Lecture 11

EMT1150
Introduction to Circuit Analysis

Department of Computer
Engineering Technology

Fall 2018

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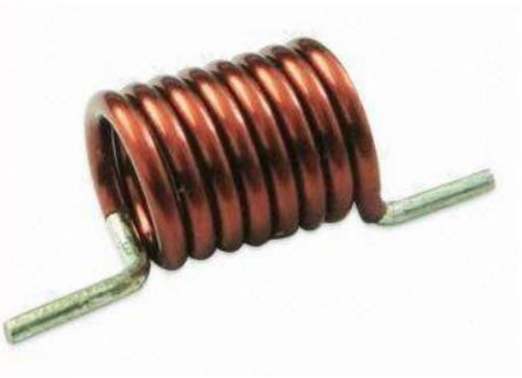
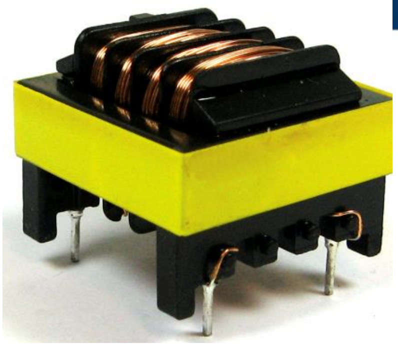
Chapter11 Inductors

- Introduction to Inductors
- The magnetic Field
- Inductance
- Inductors in Series and Parallel
- R-L Transients

Introduction

- Always compare with resistors
- Always compare with capacitors
- Two-terminal device
- Symbol



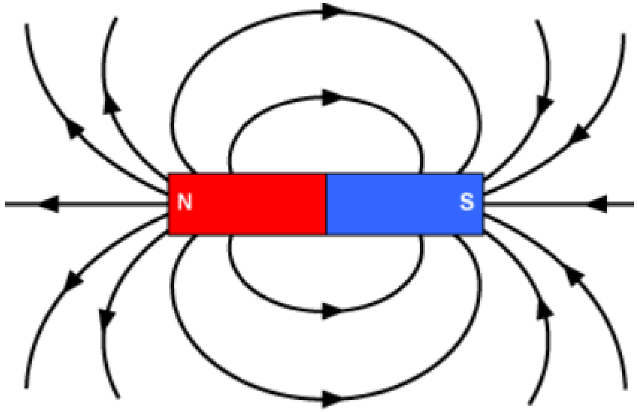


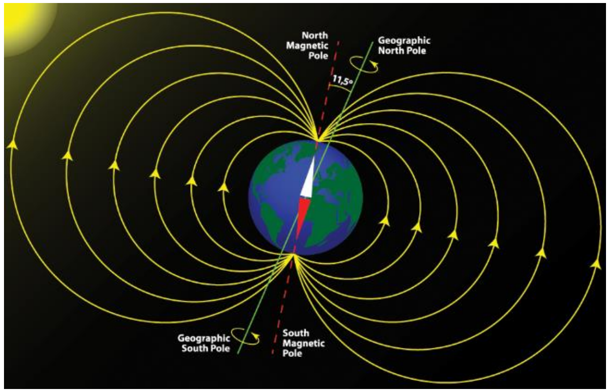
Intro Contd..

- Inductor **does not dissipate energy** as does the resistor but **store it.**
- Inductor stores energy in the form of magnetic field, but capacitor stores energy in the form of electric field.
- Inductor displays its true characteristics only when a change in the voltage or current is made in the network.

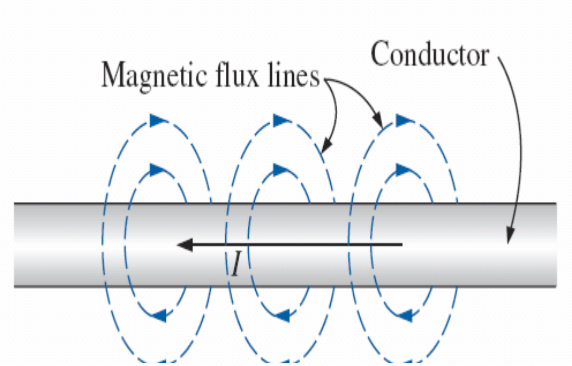
Magnetic field

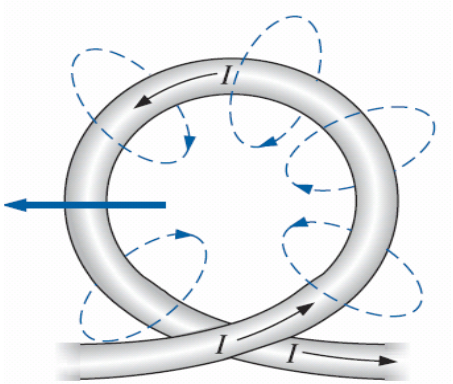
- A magnetic field is the magnetic effect of magnetic materials and electric currents.
- It is represented by magnetic flux lines, which indicate the strength of the magnetic field at any point around any charged body.
- The denser the lines of flux, the stronger is the magnetic field.

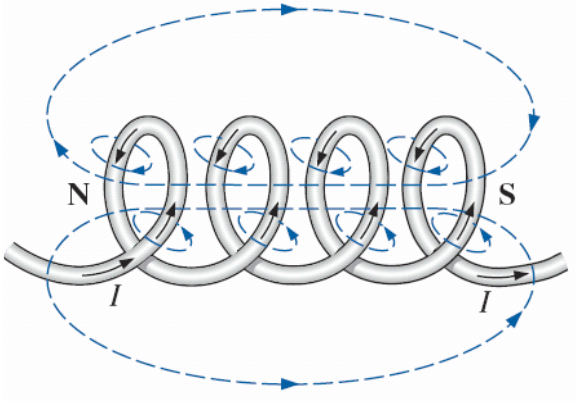


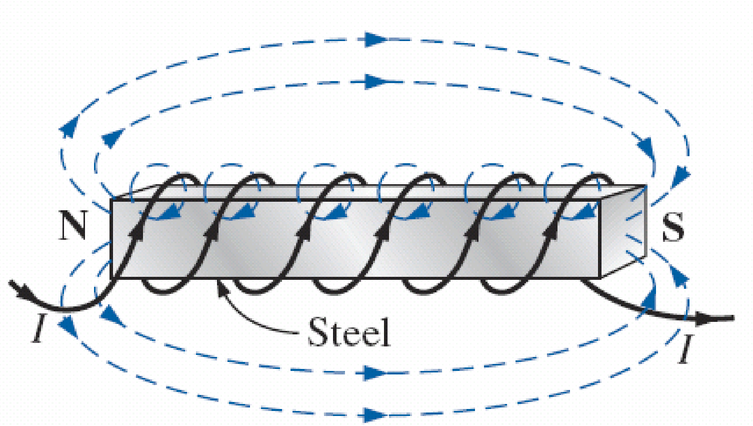


- Magnetic effect induced by flow of charge or charges are called electromagnetism.
- The direction of the magnetic flux lines follow the right hand rule.
- Electromagnet





