New York City College of Technology
Mathematics Department

MAT 2440/D646 Discrete Structures and Algorithms I - Writing Intensive

Tuesday/Thursday 10:00 am-11:40 am room N923

Instructor: Dr. Marianna Bonanome
e-mail: mbonanome@citytech.cuny.edu, phone: 718-260-5292

Office Hours: Tuesdays 12 pm - 1 pm, Thursdays 3 pm - 4 pm, room N602B.

Text: Discrete Mathematics and Its Applications, 7th edition by Kenneth H. Rosen McGraw-Hill.

Course Description: This course introduces the foundations of discrete mathematics as they apply to computer science, focusing on providing a solid theoretical foundation for further work. Topics include functions, relations, sets, simple proof techniques, Boolean algebra, propositional logic, elementary number theory, writing, analyzing and testing algorithms.

Prerequisites: MAT 1375 or higher, and CST 2403 or CST 1201

Writing Intensive: This course has been designated as a “Writing Intensive” (WI) course by City Tech. The requirements include both formal (graded) and informal (non-graded) writing assignments. Both kinds of writing assignments must be completed. A large portion of your grade in this course (20% worth) will be based on the completion and quality of these assignments, so please take them seriously and talk to me about anything you find confusing. These assignments are not intended to make your life as a student difficult. Instead, they should help you to better understand the course material, evaluate it in light of your own lived experiences, and help you to think more closely and deeply about social problems.

Grading:

93.0-100	A	77.0-79.0	C+
90.0-92.9	A-	70.0-76.9	C
87.0-89.9	B+	60.0-69.9	D
83.0-86.9	B	0-59.9	F
80.0-82.9	B-		

W = withdrawal up to 4/1/2019, WF = withdrawal after 4/1/2019 (WF = F), Note: Students who are failing should consider withdrawing officially before 4/1/2019 to avoid an F or WF.

Exams (50% of grade):

Exam questions will be modeled after problems assigned for homework. There will be 4 exams + a final exam. The lowest of the 4 exam grades will be dropped. Exams will last for 1 hour and will start at the beginning of the period. No extra time is given for students who arrive late. Exam dates are**: 2/21, 3/19, 4/11 and 5/14.**

***NO MAKE-UP EXAMS WILL BE GIVEN**

A missed exam will count as a 0 and will be the lowest grade that is dropped.

Final Exam (30% of grade):

A full period final examination is given the last class meeting of the semester **5/21**. It covers all topics studied. It **must** be taken to pass the course. No student will be allowed to take the final exam early.

Assignments (20% of grade):

It is crucial that you stay on top of the homework in this class. The course outline contains a list of homework problems for the whole course, and it can be found on our OpenLab site (see below) in the math dept's course outline under "Files" in our profile. Additionally, there will be 4 assignments given this semester which will be collected, each containing a strong writing component. They are due at the beginning of the class period on: **2/14, 3/12, 4/4 and 5/7**. Assignments late by 1 day will be penalized by 25%, 2 days late 50%, 3 days late 75%, any later and they will no longer be accepted.

Attendance:

Attendance will be taken at the beginning of every class. Lateness and students leaving before the end of the period will be recorded. It is important that you attend class and that you are on time.

Help:

Help is available during my office hours or in the Liberal Arts Learning Center AG18. Do not wait until the last minute before an exam to get help. If there is something you are having trouble with, get help as soon as possible.

OpenLab:

General information on the course, homework, assignments and tests can be found on our course OpenLab site at:

https://openlab.citytech.cuny.edu/mat2440-d646-bonanome/

You are expected to consult our OpenLab site and check your City Tech email regularly. It is strongly recommended that you become a member of our course site so that you will receive email notifications of important posts and files uploaded

