Chapter 13

EMT1150
Introduction to Circuit Analysis

Department of Computer Engineering Technology

Fall 2018

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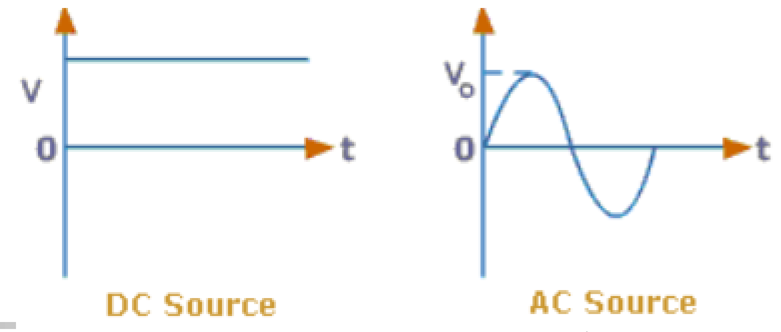
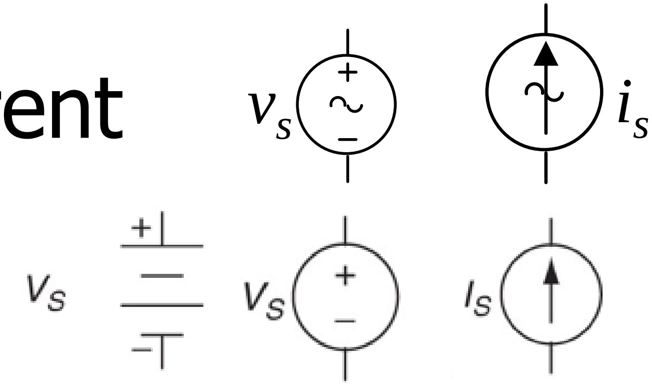
Chapter13 Sinusoidal waveform

- Identify sinusoidal waveform
- Characteristics of sinusoidal wave
- Average value, effective value(rms)

AC vs. DC

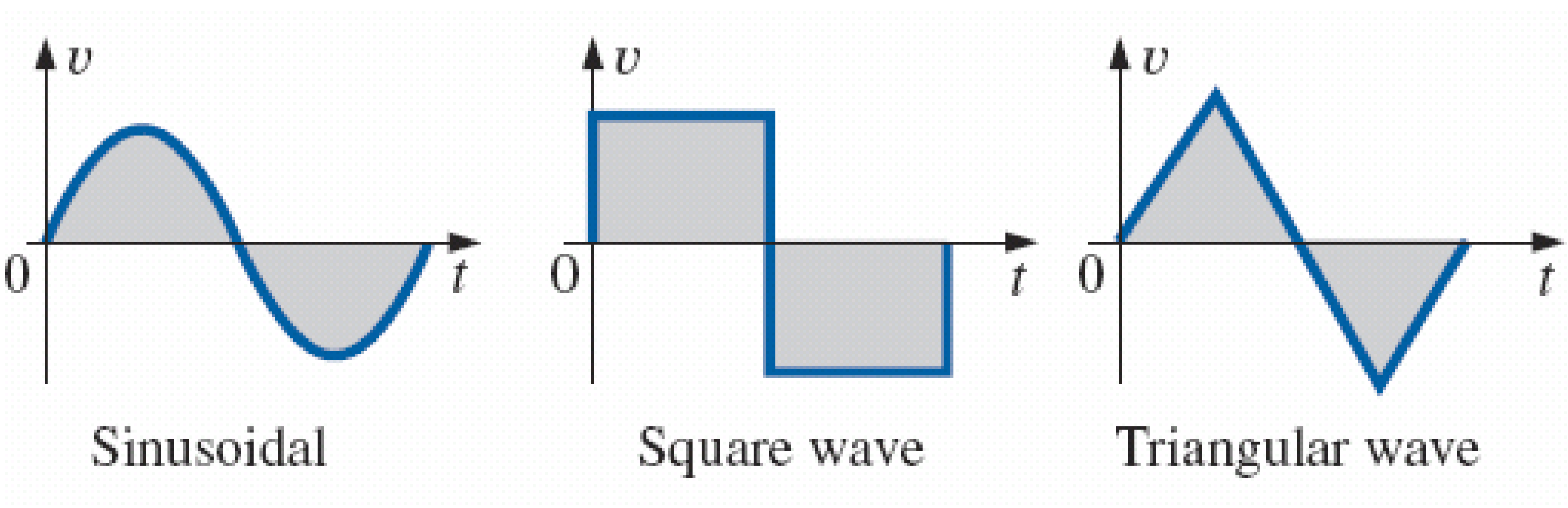


- AC: Alternating current
- DC: Direct current
- Symbol:



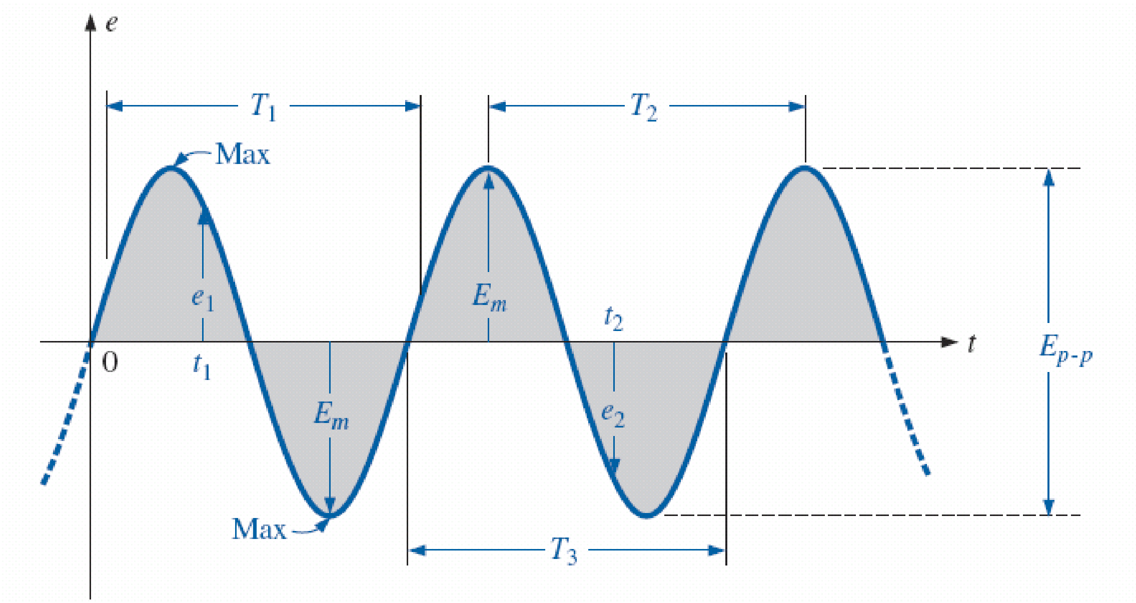
- Thomas Edison developed DC power system in 1882
- But AC power system prevailed because of its lower cost and higher transmission efficiency

- Alternating means the waveform alternates between two prescribed levels in a set time sequence.
- Since sinusoidal ac voltage is widely used in the field, it is commonly called ac voltage without confusion.