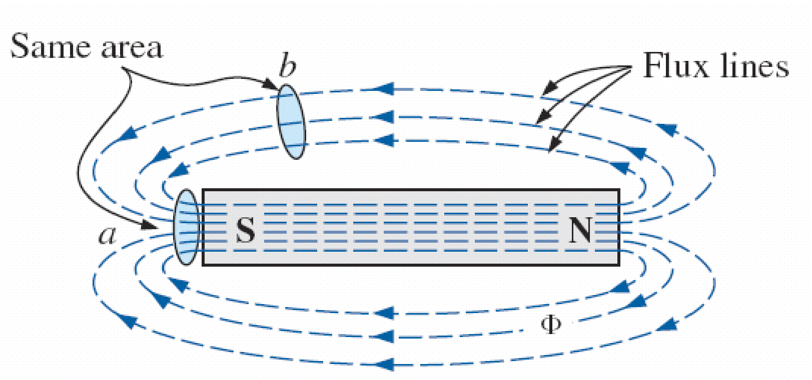


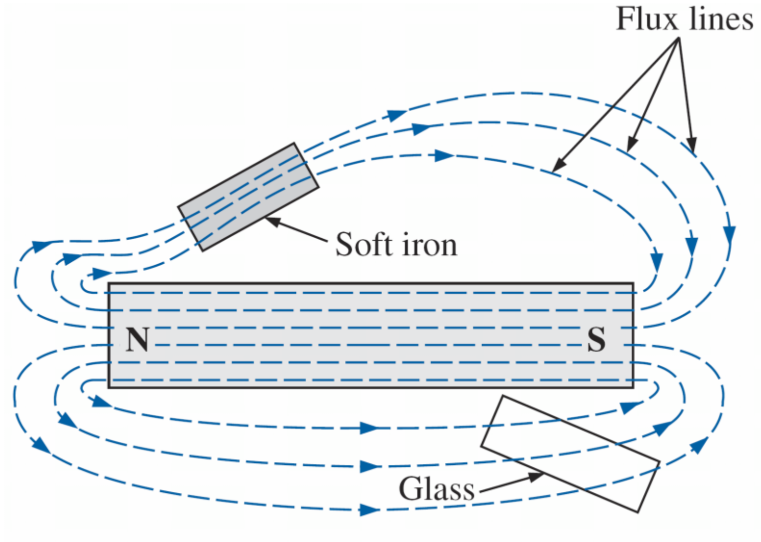


- Magnetic field is measured by magnetic flux density.

$$\boxed{B = \frac{\phi}{A}}$$

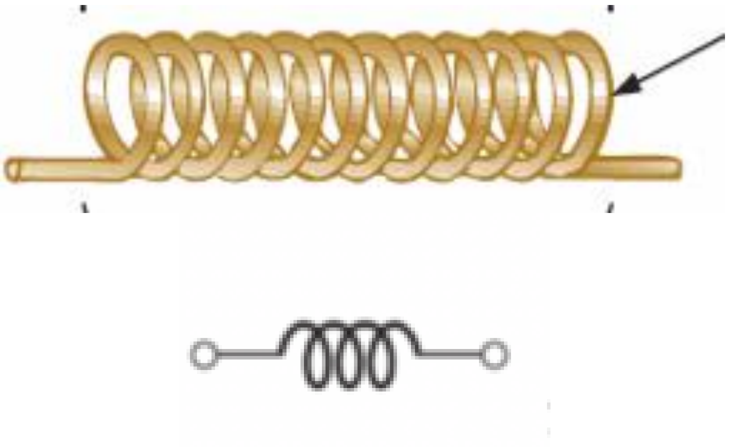
ϕ: Magnetic flux, Webers (Wb)
A: Area, m^2
B: Flux density, Tesla (T)

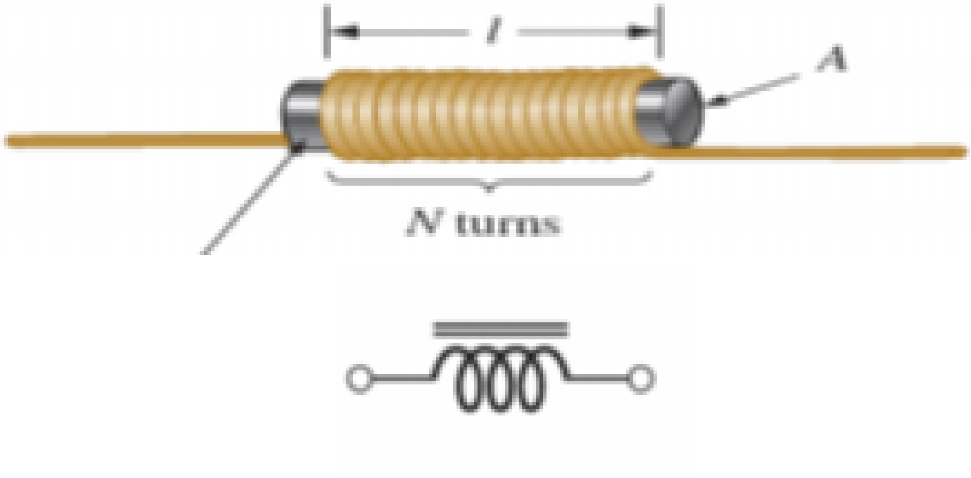




Inductor

- An inductor can set up a strong magnetic field.
- The structure of inductor consists of a coil of conducting material, typically insulated copper wire, wrapped around a core, either of air, plastic material or a ferromagnetic material.