Session	Date	Sections	Exam/Assignment
1	1/29	1.1, 1.2	
2	1/31	1.3	

Session	Date	Sections	Exam/Assignment
3	2/5	1.4, 1.5	
4	2/7	1.6	
5	2/14	1.7	Assignment #1 due
6	2/19	1.7	
7	2/21		Exam #1
8	2/26	2.1, 2.2	
9	2/28	2.3	
10	3/5	2.4	
11	3/7	2.5, 3.1	
12	3/12	3.1	Assignment #2 due
13	3/14	3.1	
14	3/19		Exam #2
15	3/21	3.2	
16	3/26	3.2	
17	3/28	3.3	
18	4/2	3.3	
19	4/4	4.1, 4.2	Assignment #3 due
20	4/9	4.3, 4.4	
21	4/11		Exam #3
22	4/16	4.5	
23	4/18	4.6	
24	4/30	4.6	
25	5/2	5.1	
26	5/7	5.1	Assignment #4 due
27	5/9	Finite state automata and the Tower of Hanoi	

Session	Date	Sections	Exam/Assignment
28	5/14		Exam #4
29	5/16	Final Exam Review	
30	5/21		Final Exam

Classroom behavior:

Inappropriate behavior will **not** be tolerated in this classroom. Sleeping during class sessions will not be overlooked. Cell phones, smart watches and devices, iPods etc. are not allowed at any time during the class session, please turn all devices off and take your headphones/ earphones off before entering the classroom.

Accommodations:

If you need any special accommodations, please proceed to take your request to Student Support Center which is located at A237. For more information, please contact the department at 718-260-5143.

Academic Integrity Policy:

Academic dishonesty is prohibited in The City University of New York. Penalties for academic dishonesty include academic sanctions, such as failing or otherwise reduced grades, and/or disciplinary sanctions, including suspension or expulsion.

2017-2018 College catalog pages 61-62

I reserve the right to make any changes to this syllabus that I see fit for your academic progress. All changes will be announced in class, and you are responsible for keeping up with such changes in case you were absent when they were announced.

MAT 2440 Assignment #1

This assignment is due on XX/XX/20XX at 10 am - at the beginning of our class period. You may submit it electronically as a pdf document or as a hard copy. Assignments late by 1 day will be penalized by 25%, 2 days late 50%, 3 days late 75% and any later they will no longer be accepted.

Part of this assignment is group work. I will be assigning groups in class. Please be sure to exchange contact information with your group members. You will be solving and writing logic puzzles. **Each** member of your team must **write** a puzzle to be solved by another team member and **solve** a puzzle by a team member in order to receive full credit.

Logic Puzzle #1
There are three neighbors living in a row. Each house has a different color and a different animal and person living in/at the house. Each person has a different profession. **Determine the color of the house that the horse lives at** given these clues: The horse lives in the first house. The doctor is Jamal's neighbor. Jamal does not have a Feret as a neighbor. Carlos does not live in the blue house and Jamal does not live in the green hourse. Ann is a lawyer and lives in the 3rd house. The professor has the horse as a neighbor. The mouse does not live in the red house. The lawyer does not live in the blue house.

Logic Puzzle #1
Three friends with different jobs and different pets live in consecutive houses on a block, their names are Sue, James and Maria (names not given in any specific order). Their houses are painted different colors. **Determine who owns a zebra** given these clues: Sue lives in the blue house. James does not live next to Sue. The owner of the first house is a professor. The dentist owns a dog. The yellow house is to the right of the red house. The owner of the yellow house owns a dog. The blue house is the last house. The parrot owning musician lives next to the dentist. The zebra's owner is a professor.

1. For **each** logic puzzle:
   (a) Decide on symbols to represent your information, for example "S" can represent "Sue" and "R" can represent "red house." [5 points per puzzle]
   (b) Make a table where the rows represent the friends and the columns represent the color of their houses, their pets and their jobs. Use logical reasoning to determine the correct entries in the table. [10 points per puzzle]
   (c) For every entry you must write at least one sentence justifiying the choice. If the information was given in the assignment prompt, say so. [5 points per puzzle]
2. Create your own logic puzzle for one member of your team to solve. Be sure to be clear and thorough in your puzzle description and solve your puzzle before sharing it! Your puzzle **MUST** contain the same categories as Logic Puzzles # 1 and #2: three neighbors, three professions, house order (1st, 2nd, 3rd), three house colors and three pets. [30 points]

