New York City College of Technology

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Final Project: Fourier Series

Final Project Progress Report

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Abstract

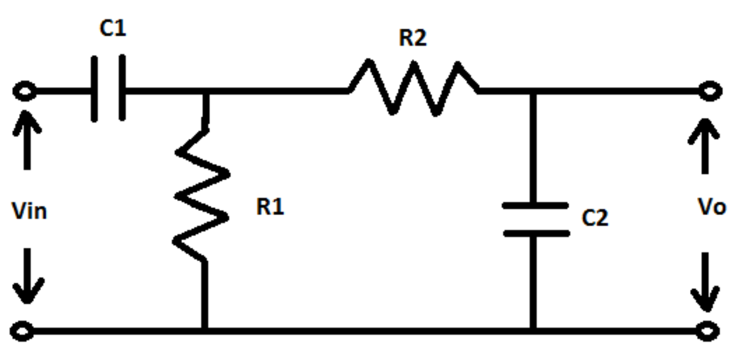
Periodic signals can be represented by a sum of sine and cosines functions. Therefore, a periodic signal can be discomposed into its components or harmonics. This has great applications for signal analysis by the implementation of theoretical analysis such as Fourier Series and by using RC or RLC circuit. By designing a tunable band pass filter, different harmonics of any periodic signal can be extracted from such signal. This circuit can be first simulated using software such as Multisim and ultimately constructed using hardware. This could be a very useful tool that can be used to experimentally obtain the different components of a periodic signal. Also, this circuit can be used to experimentally determine the Fourier Series coefficient of a periodic signal. In order to extract the components of the periodic signal, the output of the function generator is connected to the input of the filter and to a spectral analyzer which will be used to see the different components of the input signal in frequency domain. Then, the output of the filter is connected to an oscilloscope in order to see the extracted component in time domain.

<u>Background:</u>

It is very difficult to analyze a complex signal in time domain. Complex signals are compose of many other signals which overlap with each other in time domain, making it difficult to visualize the nature of such signals. Therefore, there is the need of transforming such signals into a form easier to visualize and analyze. Fourier Series is a mathematical tool use to convert signal from time domain into frequency domain. This tool is useful for signal analysis and signal processing with a wide application in engineering and scientific research. By transforming a signal from time domain into frequency domain, it is possible to visualize the manipulation of each component of the complex signal. Some of the applications where Fourier Series are use are; signal modulation for telecommunication, noise suppression, intelligent (information) recovery from modulated signals and filters design.

Band pass filters

Band pass filter is a combination of a Low pass filter and a High pass filter. A band pass filter can be obtained by connecting an RC low pass filter and an RC High pass filter. The following diagram shows an RC Band pass filter:



RC Band Pass Filter

The bandwidth of the Band pass filter is nothing more than the cut off frequencies of both the high pass filter and the low pass filter. Band pass filters allow to select signals only within the filter bandwidth such that signals out of the bandwidth of the band pass filters will be rejected. Band pass filters pass frequencies lower than the cutoff frequency of the high pass filter and higher than the cutoff frequency of the low pass filter. Band pass filters will reject frequencies higher than the cutoff frequency of the low pass filter and less than the cutoff frequency of the high pass filter. The cutoff frequency of both high and low pass filters can be calculated as follow:

$$f_{\text{CUTOFF}} = \frac{1}{2\pi RC} \quad \textbf{(Equation 1)}$$

In summary, when the low pass filter is set to a specific cutoff frequency, only signals less than the cutoff frequency will pass through the filter and the higher frequencies will be filtered out. When the High pass filter is set to a specific cut of frequency, only signals greater than the cutoff frequency will pass through the filter and the lower frequencies will be filtered out.

Basics of the spectral of a signal

One of the widely used methods for signal analysis is spectrum, analysis. Spectral analyzers analyze signals in frequency domain. When analyzing complex signals in frequency domain, complex signals are separated into their frequency components and thus the level at each frequency can be displayed. A frequency domain view of the spectrum of a signal allows to easily measure the signals' frequency, harmonic content and modulation. Spectrum analysis can be obtained by a fast Fourier transform analyzer which converts the time-domain signal to frequency domain.

Fourier series

Fourier series can represent any periodic signal as a sum of sine and cosine functions. The more terms of the Fourier series, the more the signal its approximated to a squared wave and thus can be analyze in frequency domain. Any continuous periodic function can be represented by a sum of sinusoidal functions. In this project, a squared wave signal will be the input frequency of the band pass filter and the output frequency will be represented as a sum of sine and cosine, allowing taking the different components of the output signal.

