Mathematics 1372/D552, Spring 2016 **Exam #3**
Instructor: Suman Ganguli **May 18, 2016** **Name:** ______________________

Question	Points	Score
1	20	
2	20	
3	30	
4	30	
Total:	100	

For full credit you must SHOW ALL YOUR WORK.

1. (20 points) Consider the continuous random variable:

$$X = \text{ the amount of time you wait for your subway after arriving on the platform}$$

Suppose we assume X is uniformly distributed between 0 and 10 minutes.

(a) Sketch a graph of the probability distribution of X. (Hint: It's a rectangle with area 1.) **Clearly label the scales on the horizontal and vertical axes.**

Solution: rectangle with base $(0, 10)$ and height $1/10$

(b) Compute the following probabilities:

(i) What is the probability that you wait between 2 and 5 minutes?

$P(2 < X < 5) =$

Solution: $P(2 < X < 5) = 3(0.1) = 0.3$

(ii) What is the probability that you wait more than 4 minutes?

$P(X > 4) =$

Solution: $P(X > 4) = 6(0.1) = 0.6$

(iii) What is the probability that you wait more than 11 minutes?

$P(X > 11) =$

Solution: $P(X > 11) = 0$

(c) Do you think it's reasonable to assume X is actually uniformly distributed? (Think about your own experiences waiting for the subway.). How could you get a more accurate probability distribution for X?

Solution: It's most likely not a reasonable assumption–a uniform distribution would result if the train *always* ran on regular intervals (in this case, 10 minute intervals). We all know from experience that that is not the case–in particular, there is always a probability of delays. We could get a more accurate probability distribution for X by collecting data on such waiting times.

2. (20 points) A bag contains 7 red marbles and 3 blue marbles. Five marbles are drawn from the bag *with replacement*. Let X denote the number of red marbles selected and let Y denote the number of blue marbles selected.

   (a) What are the possible values of each of the following discrete random variables?

   possible values of X:

   possible values of Y:

   possible values of $X+Y$:

   Solution: $X : \{0, 1, 2, 3, 4, 5\}, Y : \{0, 1, 2, 3, 4, 5\}, X + Y : \{5\}$

   (b) Explain why X is a binomial random variable with $n = 5$ and $p = 0.7$. (In particular, explain why it is essential that the marbles are drawn *with replacement*.)

   Solution: Each of the 5 draws has two outcomes (red or blue). Since the marbles are drawn with replacement, the result of each draw is independent of every other draw, and the probability p of drawing a red marble remains constant. If the marbles were drawn without replacement (as in the question on Exam 2), the probability does not remain constant.

   (c) Write down the probability distribution of X by using `=binomdist(i,n,p,false)` to calculate each of the probabilities $P(X = i)$. Sketch a graph of this probability distribution (i.e., a histogram).

   Solution:

i	$P(X = i)$
0	0.00243
1	0.02835
2	0.13230
3	0.30870
4	0.36015
5	0.16807

   (d) Calculate the expected value of X:

   $E[X] =$

Solution:

For a binomial random variable, $E[X] = np = 5(0.7) = 3.5$

or we can use the probability distribution:

$E[X] = 0 * 0.00243 + 1 * 0.02835 + 2 * 0.13230 + 3 * 0.30870 + 4 * 0.36015 + 5 * 0.16807 =$
$0 + 0.02835 + 0.2646 + 0.9261 + 1.4406 + 0.84035 = 3.5$

3. (30 points) Recall that Z represents a standard normal random variable, i.e., a normal random variable with mean 0 and standard deviation 1. For each of (a) and (b),

- sketch a standard normal curve and shade the area under the curve corresponding to the given probability
- use the spreadsheet command `=normsdist(x)` to find the exact value of the probability (to 5 decimal places); write down the exact spreadsheet command you use

(a) $P(Z < -0.6) =$

Solution: Using `=normsdist(-0.6)` we calculate that $P(Z < -0.6) = 0.27425$

(b) $P(Z > 1.5) =$

Solution: Using `=1 - normsdist(1.5)` (or `=normsdist(1.5)`), we calculate that $P(Z > 1.5) = 0.06681$

(c) $P(0 < Z < 3) =$

Solution: Using `=normsdist(3) - normsdist(0)`, we calculate that $P(0 < Z < 3) = 0.498650$

4. (30 points) A machine produces bolts which are supposed to be of length 4mm. Suppose the lengths of the bolts the machine actually produces are normally distributed with mean 4mm and standard deviation 0.3mm.

(a) Sketch the normal curve for this distribution. Label the points on the curve corresponding to the mean, 1 standard deviation above and below the mean ($\mu \pm \sigma$), and 2 standard deviations above and below the mean ($\mu \pm 2\sigma$).

Solution:

$$\mu - 2\sigma = 3.4, \mu - \sigma = 3.7, \mu = 4, \mu + \sigma = 4.3, \mu + 2\sigma = 4.6$$

(b) Suppose the company can only accept bolts which are between 3.7mm and 4.6mm. In your graph in part (a), shade in the areas under the curve corresponding to bolts which can be accepted.

Solution:

(c) Use the "approximate areas under a normal curve" to find the approximate proportion of bolts which will be accepted, i.e., find the the approximate value of $P(3.7 < X < 4.6)$. **As always, show your calculations.**

Solution: The approximate area under a normal curve between $\mu - \sigma$ and $\mu + 2\sigma$ is $0.34 + 0.34 + 0.135 = 0.815$, i.e., approximately 81.5% of the manufactured bolts are accepted.

(d) Now use the spreadsheet command `=normdist(b, μ, σ, true)` to compute the exact value of $P(3.7 < X < 4.6)$ (to 5 decimal places). Write down the spreadsheet command you use.

Solution: Using `=normdist(4.6,4,0.3, true) - normdist(3.7,4,0.3,true)`, we calculate that $P(3.7 < X < 4.6) = 0.81859$.