3. Choose a member of your team and solve their puzzle. Write up a **feedback report** to give to that team member. This report must be included with this assignment in order to receive credit for this part. The feedback report should contain the following information:

   (a) The solution to your team member's puzzle. (If the puzzle was not solvable, why not?) [10 points]

   (b) What suggestions would you make for improvement to the puzzle? [10 points]

   (c) Was the puzzle level appropriate? For example, the puzzle was too easy or too hard. [10 points]

Please be sure this writing is your own - do NOT borrow from a friend OR copy directly from the internet. I want to hear your own voice, not read a copy and paste of some other source!!! Paraphrasing is okay of course and make sure you cite your sources properly. It may be helpful to look at the writing resources for students on City Tech's Writing Across the Curriculum website at:
https://openlab.citytech.cuny.edu/writingacrossthecurriculum/student-resources/
and the documents posted on our OpenLab site. Please contact me for additional help and keep in mind the college's policy on academic integrity found here:
http://www.citytech.cuny.edu/academics/docs/academic_integrity_policy.pdf

MAT 2440 Assignment #2 - The Lamplighter Group L_2

1 L_2 as a dynamical system

We take our definition of dynamical system to be an "object" along with a specific set of modifications that can be performed (dynamically) upon this object. In this case, the object is a bi-infinite straight road with a lamp post at every street corner. There are two possible types of modifications: the lamplighter can walk any distance in either direction from a starting point and the lamplighter can turn the lamps "on" or "off." At any given moment the lamplighter is at a particular lamp post and a finite number of lamps are illuminated while the rest are not. We refer to such a moment, or configuration, as a "state" of the road (not to be confused with the "state" of an automaton). Any time the configuration changes, the road is in a new state. The road's state is changed over time by the lamplighter either walking to a different lamp post or turning lamps on or off (or both).

In Figure 1, the bi-infinite road is represented by a number line; the lamps are indexed by the integers. Lamps that are on are indicated by stars; lamps that are off by circles. The position of the lamplighter is indicated by an arrow pointing to an integer. The current state of the road is called the *lampstand*.

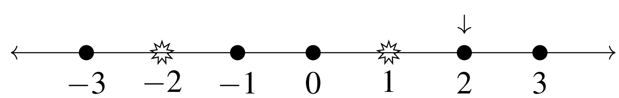

Figure 1: *A lampstand where two lamps are illuminated and the lamplighter stands at 2.*

Let us call the set of all possible lampstands $\mathscr{L}$. Now that we have a visual image, we can formalize the dynamics of changing a lampstand by specifying distinct tasks which the lamplighter can perform on any element of $\mathscr{L}$.

1. Move right to the next lamp.

2. Move left to the next lamp.

3. Switch the current lamp's status (from on to off or off to on).

4. Do nothing.

For any reconfiguration, the lamplighter performs only finitely many tasks. These tasks can be interpreted as functions τ, σ and I, whose domain and range are $\mathscr{L}$. Given a lampstand $l \in \mathscr{L}$, $\tau(l)$ is the result of performing the first task on l, $\sigma(l)$ is the result of performing the third task on l and $I(l)$ the result of performing the fourth task on l.

Proposition 1. σ is bijective.

Proof. To see that σ is onto, let l_1 be any lampstand in $\mathscr{L}$, and suppose that the lamplighter stands at lamp k. Define l_0 as the lampstand whose lamplighter stands at lamp k and whose lamps are in the same configuration as those in l_1, **except** for lamp k. If k is on in l_1, it is off in l_0; if it is off in l_1, it is on in l_0. Then $\sigma(l_0) = l_1$.

To see that σ is one-to-one, suppose that $\sigma(l_0) = \sigma(l_0') = l_1$, with the lamplighter in l_1 standing at lamp k. Since σ does not cause the lamplighter to move, the only effect it has on a lampstand is to switch the status of the current lamp. Whatever the status of lamp k is in l_1, it must be in the opposite state in both l_0 and l_0'. All other attributes of both l_0 and l_0' must match the other attributes of l_1; hence, $l_0 = l_0'$.

