Homework: Chapter 2.1

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5) Let $\mathbf{A} = \begin{bmatrix} -1 & 3 \\ 2 & 4 \\ 5 & -3 \end{bmatrix}$ $\mathbf{B} = \begin{bmatrix} 4 & -2 \\ -2 & 3 \end{bmatrix}$ $\mathbf{b1} = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$ $\mathbf{b2} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$

Compute the product of AB in two ways: (a) by the definition, where Ab1 and Ab2 are computed separately, and (b) by the row-column rule for computing AB.

Solutions.

a) **Ab1** $= \begin{bmatrix} -1 & 3 \\ 2 & 4 \\ 5 & -3 \end{bmatrix} \begin{bmatrix} 4 \\ -2 \end{bmatrix} = \begin{bmatrix} -1(4) + 3(-2) \\ 2(4) + 4(-2) \\ 5(4) + -3(-2) \end{bmatrix} = \begin{bmatrix} -10 \\ 0 \\ 26 \end{bmatrix}$

Ab2 $= \begin{bmatrix} -1 & 3 \\ 2 & 4 \\ 5 & -3 \end{bmatrix} \begin{bmatrix} -2 \\ 3 \end{bmatrix} = \begin{bmatrix} -1(-2) + 3(3) \\ 2(-2) + 4(3) \\ 5(-2) + -3(3) \end{bmatrix} = \begin{bmatrix} 11 \\ 8 \\ -19 \end{bmatrix}$

AB = **A**[**b1** **b2**] $= \begin{bmatrix} -10 & 11 \\ 0 & 8 \\ 26 & -19 \end{bmatrix}$

b) **AB** $= \begin{bmatrix} -1 & 3 \\ 2 & 4 \\ 5 & -3 \end{bmatrix} \begin{bmatrix} 4 & -2 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} -1(4) + 3(-2) & -1(-2) + 3(3) \\ 2(4) + 4(-2) & 2(-2) + 4(3) \\ 5(4) + -3(-2) & 5(-2) + -3(3) \end{bmatrix} = \begin{bmatrix} -10 & 11 \\ 0 & 8 \\ 26 & -19 \end{bmatrix}$

7) If matrix A is 5 x 3 and the product of AB is 5 x 7, what is the size of B?

Solution.

The size of B is 3 x 7 because:

a) In order for the product of two matrices to exist (not being undefined) the number of columns in A must be equal to the number of rows in B. Meaning, since A has 3 columns, B has 3 rows.

b) The size of the product, AB, is equal to the number of rows in A by the number of columns in B. Therefore, because AB has 7 columns, B has 7 columns.

9) Let **A** = $\begin{bmatrix} 2 & 3 \\ -1 & 1 \end{bmatrix}$ **B** = $\begin{bmatrix} 1 & 9 \\ -3 & k \end{bmatrix}$

What value(s) of k, if any, will make AB = BA?

Solution.

AB = $\begin{bmatrix} 2 & 3 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 9 \\ -3 & k \end{bmatrix} = \begin{bmatrix} 2(1)+3(-3) & 2(9)+3(k) \\ -1(1)+1(-3) & -1(9)+1(k) \end{bmatrix} = \begin{bmatrix} -7 & 18+3k \\ -4 & -9+k \end{bmatrix}$

BA = $\begin{bmatrix} 1 & 9 \\ -3 & k \end{bmatrix}\begin{bmatrix} 2 & 3 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} -1(4)+3(-2) & -1(-2)+3(3) \\ 2(4)+4(-2) & 2(-2)+4(3) \\ 5(4)+-3(-2) & 5(-2)+-3(3) \end{bmatrix} = \begin{bmatrix} -7 & 12 \\ -6-k & -9+k \end{bmatrix}$

In order to find the proper value of k, if it exists, one must compare each entry of AB to the corresponding entry of BA(i.e. the entries with the same (i,j) values). Only values of k consistent between all entries will make AB equal to BA. If there are no consistent values for k in all entries, then there is no value of k which would make AB equal to BA.

(1,1): -7 = -7 Any value of k will make AB at (1,1) equal BA at (1,1).

(1,2): 18 + 3k = 12 → 3k = -6 → k = -2 AB at (1,2) will equal BA at (1,2) when k is -2.

(2,1): -4 = -6 - k → 2 = -k → k= -2 AB at (2,1) will equal BA at (2,1) when k is -2.

(2,2): -9 + k = -9 + k → k = k Any value of k will make AB at (2,2) equal BA at (2,2).

The only value all 4 entries share in common is -2, therefore AB is equal to BA when k = -2.