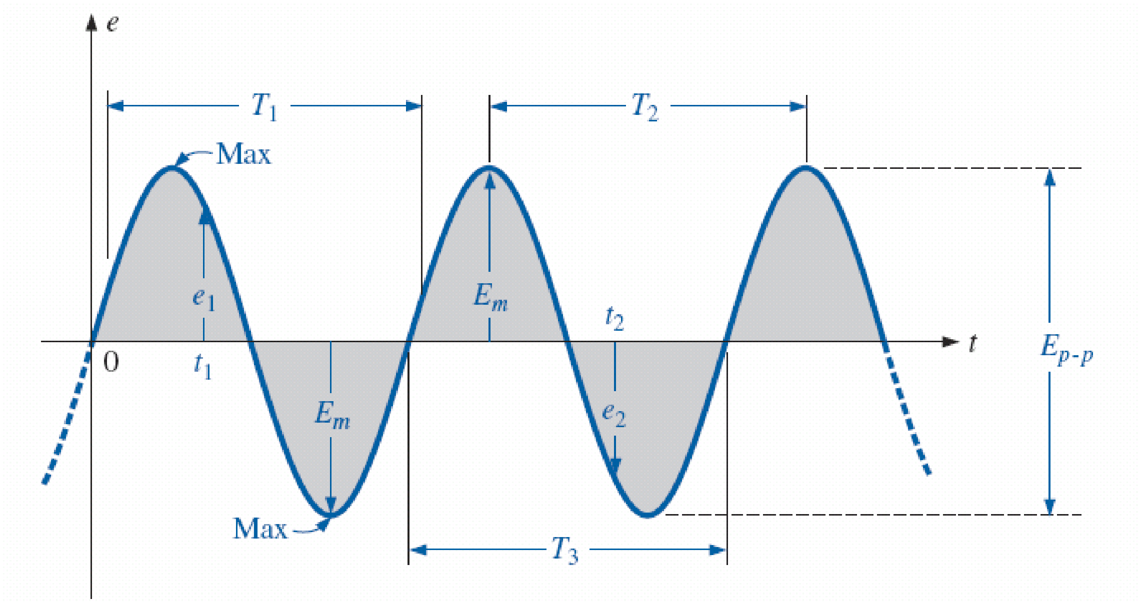
AC sinusoidal waveform characteristics

- **Waveform**: voltage or current are plotted as function of time.
- **Instantaneous value**: the magnitude at any instant of time, using lower case, $v(t)$
- **Peak amplitude:** The maximum value of a waveform as measured from its average value, denoted by uppercase letter, E_m



- Peak value: The maximum instant value of function as measured from zero volt.
- Peak-to-peak value: the full voltage range between positive peak to negative peak of waveform, denoted as $V_{p\text{-}p}$
- Cycle: a portion of waveform without any repeat
- Period: the time of one cycle, denoted as T.
- Frequency (f) : The number of cycles that occur in 1 second, unit hertz(Hz).

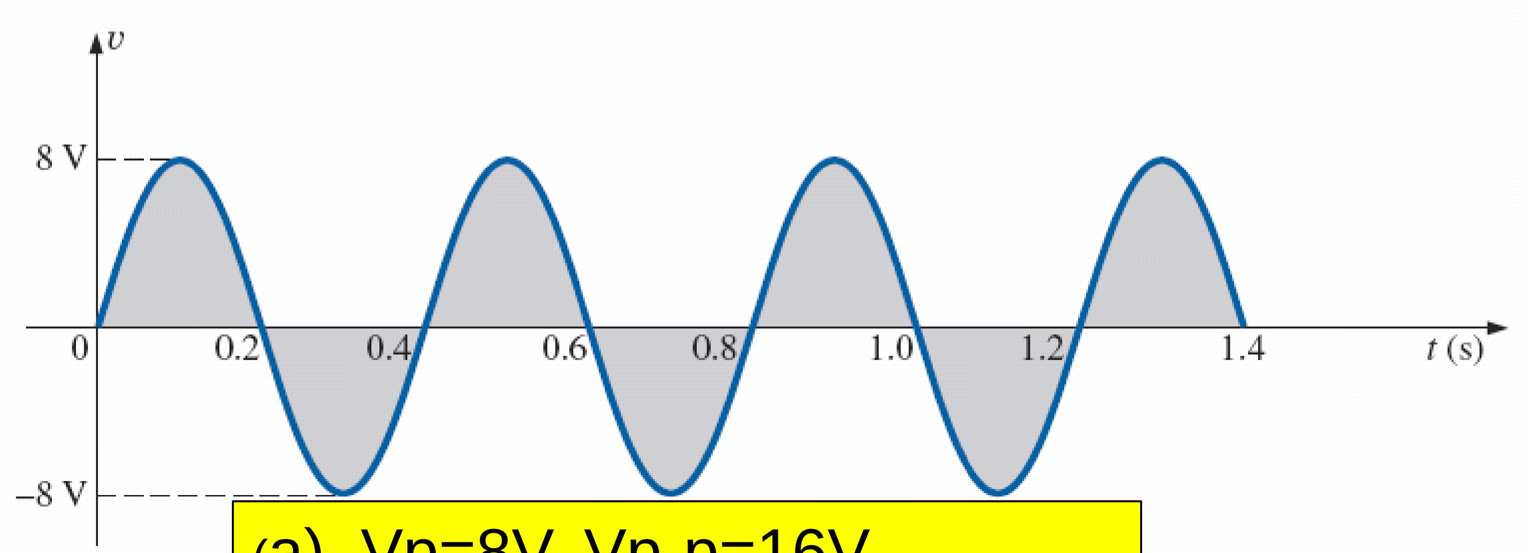
$$f = \frac{1}{T}$$



Example1: (a). What is the peak value and peak-to-peak value?

(b). What is the instant value at 0.3s and 0.6s?

