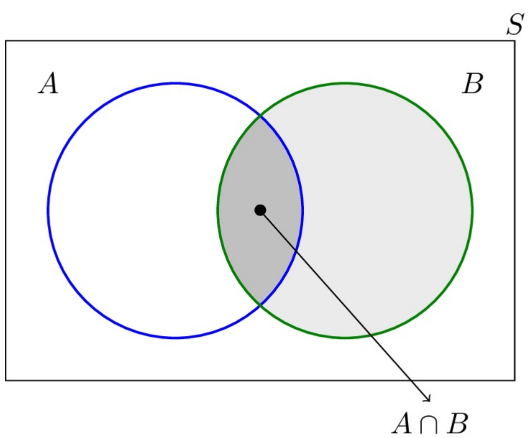
Class #12 - Monday, March 14
Section 4.5: Conditional Probability

Conditional Probability

- What is the probability of an event A given that we have some partial information about the outcome of the experiment? This is called *conditional probability.*
- notation: $P(A|B) =$ the conditional probability of A given that B has occurred (i.e., we have the information that B has occurred)
- formula for conditional probability:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

- intuition: since we know B has occurred, B is new "reduced" sample space; the formula above calculates what proportion of that new sample space is also in A



$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Examples:

- Ross Sec 4.3: Examples 4.9, 4.11

Note:

- If we reverse A and B in the formula for conditional probability, we have the following formula for the conditional probability of B given A:

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

Multiplication Rule

- take the formulas for conditional probability:

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

  and multiply through by the denominator in each equation ($P(A)$ and $P(B)$, respectively). This leads to the Multiplication Rule for $P(A \cap B)$, i.e., $P(A\&B)$, the probability that both A and B occur:

$$P(A \cap B) = P(B|A) * P(A) = P(A|B) * P(B)$$

Examples:

- Ross Sec 4.3: Example 4.12

Independence

- two events A and B are *independent* if knowing A has occurred doesn't change the probability of B, i.e.,

$$P(B|A) = P(B)$$

  Using the formula

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

  we see that A and B are independent if

$$\frac{P(A \cap B)}{P(A)} = P(B)$$

  i.e.,

$$P(A \cap B) = P(A)P(B)$$