Class #25 - Monday, May 9

Section 6.3: Normal Random Variables (cont'd)

Approximation Rules

Notation: If a random variable X is normally distributed with mean μ and standard deviation σ, we write

$$X \sim N(\mu, \sigma)$$

All normal distributions have the following approximate probabilities (from p268 of the textbook):

Approximation Rule

A normal random variable with mean μ and standard deviation σ will be

Between $\mu - \sigma$ and $\mu + \sigma$ with approximate probability 0.68

Between $\mu - 2\sigma$ and $\mu + 2\sigma$ with approximate probability 0.95

Between $\mu - 3\sigma$ and $\mu + 3\sigma$ with approximate probability 0.997

This approximation rule is illustrated in Fig. 6.6. It often enables us to obtain a quick feel for a data set.

We can also express the first rule as: "Approximately 68% of observations of a normally distributed random variable are within one standard deviation of the mean."

(Similarly: "Approximately 95% of observations of a normally distributed random variable are within two standard deviations of the mean" and "Approximately 99.7% of observations of a normally distributed random variable are within three standard deviations of the mean.")

In symbols: if $X \sim N(\mu, \sigma)$, then:

$$P(\mu - \sigma < X < \mu + \sigma) \approx 0.68$$

$$P(\mu - 2\sigma < X < \mu + 2\sigma) \approx 0.95$$

$$P(\mu - 3\sigma < X < \mu + 3\sigma) \approx 0.997$$

Recall that these approximate probabilities correspond to certain areas under a normal curve. These are illustrated in Fig 6.6 in the textbook:

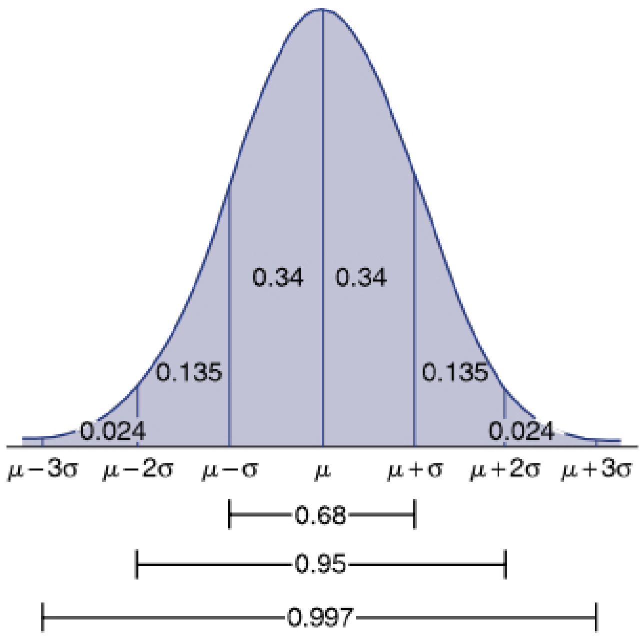

FIGURE 6.6
Approximate areas under a normal curve.

Example 1: HW #7: Sec 6.3, #1: We are told that the blood pressure of adults is normally distributed with mean 128.4 and standard deviation 19.6 (i.e., $X \sim (\mu = 128.4, \sigma = 19.6)$, where X is the random variable defined as the blood pressure of a randomly selected adult.

- Sketch the given normal distribution, labelling $\mu \pm \sigma, \mu \pm 2\sigma, \mu \pm 3\sigma$ on the horizontal axis and the approximate areas as in the Fig 6.6.
- Use these to answer the questions in the textbook.

Example 2: Use Fig 6.6 to find the approximate value of $P(X < \mu + \sigma)$, for $X \sim N(\mu, \sigma)$.

Spreadsheet command:

`=normdist(c, μ, σ, true)` returns the cumulative probability for a normal random variable with the given mean and standard deviation, i.e., for $X \sim N(\mu, \sigma)$, the spreadsheet command returns

$$P(X < c)$$

Example 3: Use the spreadsheet command to find the value of $P(X < \mu + \sigma)$, for $X \sim N(\mu, \sigma)$. (You can use any values for μ and σ; just make sure you enter $c = \mu + \sigma$).

Standard normal random variables:

A standard normal random variable has mean $\mu = 0$ and standard deviation $\sigma = 1$. The letter Z is usually used to refer to a standard normal random variable. Thus:

$$Z \sim N(0, 1)$$

The standard normal curve is shown below:

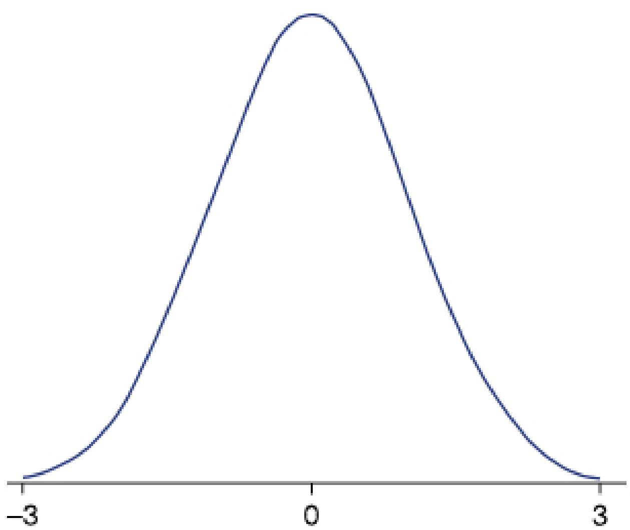

FIGURE 6.5
The standard normal curve.

Example 4: Write in the approximate areas (as in Fig 6.6) on the standard normal curve above. (Use this for Sec 6.3, #3-7.)