(c) What is the period of waveform, frequency? How many cycles are shown?



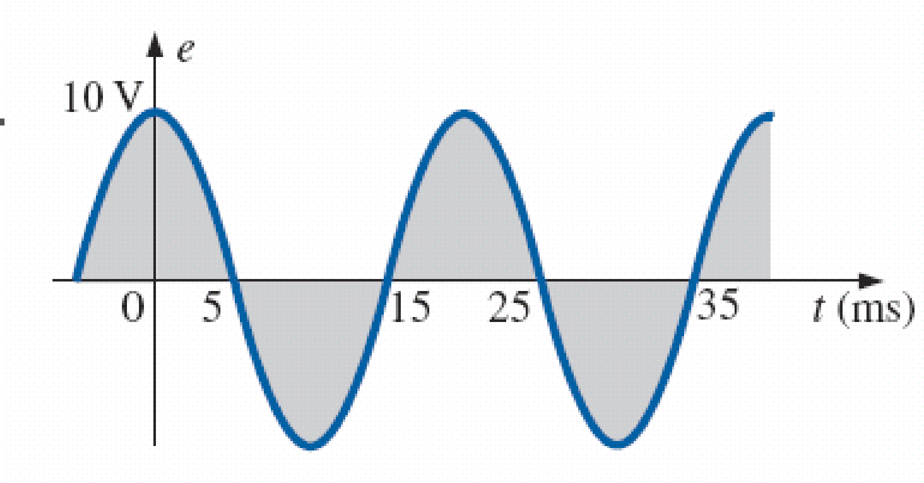
(a). Vp=8V, Vp-p=16V

(b). *v*(0.3s)=-8V, v(0.6s)=0V

(c). T=0.4s, total 3.5 cycles are shown,

$$f = \frac{1}{T} = \frac{1}{0.4s} = 2.5(Hz)$$

Example2: Find the frequency of the waveform.



T= 20ms

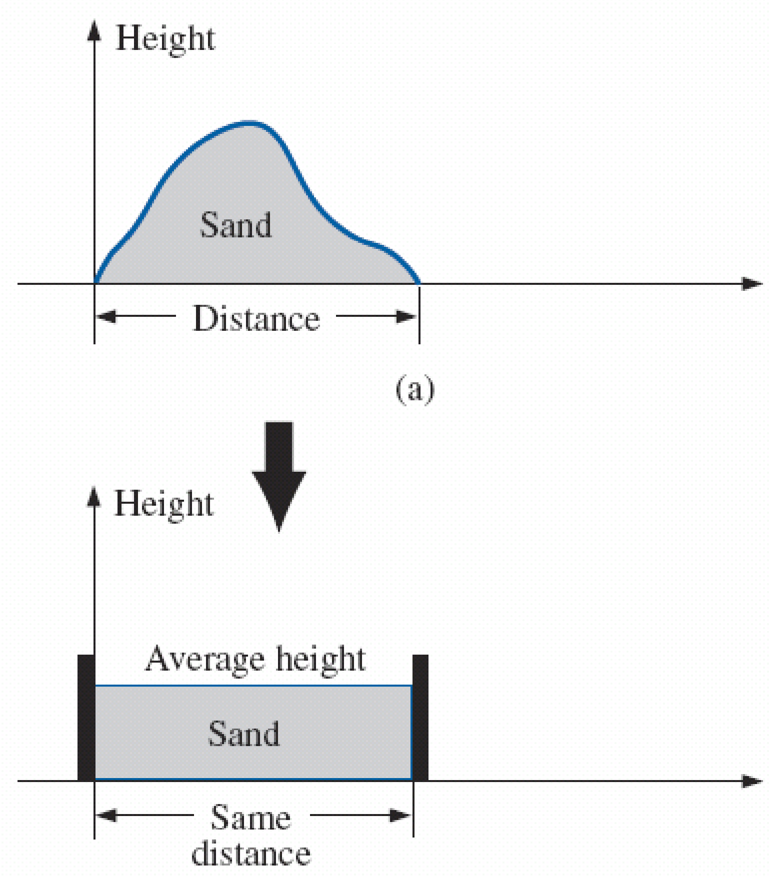
$$f = \frac{1}{T} = \frac{1}{20ms} = 50(Hz)$$