Inductance (L) is a measure of a inductor’s ability to store energy in its coils.

Unit: Henry (H)

Inductance is a measure of the amount of magnetic flux produced for a given electric current.

$$L = \frac{\phi}{i}$$

ϕ : magnetic flux, Webbers (Wb)

i: current, Amperes (I))

L: Inductance, Henry (L)

$$L = \frac{d\phi}{di}$$

For general case, with ferromagnetic core

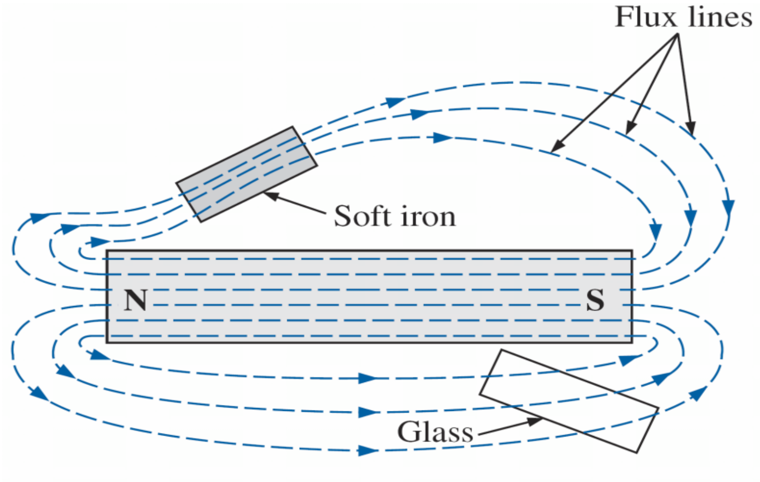
Inductance

- The inductance of any inductor is due primarily to four factors:
  - Magnetic Permittivity (core material)
  - Length of wire
  - Area within the coil
  - Number of turns

$$L = \frac{\mu N^2 A}{l}$$

L: Henry(H)
μ : Magnetic Permittivity (Wb/Am)
N: number of turns
A: m^2
l: m

- **Magnetic Permittivity** is the measure of the ease with which magnetic flux lines can be established in the material.

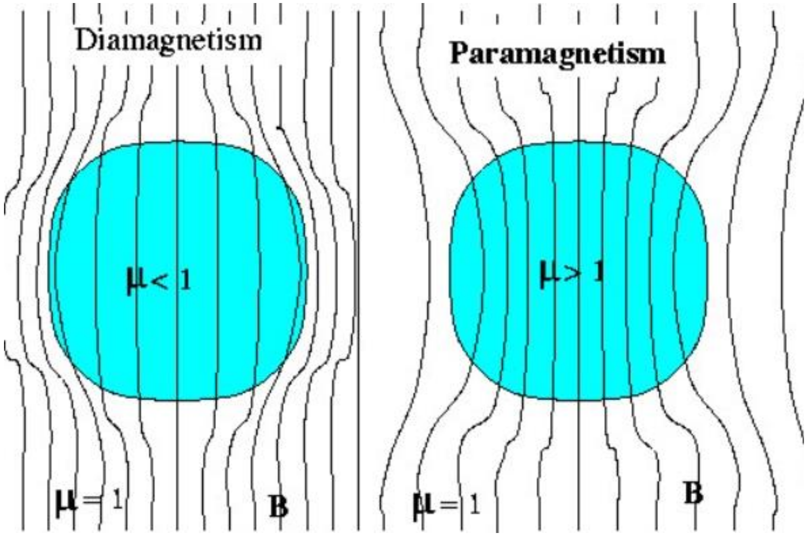


- In general, magnetic permittivity of other materials can compare to the magnetic permittivity of air.

$$\boxed{\mu_r = \frac{\mu}{\mu_0}}$$

$$\mu_0 = 4\pi \times 10^{-7}\ \text{Wb/Am}$$

- If μ_r is slightly less than 1, materials are called diamagnetic, such as wood, glass.
- If μ_r is slightly greater than 1, materials are called paramagnetic, such as copper, aluminum.
- If μ_r is hundreds or thousands larger than 1, materials are called ferromagnetic, such as iron.

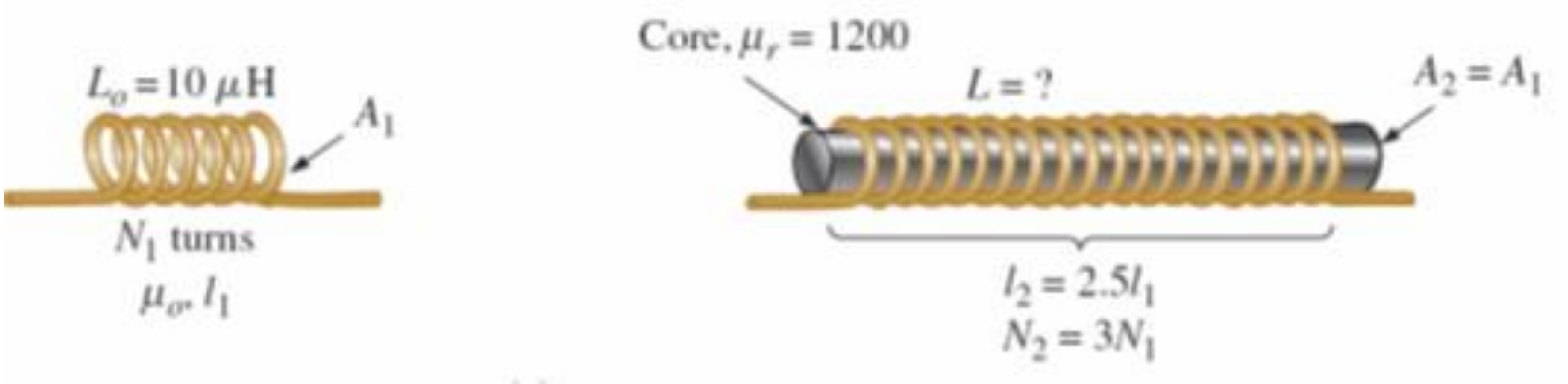


$$L = \frac{\mu_r \mu_0 N^2 A}{l}$$

Example1: Determine the inductance on the right side.

