians-alg2/v/radian-and-degree-conversion-practice

3. ▷ Converting degrees to radians (7:01) https://www.khanacademy.org/math/algebra2/trig-functions/intro-to-radians-alg2/v/we-converting-degrees-to-radians

4. ▷ Converting radians to degrees (3:19) https://www.khanacademy.org/math/algebra2/trig-functions/intro-to-radians-alg2/v/we-converting-radians-to-degrees

5. ⋆ Converting from radians to degrees and visa versa https://www.khanacademy.org/math/algebra2/trig-functions/intro-to-radians-alg2/e/degrees_to_radians

6. ▷ Radians and Quandrants (3:51) https://www.khanacademy.org/math/algebra2/trig-functions/intro-to-radians-alg2/v/rotation-by-radians-and-quadrants

A DIVERGENCE IN APPROACHES the current MAT 1275 syllabus contains some high school geometry (which is also covered in MAT 1175) as if it is new material. The video sequence acknowledges that that material, i.e., SOH-CAH-TOA, should be review.

Unit Circle: Defining the trigonometric functions sine and cosine for all real numbers

1. ▷ Unit circle definition of trig functions (9:03)

   How to extend SOH-CAH-TOA to define trigonometric functions (sine, cosine, and tangent) for all real numbers.

2. ▷ The trig functions and right angle trig ratios (6:57) Pause the video and work it out on your own!

   Sal shows how, for acute angles, the two different definitions of the trigonometric values (SOH-CAH-TOA and the unit circle definition) result in the same values.

Videos and exercises for defining trigonometric functions for the whole unit circle

1. ▷ Unit circle definition of trig functions (9:03) https://www.khanacademy.org/math/algebra2/trig-functions/unit-circle-definition-of-trig-functions-alg2/v/unit-circle-definition-of-trig-functions-1

2. ▷ The trig functions and right angle trig ratios (6:57) https://www.khanacademy.org/math/algebra2/trig-functions/unit-circle-definition-of-trig-functions-alg2/v/matching-ratios-trig-functions

Special Triangles

Here is how to find the trigonometric values of some special angles without using a calculator to approximate them.

1. ▷ Trig values of special angles (7:53) Sal finds the trig values of $\frac{\pi}{4}$ on the unit circle using triangles.

   One might wish to do the triangle that is 30-60-90 in class.

2. ⋆ Trig values of special angles Students are asked to find the trig values of angles such as $-\frac{2\pi}{3}$ without using a calculator.

Videos for special triangles

1. ▷ Trig values of special angles (7:53) https://www.khanacademy.org/math/algebra2/trig-functions/trig-values-special-angles-alg2/v/solving-triangle-unit-circle
2. ⋆ Trig values of special angles https://www.khanacademy.org/math/algebra2/trig-functions/trig-values-special-angles-alg2/e/trigonometric-functions-of-special-angles

Graphs of Sine, Cosine, and Tangent Functions

1. ▷ Graph of $y = \sin(x)$ (9:22) Includes finding the domain and range of the sine function. https://www.khanacademy.org/math/algebra2/trig-functions/graphs-of-sine-cosine-tangent-alg2/v/we-graph-domain-and-range-of-sine-function
2. ▷ Graph of $y = \tan(x)$ (10:15) Shows how to graph the tangent function from the unit circle https://www.khanacademy.org/math/algebra2/trig-functions/graphs-of-sine-cosine-tangent-alg2/v/tangent-graph
3. ▷ At how many points do $\sin(x)$ and $\cos(x)$ intersect in the interval $0 \leq x \leq 2\pi$. (11:06) https://www.khanacademy.org/math/algebra2/trig-functions/graphs-of-sine-cosine-tangent-alg2/v/we-graphs-of-sine-and-cosine-functions
4. *rhd* Finding amplitude and midline of sinusoidal functions https://www.khanacademy.org/math/algebra2/trig-functions/intro-to-amplitude-and-midline-of-sinusoids-alg2/v/midline-amplitude-period

Gaining information from the formula for a trigonometric function https://www.khanacademy.org/math/algebra2/trig-functions/amplitude-and-midline-of-sinusoids-from-formulas-alg2/e/find-midline-of-a-sinusoid-from-formula

Period of sinusoidal functions https://www.khanacademy.org/math/algebra2/trig-functions/period-of-sinusoids-alg2/e/period-of-trig-functions

Graphing (including translating) sinusoidal functions https://www.khanacademy.org/math/algebra2/trig-functions/graphing-sinusoids-alg2/v/example-amplitude-and-period-transformations

Constructing Trig functions from graphs and modeling with trig functions

https://www.khanacademy.org/math/algebra2/trig-functions/constructing-sinusoids-alg2/v/trig-function-equation

Fundamental Identities of Trigonometry

▷ Horizontal and vertical symmetry properties of sine and cosine derived from the unit circle (7:57) https://www.khanacademy.org/math/algebra2/trig-functions/trig-identities-alg2/v/trigonometry-unit-circle-symmetry