The Fourier series expansion can be expressed as follow:

$$\boldsymbol{f(t) = a_0 + \sum_{i=1}^{N} a_i \cos(\omega i t) + \sum_{i=1}^{N} b_i \sin(\omega i t)}$$ **Equation 1**

Where $\omega i = 2\pi f_1$, f_1 *is the fundamental frequency*

The expansion for the furrier series is as follow:

$$\boldsymbol{f(t) = a_0 + a_1 \, cos(t) + a_2 \, cos(2t) + a_3 \, cos(3t) + \cdots + b_1 \, sin(t) + b_2 \, sin(2t) + \cdots}$$

(Equation 2)

Finding the coefficients a_n and b_n in terms of f. By assuming that the trigonometric series converges and that it is continuous from the intervals of $-\pi$ *to* π, by integrating both sides of the power series as follow;

$$\boldsymbol{\int_{-\pi}^{\pi} f(t)dt = \int_{-\pi}^{\pi} a_0 \, dt + \int_{-\pi}^{\pi} \sum_{n=1}^{\infty} (a_n \, cos(nt) + b_n \, sin(nt)dt}$$

$$= 2\pi a_0 + \sum_{n=1}^{\infty} a_n \int_{-\pi}^{\pi} \cos(nt)\, dt + \sum_{n=1}^{\infty} b_n \int_{-\pi}^{\pi} \sin(nt)\, dt$$

Recall that:

$$\int_{-\pi}^{\pi} cos(nt)\, dt = \frac{1}{n} \; sin\,(nt)\Big]_{-\pi}^{\pi} \quad = \frac{1}{n} \; [sin\,(n\pi) - sin(-n\pi)] = (0)$$

Since n is an integer , also $\int_{-\pi}^{\pi} sine(nx)\, dx = 0$ and therefore $\int_{-\pi}^{\pi} f(t)\, dt = 2\pi a_0$

From this, the coefficient a_0 ca be obtained as follow;

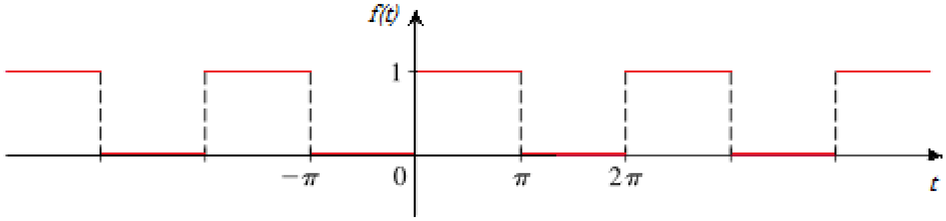
$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(t) dt \quad \textbf{Equation (3)}$$

The coefficients a_n and b_n can be obtained by multiplying equation 2 by (cos(mx)) where m is an integer greater or equal to one and then integrating from the interval of $-\pi$ *to* π. Taking the same approach of multiplying equation 2 by (sin(mx)) will give the coefficients of b_n .

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(t) \cos(nt)\, dt \qquad n = 1, 2, 3, \ldots \qquad \textbf{Equation (4)}$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(t) \sin(nt)\, dt \qquad n = 1, 2, 3, \ldots \qquad \textbf{Equation (5)}$$

For Example, the following square wave function $f(t) = \begin{cases} 0 \; if \; -\pi \le t < 0 \\ 1 \; if \; 0 \le t < \pi \end{cases}$



$$\boldsymbol{a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(t)dt = a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} 0\, dt + \frac{1}{2\pi}\int_{-\pi}^{\pi} 1\, dt = 0 + \frac{1}{2\pi}(\pi) = {}^{1}/_{2}}$$

$$\boldsymbol{a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\cos(nt)\, dt = \frac{1}{\pi}\int_{-\pi}^{0} 0\, dt + \frac{1}{\pi}\int_{0}^{\pi} cos(nt)dt}$$

$$= 0 + \frac{1}{\pi}\left.\frac{\sin nt}{n}\right]_{0}^{\pi} = \frac{1}{n\pi}(\sin(n\pi) - \sin(0)) = 0$$

$$\boldsymbol{b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\sin(nt)\, dt = \frac{1}{\pi}\int_{-\pi}^{0} 0\, dt + \frac{1}{\pi}\int_{0}^{\pi} \sin(nt)\, dt}$$

$$= -\frac{1}{\pi}\left.\frac{\cos(nt)}{n}\right]_{0}^{\pi} = -\frac{1}{n\pi}(\cos(nt) - \cos(0))$$