(a) $L = 2^2 \times (20\mu H)$
$=80(\mu H)$

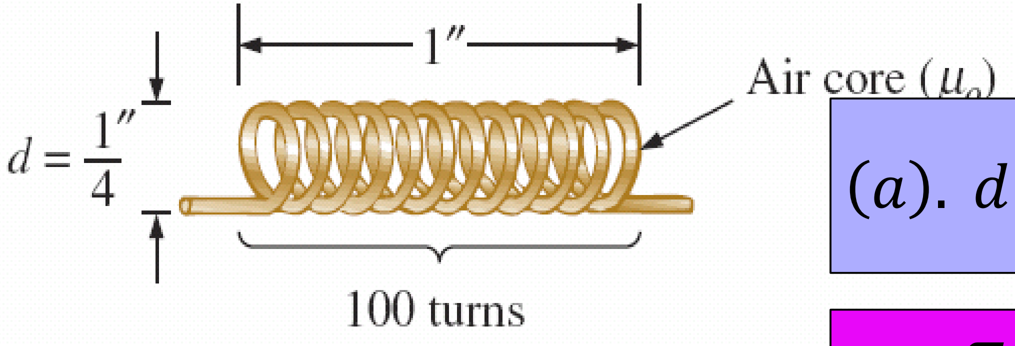
(b) $L = 3 \times (1/2)^2 \times$
$(16\mu H) = 12\ (\mu H)$



(c) $L = 1200 \times 3^2 \times (10\mu\text{H})/2.5$

$=43200(\mu\text{H}) = 43.2\ \text{mH}$

Example2: (a)Find the inductance of inductor. (b) If the metal core with μ_r =2000 is used, find the inductance.



$$(a).\ d = \frac{1}{4} in \left(\frac{1m}{39.37in}\right) = 6.35(mm)$$

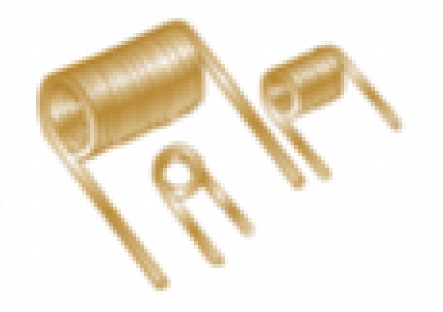
$$A = \frac{\pi d^2}{4} = \frac{\pi(6.35mm)^2}{4} = 31.67\ \mu m^2$$

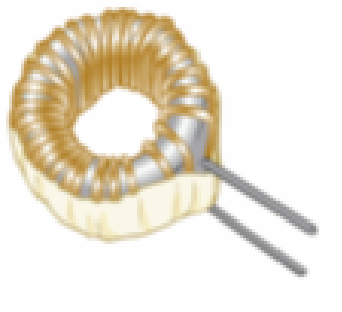
$$l = 1in\left(\frac{1m}{39.37in}\right) = 25.4(mm)$$

$$L = \frac{\mu_r\mu_0 N^2 A}{l} = \frac{4\pi\times10^{-7}\times100^2\times31.67}{25.4} = 15.68(\mu H)$$

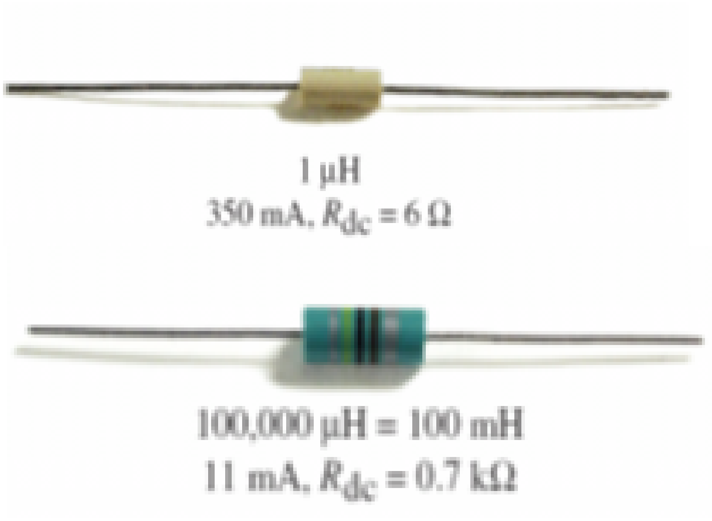
$$(b).\ L = \frac{\mu_r\mu_0 N^2 A}{l} = 2000 \times 15.68 = 31.36(mH)$$

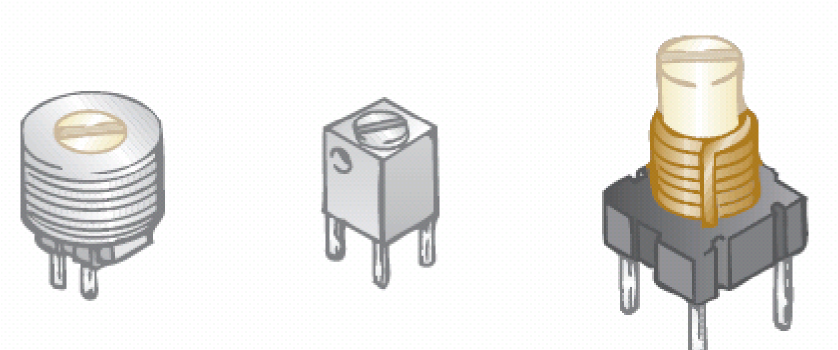
Type of inductors





- Fixed inductor
  - *Air-core inductors*
  - *Toroid coil*
  - Phenolic (resin of plastic core)
  - Ferrite core inductor
- Variable Inductor





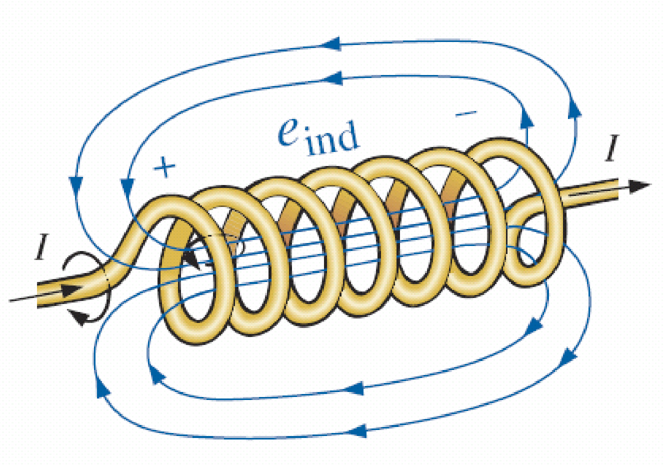
Properties of an Inductor

- Acts like an short circuit at steady state when connected to a DC voltage or current source.
- Current through an inductor must be continuous
  - There are no abrupt changes to the current, but there can be abrupt changes in the voltage across an inductor.
- An ideal inductor does not dissipate energy, it takes power from the circuit when storing energy and returns it when discharging.