□ □

The reader will prove that τ is also bijective in Exercise 2 at the end of this chapter. Hence, both σ and τ have inverses. $\tau^{-1}(l)$ is the result of performing the second task on l. Note that σ is its own inverse. Thus $\sigma^2 = 1$.

If we let the lamplighter stand at 0 with all the lamps turned off, this configuration is called the *empty lampstand* and is denoted e. See Figure 2.

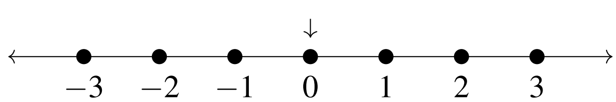

Figure 2: *The empty lampstand e*

Example 1. Consider the lampstand l_1 in Figure 3.

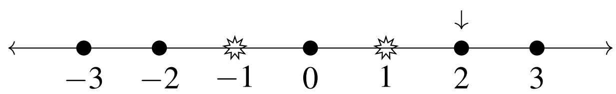

Figure 3: *The lampstand l_1*

Starting with the empty lampstand e, we can apply a composition of functions τ, τ^{-1}, σ and I to achieve l_1. For instance the composition $\tau\sigma\tau\tau\sigma\tau^{-1}$ (or $\tau \circ \sigma \circ \tau \circ \tau \circ \sigma \circ \tau^{-1}$) applied to e yields the lampstand configuration l_1. In keeping with standard function notation, the order of the composition is such that τ^{-1} is applied to e first and so on, reading from right to left. Figure 4 shows the details of the transformation from e to l_1.

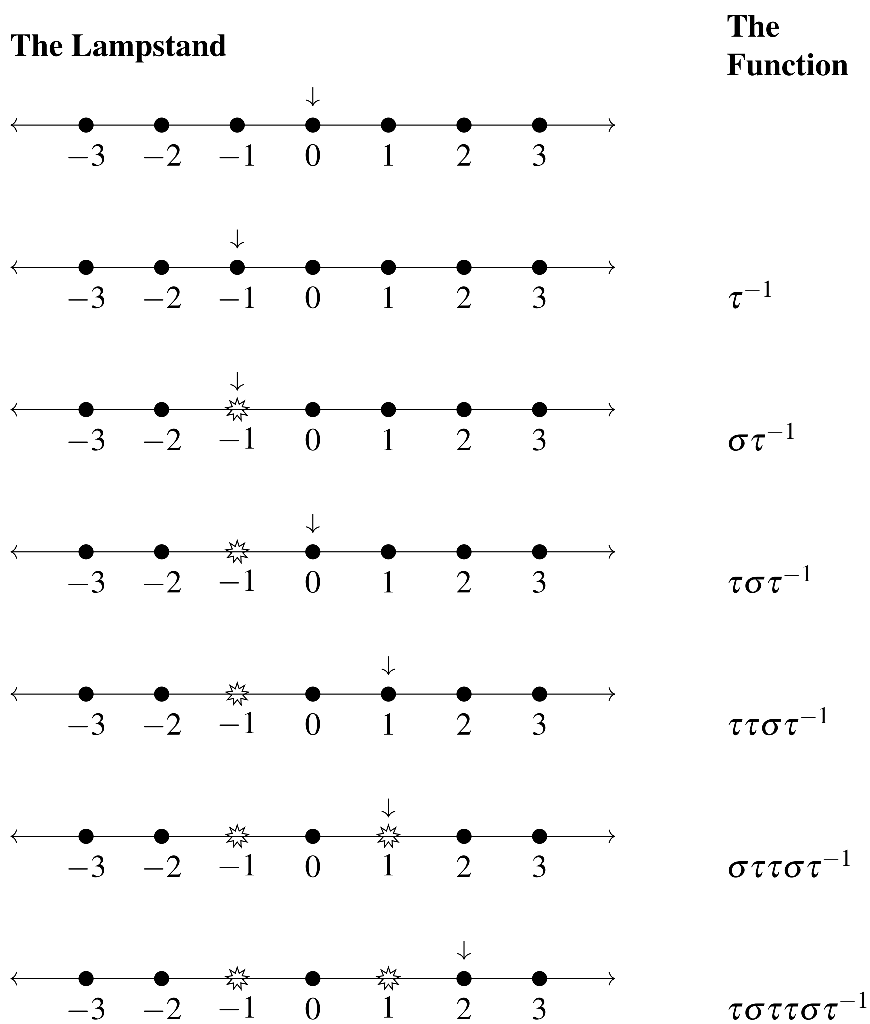

Figure 4: *A sequence of lampstands from the empty lampstand to* l_1 $\diamond$

To get the same lampstand l_1 (Figures 3 and 4), we could easily have applied a different function composition to e, for instance