Average value

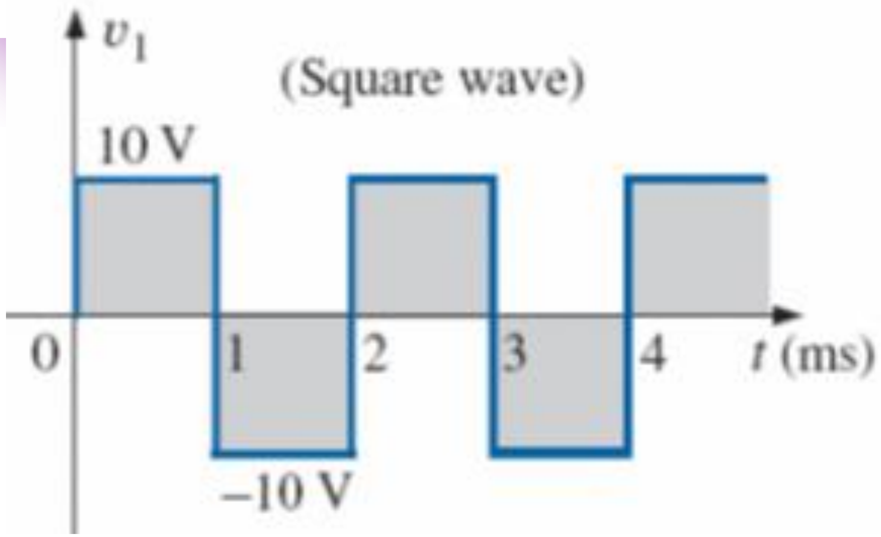
- Average of numbers
- Average of function

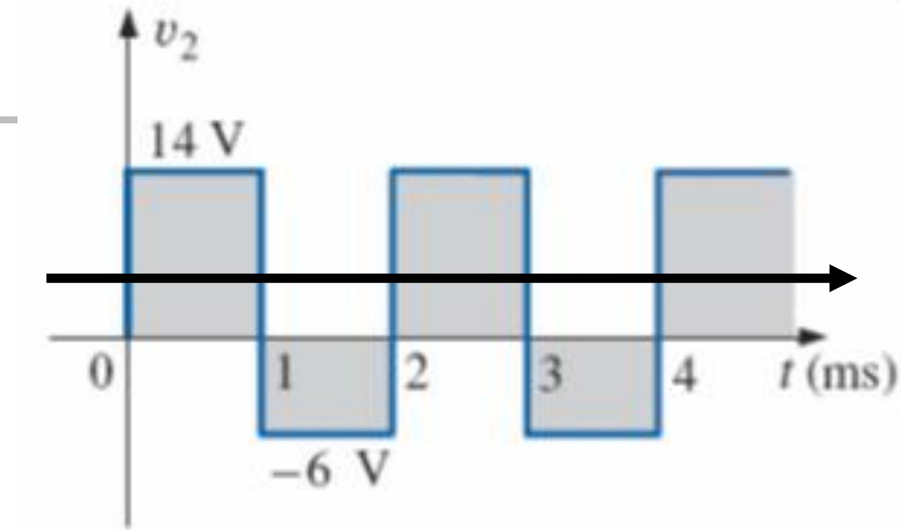
$$Average\ value = \frac{Algebraic\ sum\ of\ areas}{length\ of\ curve}$$

$$Average\ value = \frac{\int_a^b f(x)dx}{length\ of\ curve}$$



Example3: Find the average value of waveforms in one cycle.