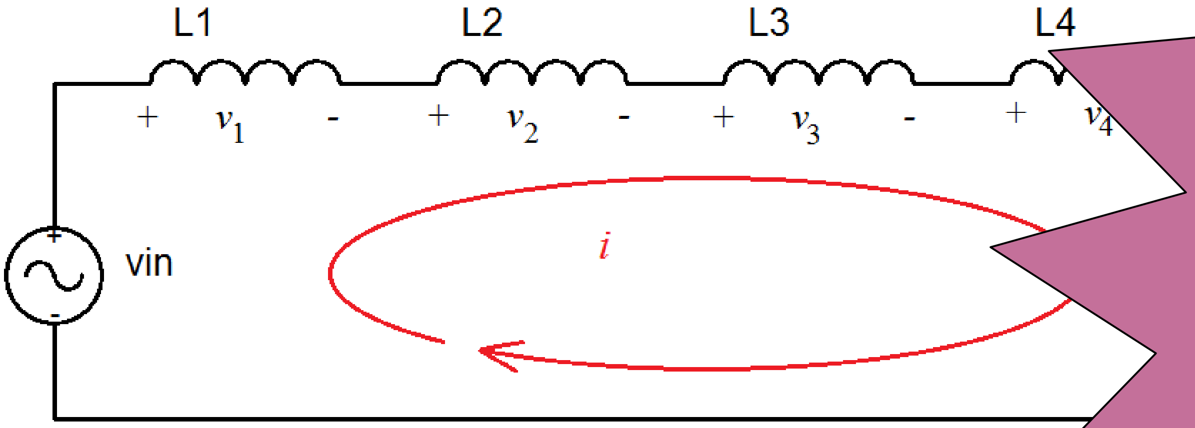
Faraday's Law

- Any change in the magnetic field of a coil will cause a voltage to be "induced" in the coil.
- The induced effect always oppose the cause that produced it

$$\boxed{v_L = L\frac{di_L}{dt}}$$



Inductors in Series



Similar format to series-connected resistor

Apply KVL to CP

$$v_{in} = v_1 + v_2 + v_3 + v_4$$

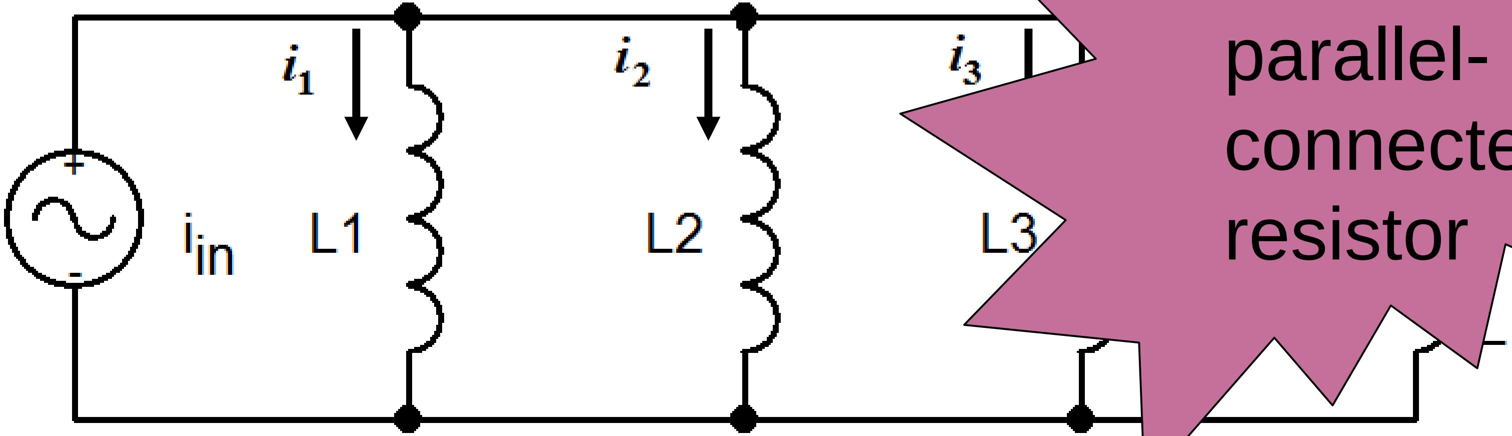
$$v_1 = L_1 \frac{di}{dt}$$

$$v_{in} = L_1 \frac{di}{dt} + L_2 \frac{di}{dt} + L_3 \frac{di}{dt} + L_4 \frac{di}{dt}$$

$$v_{in} = L_T \frac{di}{dt}$$

$$L_T = L_1 + L_2 + L_3 + L_4$$

Inductors in parallel

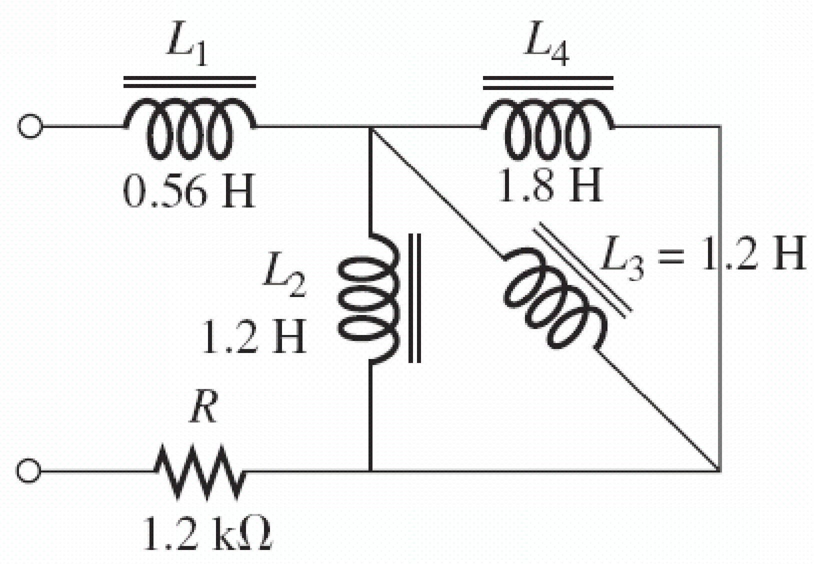


Similar format to parallel-connected resistor

$$\frac{1}{L_T} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \frac{1}{L_4}$$