▷ Tangent identities — symmetry (7:15) https://www.khanacademy.org/math/algebra2/trig-functions/trig-identities-alg2/v/tan-symmetries-unit-circle

▷ Sine and cosine — periodicity (6:12) https://www.khanacademy.org/math/algebra2/trig-functions/trig-identities-alg2/v/trig-angle-rotations

▷ Tangent — periodicity (4:02) https://www.khanacademy.org/math/algebra2/trig-functions/trig-identities-alg2/v/tan-periodicity

Pythagorean Identity

▷ Proof of the pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ (6:12) https://www.khanacademy.org/math/algebra2/trig-functions/pythagorean-identity-alg2/v/pythagorean-trig-identity-from-unit-circle

▷ Using the Pythagorean trig identity (6:15) Given that $\sin(\theta) = \frac{1}{2}$ and that θ is in the third quadrant, Sal finds $\tan(\theta)$. https://www.khanacademy.org/math/algebra2/trig-functions/pythagorean-identity-alg2/v/using-the-pythagorean-trig-identity

⋆ Solving problems of the type: given that θ is in the third quadrant and that $\cos(\theta) = \frac{-12}{13}$, find $\sin(\theta)$. https://www.khanacademy.org/math/algebra2/trig-functions/pythagorean-identity-alg2/e/circles-and-pythagorean-identities

(Basic) Trigonometric Equations

The link to this series of videos: https://www.khanacademy.org/math/trigonometry/trig-equations-and-identities/basic-sinusoidal-equations/e/solve-basic-sinusoidal-equations

And now the specifics...

▷ Solving sinusoidal equations of the form $\sin(x) = d$. (6:10) https://www.khanacademy.org/math/trigonometry/trig-equations-and-identities/basic-sinusoidal-equations/v/sine-solutions

▷ Solving $\cos(\theta) = 1$ and $\cos(\theta) = -1$ (5:18) https://www.khanacademy.org/math/trigonometry/trig-equations-and-identities/basic-sinusoidal-equations/v/we-graph-of-cosine-function

⋆ Solve equations like $\sin(x) = 1$ https://www.khanacademy.org/math/trigonometry/trig-equations-and-identities/basic-sinusoidal-equations/

e/solve-basic-sinusoidal-equations

⋆ Solve equations of the type $a*\sin(bx+c)=d$ or $a*\cos(bx+c)=d$ https://www.khanacademy.org/math/trigonometry/trig-equations-and-identities/advanced-sinusoidal-equations/e/solve-advanced-sinusoidal-equations

Law of Sines

▷ Solving for a side of a triangle with the Law of Sines (5:57)

One angle of measure 30 degrees across from a side of length two, another angle measuring 45 degrees. Sal finds that the side across from the 45 degree angle is approximately 2.83 and the one across from the other (105 degree) angle is approximately 5.86. https://www.khanacademy.org/math/geometry/right-triangles-topic/law-of-sines-geo/v/law-of-sines

▷ Solving for an angle using the law of sines (5:33) Distance between two people is 40 m, find the angle between one of the people and the kite she is flying, which is 30 m away from her. The angle between the ground near the friend and the kite is 40 degrees. https://www.khanacademy.org/math/geometry/right-triangles-topic/law-of-sines-geo/v/law-of-sines-example

⋆ Find the missing triangle measures using the law of sines. https://www.khanacademy.org/math/geometry/right-triangles-topic/law-of-sines-geo/e/law_of_sines

▷ Proof of the law of sines (6:34) https://www.khanacademy.org/math/geometry/right-triangles-topic/law-of-sines-geo/v/proof-law-of-sines

Law of Cosines

▷ Solving for a side of the triangle with the law of cosines. (4:38) Side b of a triangle is 12 and side c of a triangle is 9, and the angle contained between them is 87 degrees. What is the length of the other side of the triangle? Solution: $a \approx 14.62$ https://www.khanacademy.org/math/geometry/right-triangles-topic/law-of-cosines-geo/v/law-of-cosines-example

▷ Solving for an angle with the law of cosines. (6:41) Given a triangle with base 60 m and other sides 50 m and 20 m, find the angle θ between the base and the longer side of the triangle. Solution: $\theta \approx 18.19$ degrees. https://www.khanacademy.org/math/geometry/right-triangles-topic/law-of-cosines-geo/v/law-of-cosines-missing-angle

⋆ Solve a triangle using the law of cosines https://www.khanacademy.org/math/geometry/right-triangles-topic/law-of-cosines-geo/e/law_of_cosines

▷ Proof of the law of cosines (9:23) https://www.khanacademy.