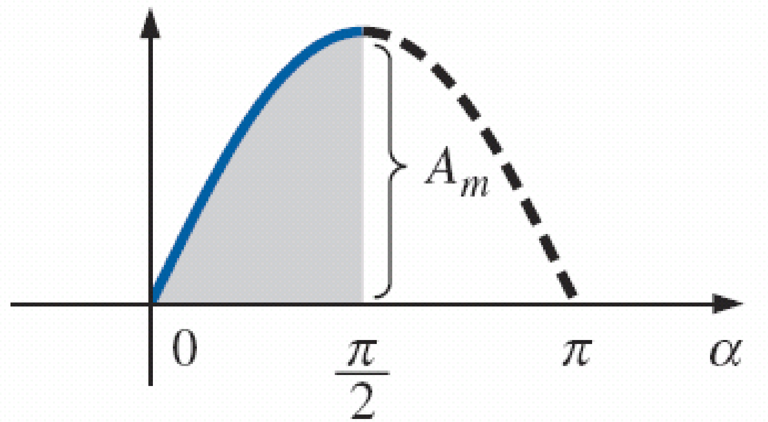


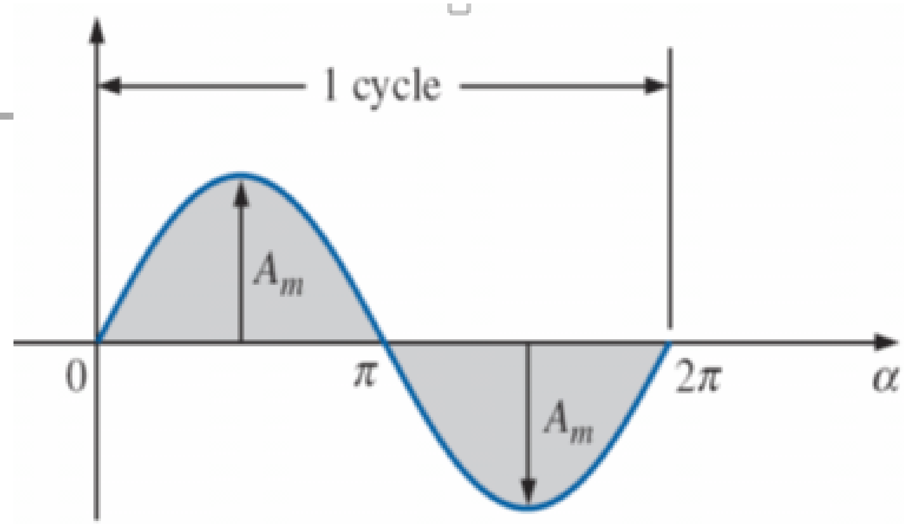
$$A = \frac{10V \times 1ms - 10v \times 1ms}{2ms} = 0(V)$$

$$A = \frac{14V \times 1ms - 6v \times 1ms}{2ms} = 4(V)$$

When the average value isn’t zero, then the waveform has DC offset/ DC shift.

Example4: Find the average value of sin wave in half cycle and one cycle.

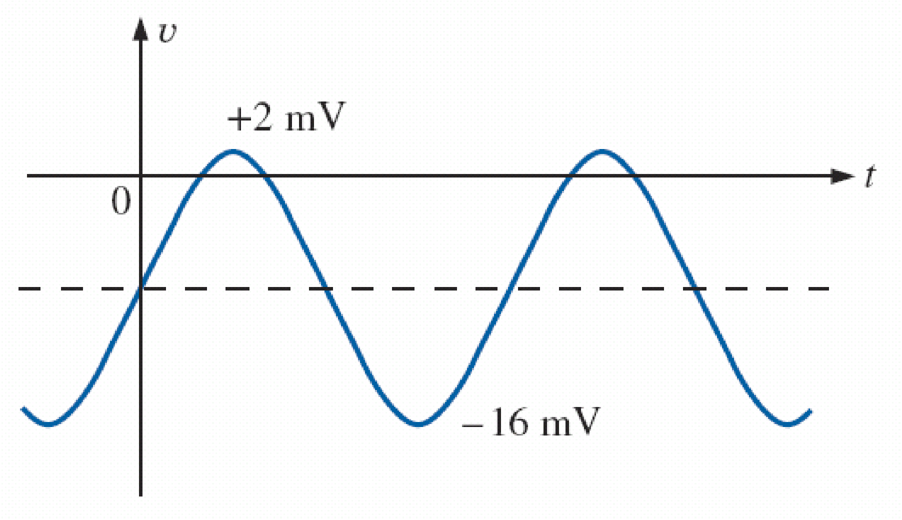




$$A = \int_0^{\pi} A_m \sin(\alpha)\, d\alpha$$
$$= A_m[-\cos(\alpha)]_0^{\pi}$$
$$= -A_m(\cos(\pi) - \cos(0))$$
$$= 2A_m$$