$$L_T = \frac{L_1 L_2}{L_1 + L_2}$$

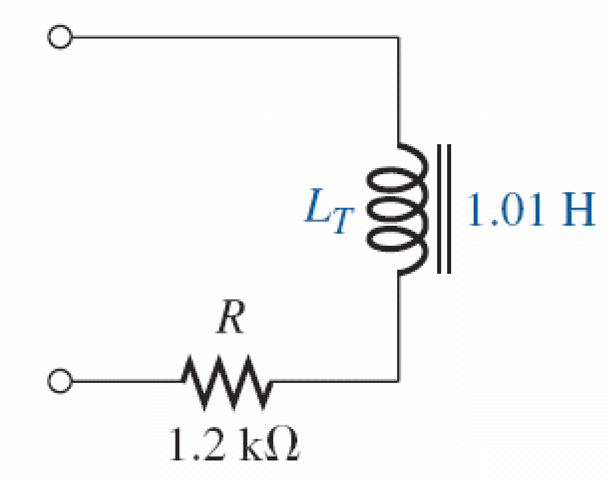
Example3: Reduce the network to its simplest form.



$$\frac{1}{L'_T} = \frac{1}{L_2} + \frac{1}{L_3} + \frac{1}{L_4}$$

$$\frac{1}{L'_T} = \frac{1}{1.8H} + \frac{1}{1.2H} + \frac{1}{1.2H}$$

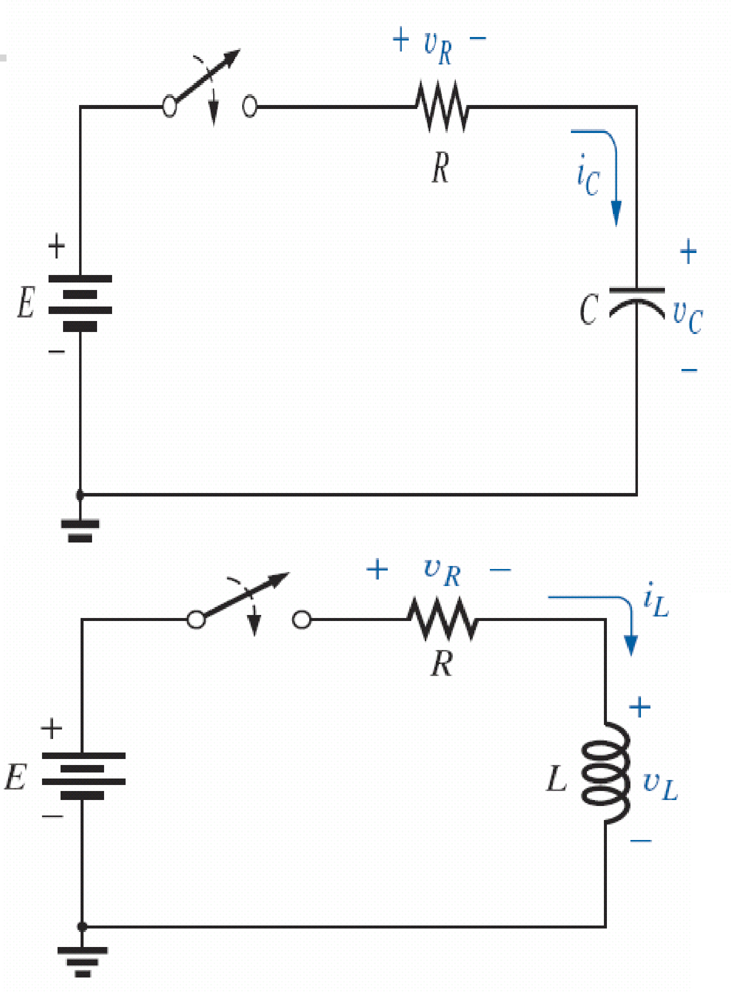
$$L'_T = 0.45H$$

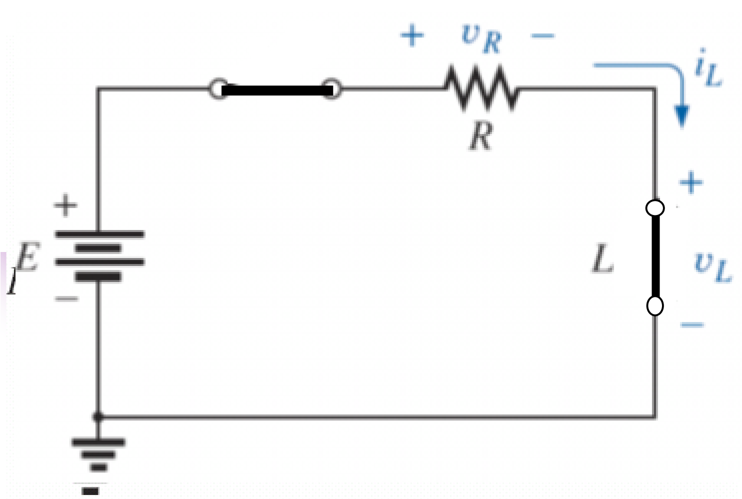


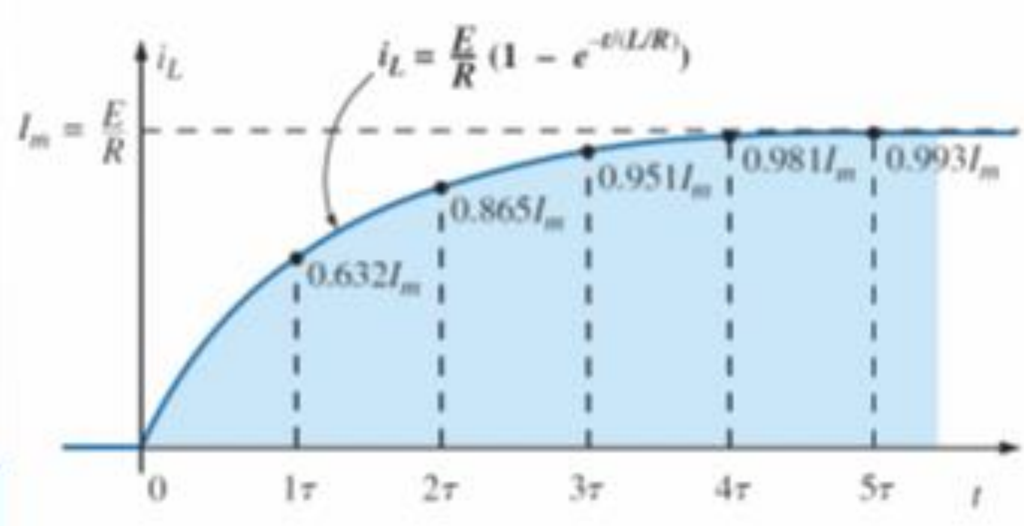