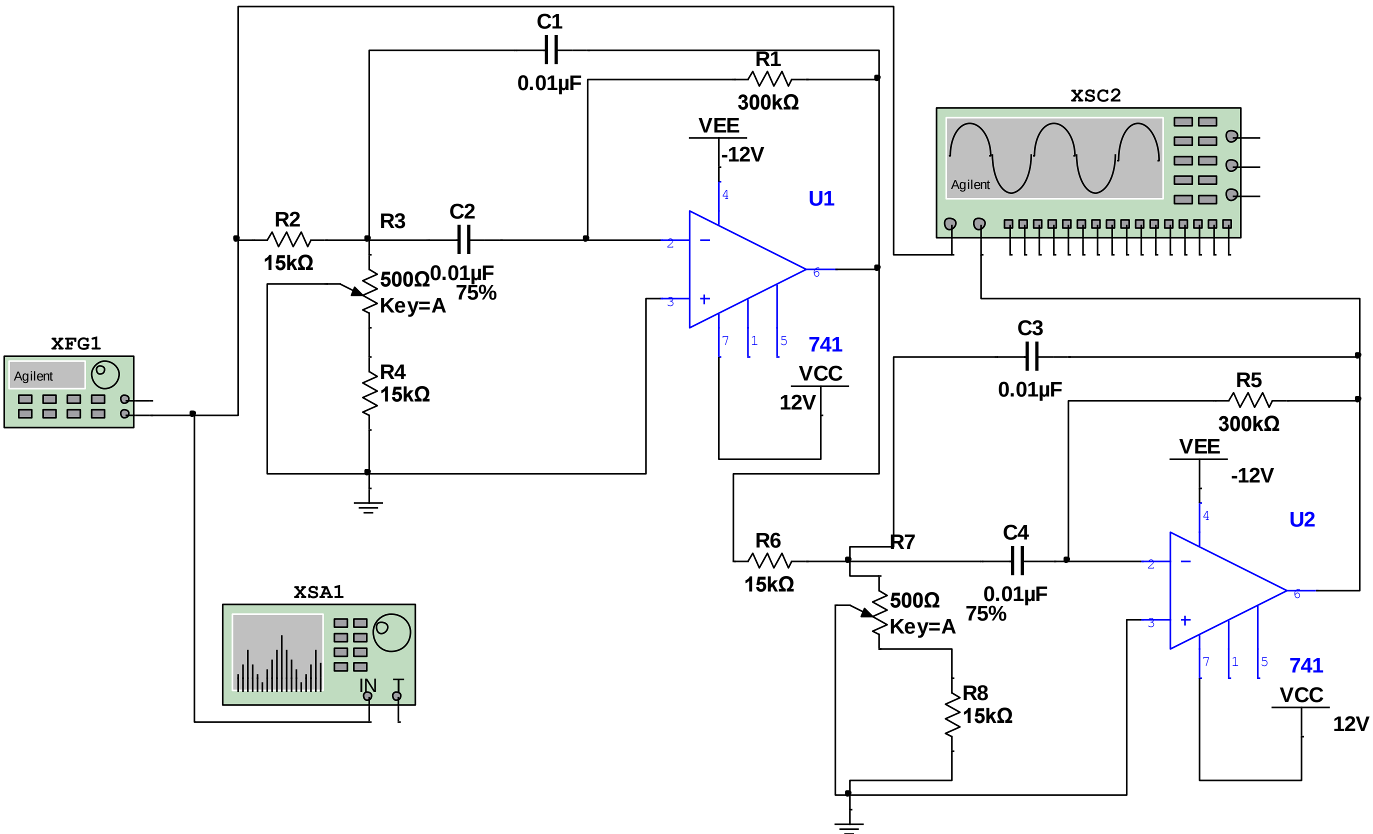
If n is even, a_n = 0 if n is odd then $a_n = \frac{2}{n\pi}$

Therefore, the Fourier series will only contain the odd a_n coefficient components.