$$A = 2A_m - 2A_m = 0$$

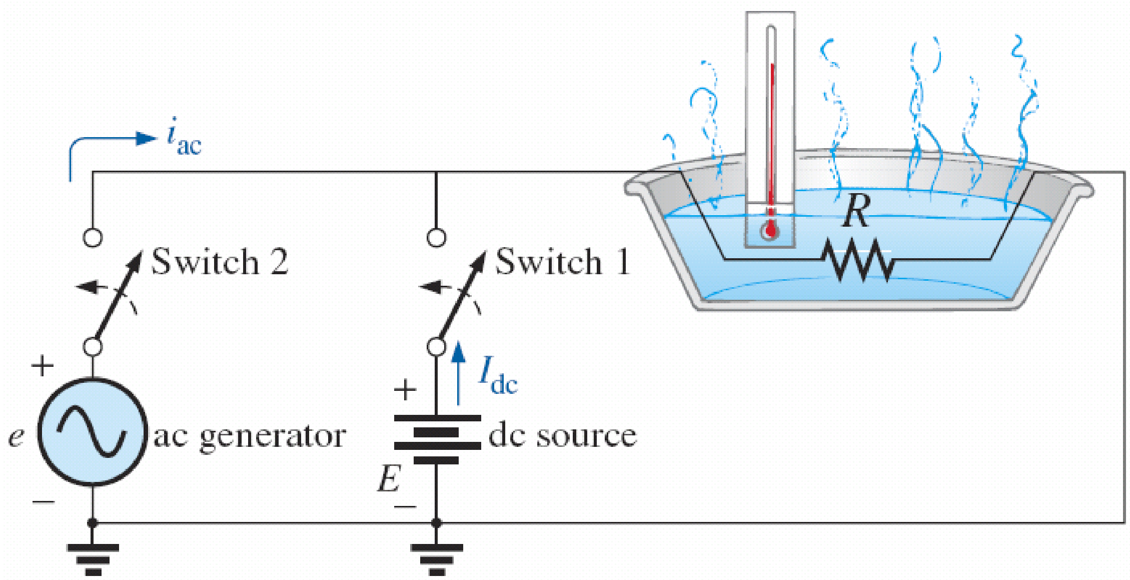
Example5: Find the average value of waveform.

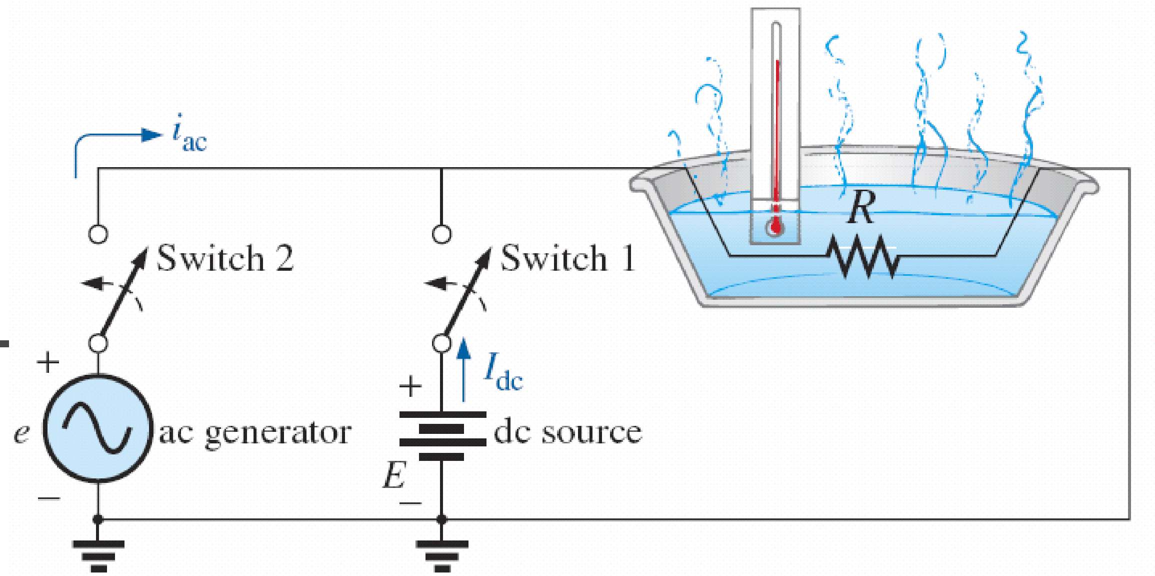


V_{av} = (-16mV-2mV)/2-2=-7(mV)

Effective value(root-mean-square value)

- Effective value of AC is the amount of AC that produces the same heating effect as an equal amount of DC.





$$P_{dc} = I_{dc}^2 R$$

$$P_{ac} = i_{ac}^2 R = (I_m \sin(\varpi t))^2 R = I_m^2 sin^2(\varpi t) R$$
$$= I_m^2 [\frac{1}{2}(1 - cos(2\varpi t)] R = \frac{I_m^2 R}{2} - \frac{I_m^2 R}{2} \cos(2\varpi t)$$

$$P_{av(ac)} = \frac{I_m^2 R}{2}$$

$$P_{av(ac)} = P_{dc}$$

$$\frac{I_m^2 R}{2} = I_{dc}^2 R$$

$$I_{dc} = \frac{I_m}{\sqrt{2}} = 0.707 I_m$$

- The equivalent dc value of a sinusoidal current or voltage (root-mean-square value, rms value) is 0.707 of its peak value.

