Homework Assignment #5
Page 115 #11-31 odd

Ying Huang
Section 1594

In exercises 11 and 12, the matrices are all nxn. Each part of the exercises is an implication of the form "If (statement1), then (statement 2)." Mark an implication as True if the truth of (statement 2) always follows whenever (statement 1) happens to be true. An implication is False if there is an instance in which (statement 2) is false but (statement 1) is true. justify each answer.

11. a. If the equation Ax=0 has only the trivial solution, then A is row equivalent to the n x n identity matrix.
    True

    b. If the columns of A span R^n, then the columns are linearly independent.
    True

    c. If A is an n x n matrix, then the equation Ax=b has at least one solution for each b in R^n.
    True

    d. If the equation Ax=0 has a nontrivial solution, then A has fewer than n pivot positions.
    True

    e. If A^T is not invertible, then A is not invertible.
    True

13. An m x n upper triangular matrix is one whose entries below the main diagonal are 0's as in Exercise 8). When is a square upper triangular matrix invertible? Justify your answer.
    It is invertible if and only if all the entries on the diagonal are nonzero because it is the only way this matrix will has n pivot positions.

15. Is it possible for a 4 x 4 matrix to be invertible when its columns do not span R^4? Why or why not?
    No, a n x n matrix only invertible when it span R^n, so it will get 4 pivot positions, it is the only way this matrix is invertible.

17. Can a square matrix with two identical columns be invertible? Why or why not?
No, this will make the pivot positions of the matrix not equal to n, there can be only one pivot position in every columns, if there are two identical columns, only the left column count because it is in front of the same row, pivot position is the leading number.

19. If the columns of a 7 x 7 matrix D are linearly independent, what can be said about the solutions of Dx=b? Why?
The equation Dx=b has at least one solution for each b in R^7, because Theorem 8 g "The equation Ax=b has at least one solution for each b in R^n."

21. If the equation Cu=v has more than one solution for some v in R^n, can the columns of the n x n matrix C span R^n? Why or why not?
No, it only does if Cu=v has an unique solution, because Theorem 8 d "The equation Ax=0 has only the trivial solution."

23. Assume that F is an n x n matrix. If the equation Fx=y is inconsistent for some y in R^n,what can you say about the equation Fx=0? Why?
It will has a nontrivial solution because the equation Fx=y is inconsistent for some y, that means it may has no solution that it contradict with Theorem 8 d "The equation Ax=0 has only the trivial solution." so it is true if both of them is false.

25. Verify the boxed statement preceding Example 1.
A= $\begin{bmatrix} 1 & 0 & -2 \\ 3 & 1 & -2 \\ -5 & -1 & 9 \end{bmatrix}$ Echelon form A= $\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 4 \\ 0 & 0 & 3 \end{bmatrix}$
so A has 3 pivot positions which fulfill Theorem 8 c "A has n pivot positions."

27. Let A and B be n x n matrices. Show that if AB is invertible, so is A. You cannot use Theorem 6(b), because you cannot assume that A and B are invertible. [Hint: There is a matrix W such that ABW=1. Why?]
If AB=I, then A and B are both invertible, with B= A^{-1} and A= B^{-1} which also true for ABW=1 because AB=I so ABW=IW=1

29. If A is an n x n matrix and the transformation x→ Ax is one-to-one, what else can you say about this transformation? Justify your answer.
So, the linear transformation x→ Ax maps R^n onto R^n and it is invertible, because the question says that Theorem 8 f, which same as the question, than we could say that Theorem 8 i "The linear transformation x→ Ax maps R^n onto R^n" also true because both true make it true, and if there are true, then it is invertible.

31. Suppose A is an n x n matrix with the property that the equation Ax=b has at least one solution for each b in R^n. Without using Theorems 5 or 8, explain why each equation Ax=b has in fact exactly one solution.

If Ax=b has at least one solution for each b in R^n, then there will be a pivot position in each row of A, therefore Ax=b doesn't has a space for a free variable, then Ax=b has only one solution.