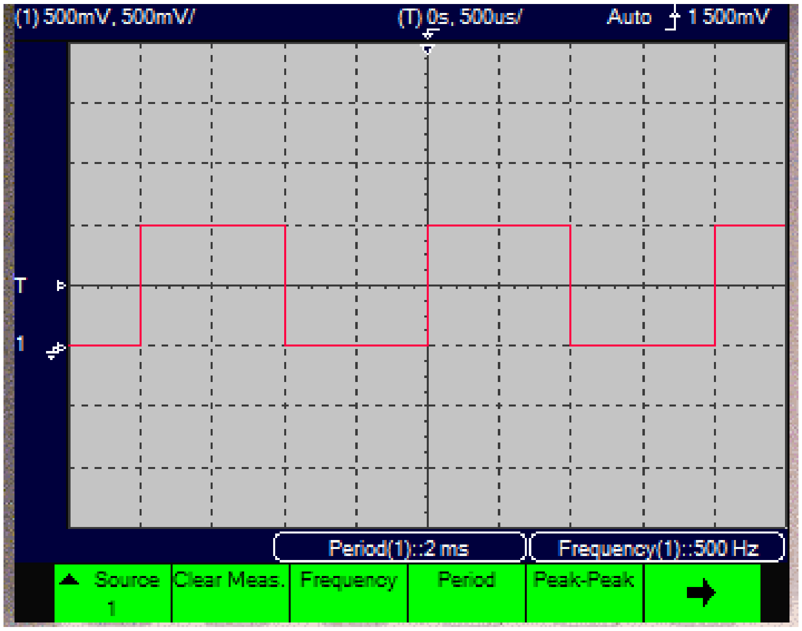
Experimental:

Multisims Simulation

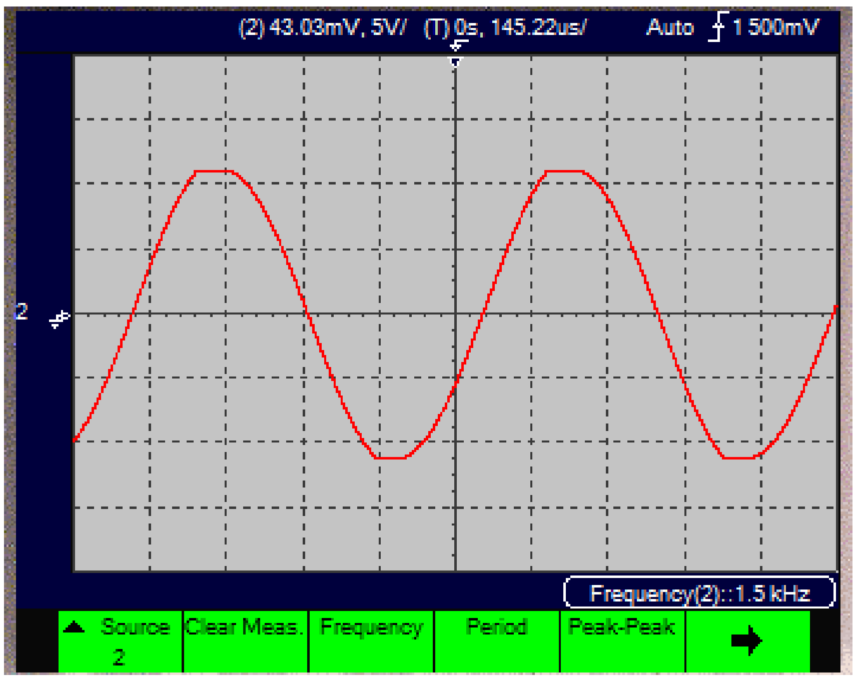
Active Band Pass Filter



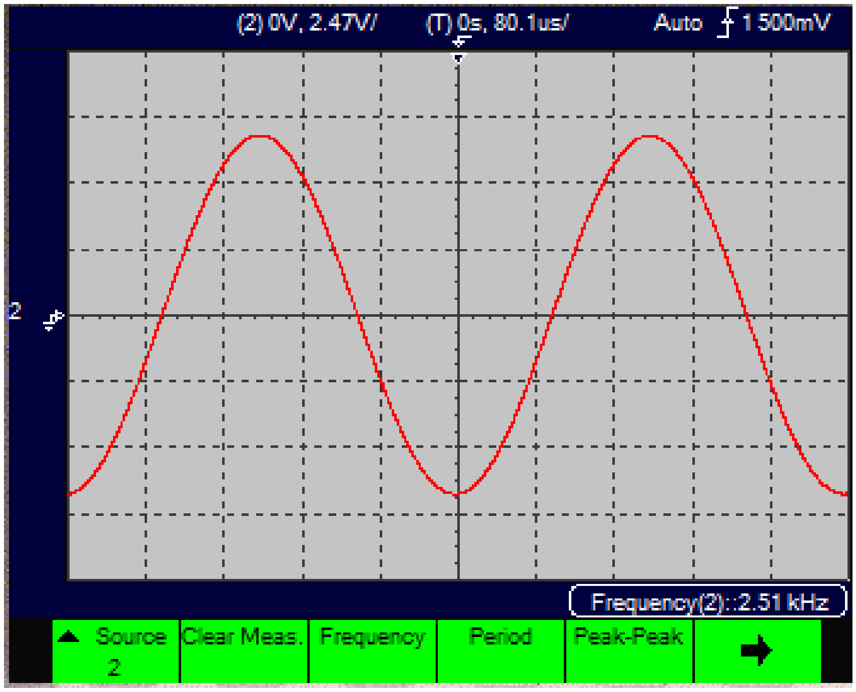
Input signal



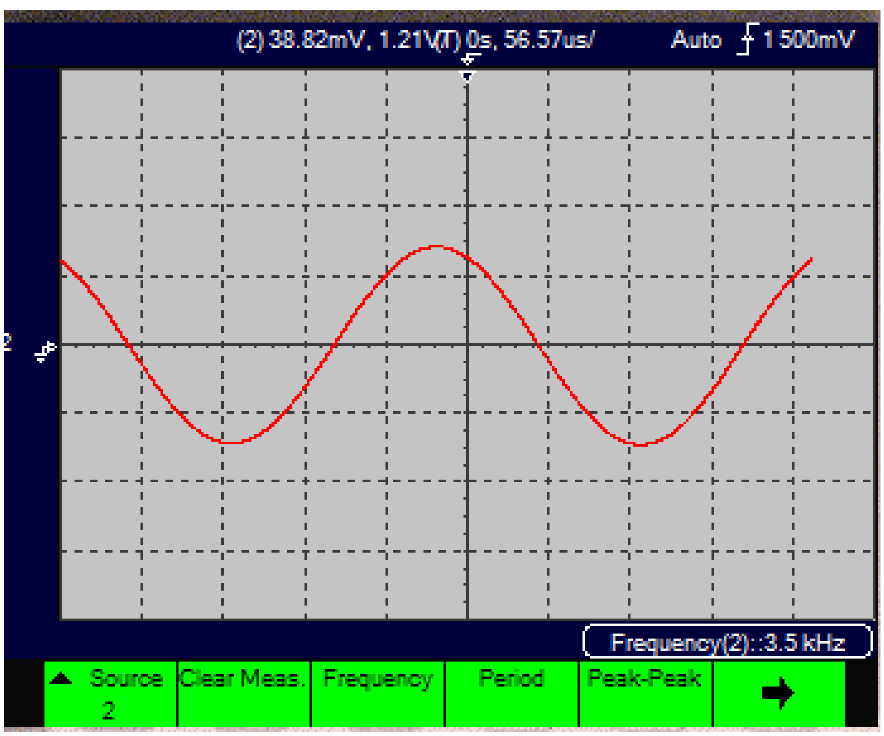
Third Harmonic of the Fourier Series (1.5Hhz)



Fifth harmonic of the Fourier Series (2.5KHz)



Seventh harmonic of the Fourier Series (3.5KHz)



Conclusion:

In conclusion, any periodic signal such as a square wave can be represented as a function of sine and cosine wave. Fourier series allows obtaining each component of the signal and represented as a sum of cosines and sine wave. The more components the Fourier series have, the most accurate the representation of the function will be. The first harmonic of the signal is the fundamental frequency and the coefficient a_0 represents the DC offset of the signal. The coefficients a_1, a_2,….a_n and b_1, b_2,…b_n are the amplitude of the frequency counter which are the amplitude in relation to the energy. Most of the energy is in lower frequency therefore, as the frequency increases, the coefficients a_n decreases. The highest the frequency of the signal is, the lower the energy carries. A tunable band pass filter was simulated using Multisims. Different harmonics of the periodic signal were extracted from the signal. Fourier series is a very useful tool that is used to experimentally obtain the different components of a periodic signal

References:

Rogawski, Jon. *Calculus*. New York: W.H. Freeman, 2008. Print.

"Fourier Series - The Fourier Coefficients." *TheFourierTransform.com*. Web. 20 Nov. 2014. <http://www.thefouriertransform.com/series/coefficients.php>.

Spectrum Analyzer Papers. Calif.: Hewlett Packard, 1971. Web. 19 Nov. 2014.

"Active Band Pass Filter - Op-amp Band Pass Filter." *Basic Electronics Tutorials*. N.p., 14 Aug. 2013. Web. 18 Nov. 2014.