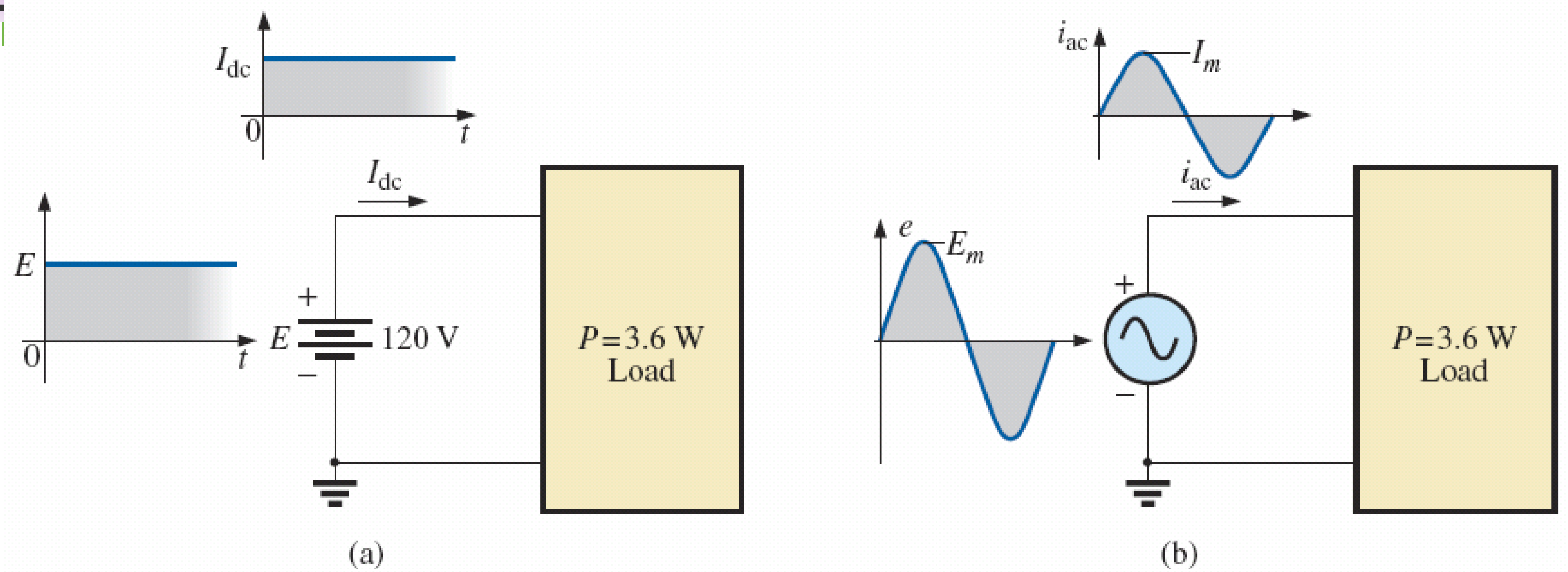
$$I_{rms} = \frac{I_m}{\sqrt{2}} = 0.707 I_m$$

$$V_{rms} = \frac{V_m}{\sqrt{2}} = 0.707 V_m$$

$$I_m = \sqrt{2}\, I_{rms} = 1.414 I_m$$

$$V_m = \sqrt{2}\, V_{rms} = 1.414 V_m$$

Example6: Find the peak value of applied ac voltage and current to have the same power as the dc source.



$$I_{dc} = \frac{P_{dc}}{V_{dc}} = \frac{3.6W}{120V} = 30(mA) = I_{rms}$$

$$V_{dc} = V_{rms} = 120V$$

$$I_m = \sqrt{2}I_{rms} = (1.414)(30mA) = 42.42(mA)$$

$$V_m = \sqrt{2}V_{rms} = (1.414)(120V) = 169.68(V)$$