Class #9 - Wednesday March 2
Probability: Basic Concepts & Properties

Textbook Readings:

- **Section 4.2**: Sample Spaces & Events
- **Section 4.3**: Properties of Probability & the Addition Rule
- **Section 4.3**: Experiments Having Equally Likely Outcomes

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What is the probability $P(A)$ of an event A? There are a few different ways of defining probability:

- "theoretical" or "classical" probability: in experiments where every outcome is equally likely, define the probability of an event A as

$$P(A) = \frac{\#\text{ of outcomes in the event} A}{\#\text{ of total outcomes in the sample space } S}$$

- "empirical" or "frequentist" probability: define $P(A)$ as the long-run relative frequency of A occurring (i.e., if the experiment is repeated a large number of times, what is the proportion of times that A occurs)
- "subjective" probability: a measure of an individual's belief that an event will occur

We will focus on "classical probability" for now, but we will discuss "empirical probability" in connection with statistics later in the course.

Basic Properties of Probability: For an experiment with sample space S, we assume that for each event $A \subseteq S$ there is a number $P(A)$, called the probability of A, with the following properties:

(1) The probability of any event A is between 0 and 1: $0 \leq P(A) \leq 1$

(2) The probability of the entire sample space S is 1: $P(S) = 1$

(3) For any two disjoint events A and B: $P(A \cup B) = P(A) + P(B)$

If we apply property (3) to any A and its complement A^C (which are obviously disjoint), we see that $P(A \cup A^C) = P(A) + P(A^C)$. But $A \cup A^C = S$ and $P(S) = 1$. So we have another property, which is a consequence of (2) and (3):

(4) $P(A) + P(A^C) = 1$, i.e., $P(A^C) = 1 - P(A)$

Addition Rule: For any events A and B,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Example 1: Consider the probability experiment of first flipping a coin and then rolling a 6-sided die.

(i) What is the sample space? How many outcomes are there in the sample space?

(ii) An example of an event A is $A =$ "Rolling a 4" $= \{$ H4, T4 $\}$. Another example of an event is $B =$ "Tossing heads and rolling an even number." List the outcomes that make up the latter event.

(iii) Calculate $P(A)$ and $P(B)$.

(iv) What is the complement of A, i.e., A^C? What is $A \cup B$? What is $A \cap B$? Are A and B disjoint?

(v) Calculate $P(A^C)$ and $P(A \cup B)$.

Example 2: The most recent US Census Bureau population estimates for NYC's five boroughs are show below (via `http://www.nyc.gov/html/dcp/html/census/popcur.shtml`):

Bronx	1,438,159
Brooklyn	2,621,793
Manhattan	1,636,268
Queens	2,321,580
Staten Island	473,279
Total NYC population	8,491,079

Note that this table is a frequency table! (Instead of classes consisting of numeric intervals, we have different categories–in this case the five boroughs.)

(a) Suppose a New York City resident is *randomly selected*. What is the sample space of this probability experiment?

(b) Calculate the probabilities of selected a resident of (i) the Bronx, (ii) Brooklyn, (iii) Manhattan (iv) Queens, (v) Staten Island.

(Note the connection with relative frequencies!)