$$\tau I \tau \tau I \sigma \tau^{-1} \tau^{-1} \sigma \tau.$$

For that matter, pick any $l \in \mathscr{L}$ as input. These two different-looking functions always have the same output.

$$\tau\sigma\tau\tau\sigma\tau^{-1}(l) = \tau I\tau\tau I\sigma\tau^{-1}\tau^{-1}\sigma\tau(l).$$

It doesn't matter that there are different function compositions representing the same lampstand, since two functions are defined to be the same function as long as the domains are the same and the outputs are the same. However, some function compositions are clearly "shorter" than others. Here "shorter" refers to the number of tasks in the function composition. This begs the question, is there a "shortest" function composition for a given lampstand configuration? You will explore this in the Exercise 4 below.

2 Assignment #2

This assignment is due on XX/XX/20XX at 10 am - at the beginning of our class period. You may submit it electronically as a pdf document or as a hard copy. Assignments late by 1 day will be penalized by 25%, 2 days late 50%, 3 days late 75% and any later they will no longer be accepted. You must submit your OWN work, you may not submit a group report.

Exercise 1. Check that the lampstand l_2 shown in Figure 5 can be arrived at by starting with the empty lampstand and applying this composition of functions - show/draw all steps: [20 points]

$$\sigma\tau\sigma\tau^{-1}\sigma\tau^{-1}\tau^{-1}\sigma\tau\sigma.$$

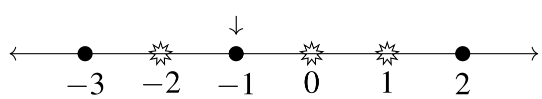

Figure 5: *The lampstand l_2*

Exercise 2. Prove that τ is bijective. Please be sure to write out your reasoning and show all of your steps. [30 points]

Exercise 3. Draw your own lampstand configuration and write down its function composition (consisting of τ, τ^{-1} and σ) and **share it** with one of your team members. You must also include: **the drawing of the lampstand** and **its function composition** here with this assignment in order to receive credit for this part. [10 points]

Exercise 4. a. Consider the lampstand shared with you by your fellow team member. Write down **two more** distinct function compositions (consisting of τ, τ^{-1} and σ) besides the one shared with you to represent your fellow team member's lampstand. [10 points]

b. Of your three different function compositions (from part a.) representing the lampstand shared with you, which is the "shortest"? How many function compositions of τ, τ^{-1} and σ does it have? [5 points]

c. What is the minimum number of function compositions necessary to express the lampstand shared with you? [5 points]

d. Describe a general method for finding the shortest function composition to represent *any* lampstand l where the lamplighter stands to the left of 0, with finitely many positive and negative lamps lit. [10 points]

e. How would you modify your method if the starting position of the lamplighter was to the right of 0? [10 points]

Please be sure this writing is your own - do NOT borrow from a friend. I want to hear your own voice, not read a copy and paste of some other source!!!

The Fibonacci Sequence - Activity

Leonardo Fibonacci discovered the sequence in the 13th century in connection with this problem: A rabbit colony begins with one pair of adult rabbits (one male, one female). Each adult pair produces one pair of babies (one male, one female) every month. Each pair of baby rabbits becomes adult and produces the first offspring at age two months. Assuming that no rabbits die, how many adult pairs of rabbits are in the colony at the end of n months ($n = 1, 2, 3, \ldots$). Explain your reasoning.

(Hint: It may be helpful to make up a chart listing for each month the number of adult pairs, the number of one-month-old pairs, and the number of baby pairs.)