$$L_T = L'_T + L_1 = 0.56H + 0.45H$$
$$= 1.01H$$

Transients in RL Networks

- Storage phase
  - similarities exist between the analyses of inductive and capacitive networks.
  - The behavior for the voltage of a capacitor is the same for the current of an inductor

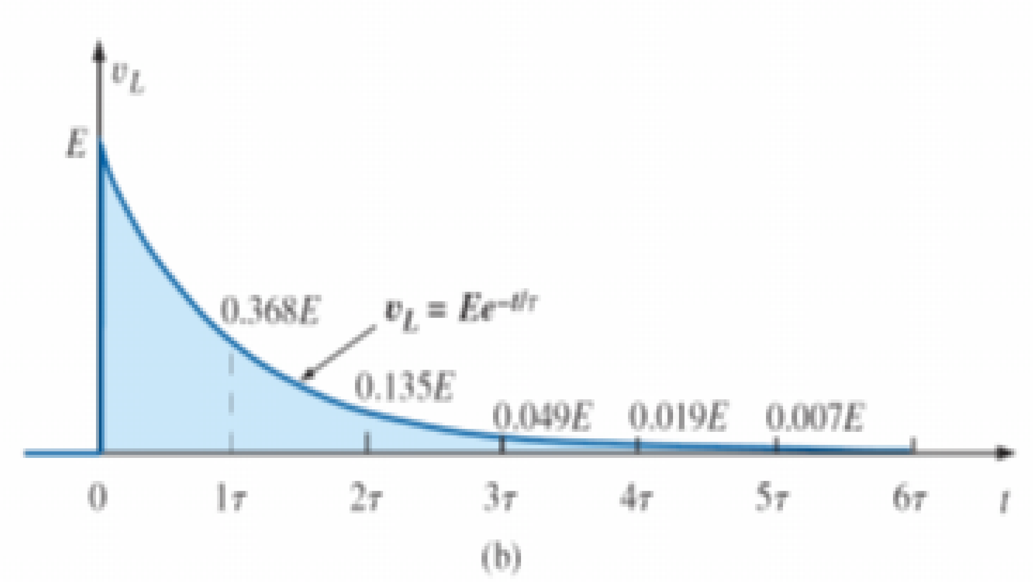






$$i_L = \frac{E}{R}(1 - e^{-t/\tau})$$

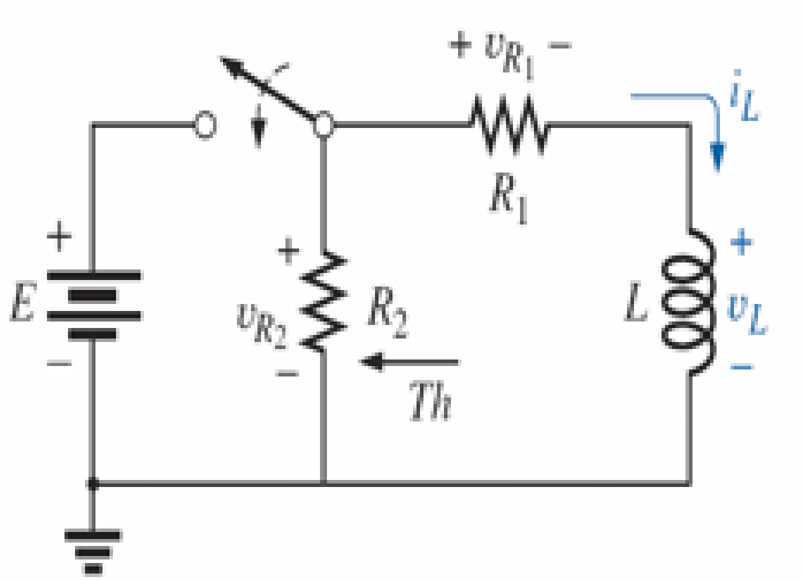
$$\tau = \frac{L}{R}$$



(b)