org/math/geometry/right-triangles-topic/law-of-cosines-geo/v/law-of-cosines

Solving General Triangles

▷ Trig word problem about stars (6:00) Uses the law of cosines. https://www.khanacademy.org/math/geometry/right-triangles-topic/solving-general-triangles-geo/v/law-of-cosines-word-problem

⋆ Solving for either a side or an angle of a general triangle using either the law of sines or the law of cosines. https://www.khanacademy.org/math/geometry/right-triangles-topic/solving-general-triangles-geo/e/law-of-sines-and-cosines-word-problems

Exponential and Logarithmic Functions

Exponential Functions

Logarithmic Functions

▷ Introduction to Logarithms (7:01) Gives the definition of the logarithm after giving several examples such as $2^x = 8$ is equivalent to $\log_2 8 = 3$. What power do I need to raise 2 to in order to get 8? Three. Several examples, including $\log_x 1 = 0$ for all bases. https://www.khanacademy.org/math/algebra2/exponential-and-logarithmic-functions/introduction-to-logarithms/v/logarithms

▷*and*⋆ together! Really it's an interactive text document covering similar material to the above video, but requiring input from the student. https://www.khanacademy.org/math/algebra2/exponential-and-logarithmic-functions/introduction-to-logarithms/a/intro-to-logarithms

⋆ Evaluate a logarithm such as $\log_5 125$. https://www.khanacademy.org/math/algebra2/exponential-and-logarithmic-functions/introduction-to-logarithms/e/logarithms_1

▷ Evaluating logarithms (more advanced) (4:20) Includes $\log_2 8$, $\log_8 2$, $\log_2 \frac{1}{8}$, and $\log_8 \frac{1}{2}$. https://www.khanacademy.org/math/algebra2/exponential-and-logarithmic-functions/introduction-to-logarithms/v/fancier-logarithm-expressions

⋆ Evaluate a logarithm (more advanced practice) such as $\log_{\frac{1}{2}} 32$. https://www.khanacademy.org/math/algebra2/exponential-and-logarithmic-functions/introduction-to-logarithms/e/logarithms_1.5

▷ Relationship between exponentials and logarithms (1:42) Sal rewrites $100 = 10^2$ as $\log_{10} 100 = 2$ and $\log_5 \frac{1}{125} = -3$ as $5^{-3} = \frac{1}{125}$. https://www.khanacademy.org/math/algebra2/exponential-and-logarithmic-functions/introduction-to-logarithms/v/exponential-to-logarithmic-form

▷ Relationship between exponentials and logarithms: graphs (4:10) Deducing equations of exponentials and logarithms from their graphs. https://www.khanacademy.org/math/algebra2/exponential-and-logarithmic-functions/introduction-to-logarithms/v/plotting-exponential-logarithm

▷ Relationship between exponentials and logarithms: tables

(5:58) Given incomplete tables of values of b^x and its corresponding inverse function, $\log_b(y)$, Sal uses the inverse relationship of the functions to fill in the missing values. https://www.khanacademy.org/math/algebra2/exponential-and-logarithmic-functions/introduction-to-logarithms/v/logarithm-exponential-deductions

⋆ Relationship between exponentials and logarithms. Students are asked to solve various problems that focus on the relationship between $a^x = b$ and $\log_a(b) = x$ https://www.khanacademy.org/math/algebra2/exponential-and-logarithmic-functions/introduction-to-logarithms/e/understanding-logs-as-inverse-exponentials

Properties of Logarithms

https://www.khanacademy.org/math/algebra2/exponential-and-logarithmic-functions/properties-of-logarithms/v/introduction-to-logarithm-properties

Change of base formula https://www.khanacademy.org/math/algebra2/exponential-and-logarithmic-functions/change-of-base-formula-for-logarithms/v/change-of-base-formula

Compound Interest

▷ The constant e and compound interest (11:38) Sal discusses borrowing a dollar for a year at 100% interest compounded once, twice, monthly, daily, and continuously (approximately). https://www.khanacademy.org/math/algebra2/exponential-and-logarithmic-functions/e-and-the-natural-logarithm/v/e-through-compound-interest

▷ The constant e as a limit. (5:39) https://www.khanacademy.org/math/algebra2/exponential-and-logarithmic-functions/e-and-the-natural-logarithm/v/e-as-limit

▷ Evaluating the natural logarithm with a calculator.(3:37) Sal evaluates $\log_3 67$ $\ln 67$ with a TI graphing calculator. https://www.khanacademy.org/math/algebra2/exponential-and-logarithmic-functions/e-and-the-natural-logarithm/v/natural-logarithm-with-a-calculator

Exponential Equations

Solving exponential equations with logarithms https://www.khanacademy.org/math/algebra2/exponential-and-logarithmic-functions/solving-exponential-equations-with-logarithms/v/exponential-equation

Modeling with exponential functions https://www.khanacademy.org/math/algebra2/exponential-and-logarithmic-functions/solving-exponential-models/v/solving-exponential-model-word-problems-1