Class #16 - Wednesday, March 30
Section 5.2: Random Variables & Probability Distributions

HW #5 (due next Wednesday, April 6):

- **Section 4.7**: #1, 2, 3, 9, 10, 11 (For #9: first use the formula for $\binom{n}{r}$ & then check your answer using `=combin(n,r)` in a spreadsheet)
- **Section 5.2**: #3, 4, 5, 12, 13, 14 (for #4 & #5 you might find this listing of the sample space for rolling a pair of dice helpful:

(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

For a given probability experiment, often we're not interested in all the details of the outcome of the experiment, but instead we're more interested in the value of some *numerical value* determined by the outcome.

Such a numerical value associated with each outcome of a probability experiment is called a **random variable**:

Definition: A **random variable** is a numerical value associated with each outcome of a probability experiment. Random variables are typically denoted by symbols such as X and Y.

- You can think of a random variable as **a measurement** taken on the outcome of a probability experiment.
- Random variables can be discrete or continuous, depending on what is being measured. First we will study discrete random variables (Ch5 of the textbook). Later we will study continuous random variables (Ch6)

Example 1: For each of the following experiments, give a couple examples of random variables:

(i) If the experiment is rolling two dice, let $X =$

(ii) If the experiment is choosing a voter at random, let $X =$

(iii) If the experiment is a basketball game, let $X =$

(iv) If the experiment is flipping a coin n times, let $X =$

Definition: A **probability distribution** for a discrete random variable X is the collection of probabilities $P(X = x_i)$ for each possible value $x_1, x_2, \ldots, x_n$ that X can take on.

A discrete probability distribution must satisfy the following conditions:

- The probability of each value is between 0 and 1: $0 \leq P(X = x_i) \leq 1$
- The sum of all the probabilities is 1: $\Sigma_{i=1}^{n} P(X = x_i) = 1$

Example 2: Suppose the random variable X is the number of days that it will rain over the next 3 days. The following is a possible probability distribution for X. Verify that $\Sigma P(X = x_i) = 1$. Graph the distribution using a histogram.

Days of rain, x_i	**Probability** $P(X = x_i)$
0	0.21
1	0.44
2	0.29
3	0.06

Example 3: Consider (again) the probability experiment consisting of flipping a coin 3 times. Let the random variable X = the total number of heads observed.

(i) What is the sample space of the experiment? (Recall from Exam #1 and HW #3!)

(ii) What is the probability distribution of X? For the exam you were asked to calculate $P(X = 2)$ and $P(X = 3)$. So we also need to calculate $P(X = 0)$ and $P(X = 1)$:

Number of heads, x_i	Outcomes	**Probability** $P(X = x_i)$
0	$\{TTT\}$	
1	$\{HTT, THT, TTH\}$	
2	$\{HHT, HTH, THH\}$	$3/8 = 0.375$
3	$\{HHH\}$	$1/8 = 0.125$

(iii) Graph the probability distribution using a histogram.