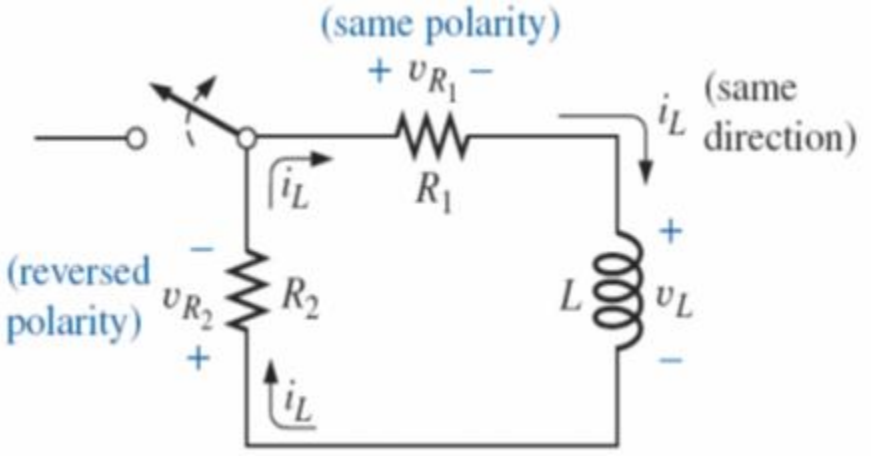
$$v_L = Ee^{-t/\tau}$$

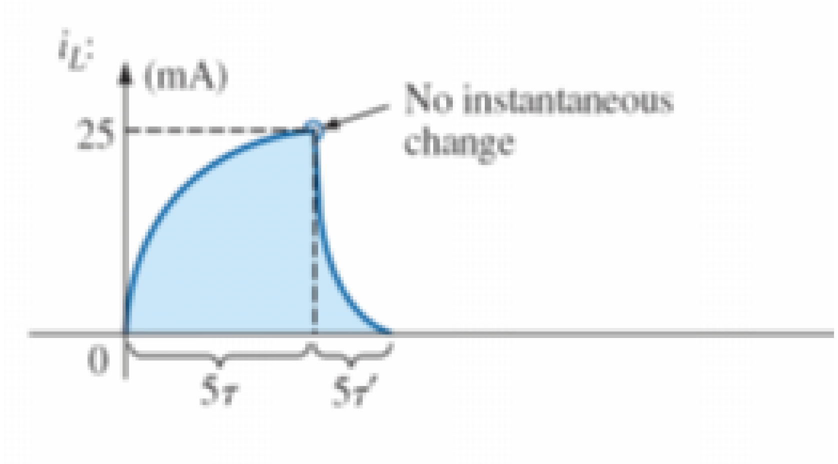
Release phase

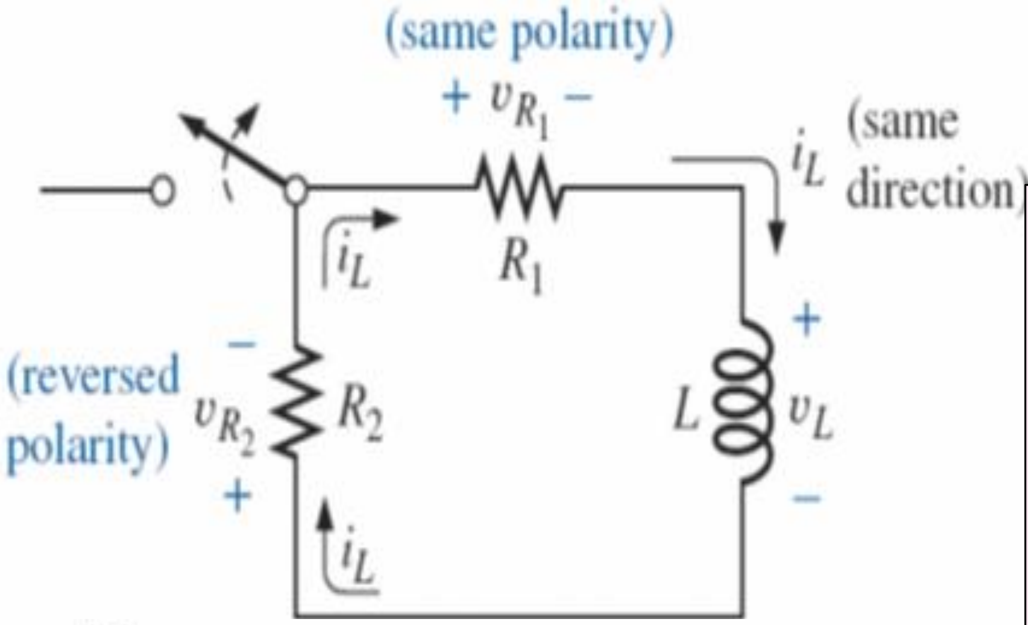


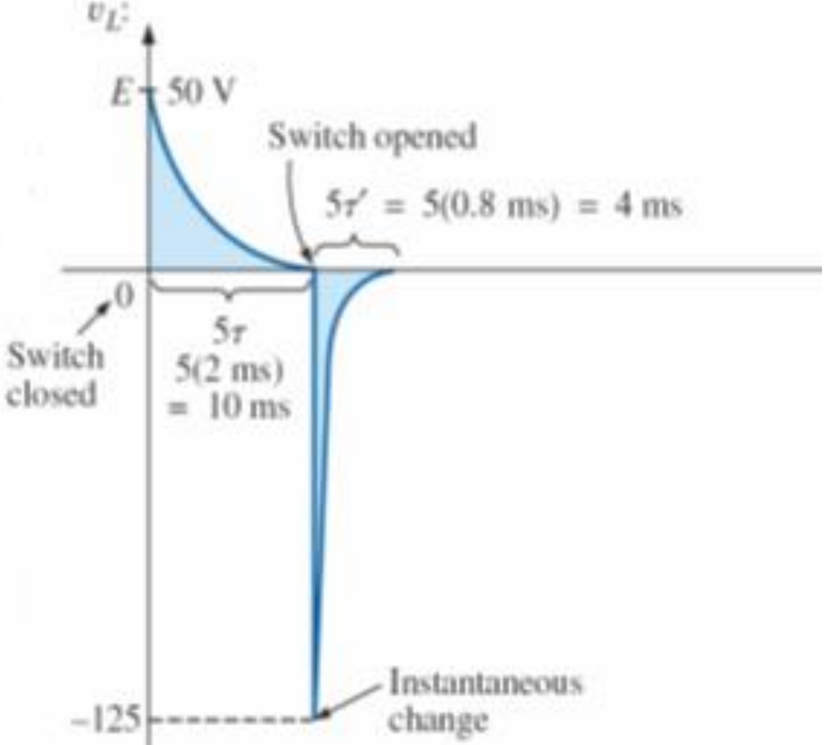
$$i_L = \frac{E}{R_1} e^{-\frac{t}{\tau'}}$$

$$\tau' = \frac{L}{R_1 + R_2}$$









$$
\begin{aligned}
v_L &= -(v_1 + v_2) \\
&= -i_L(R_1 + R_2) \\
&= -\frac{E}{R_1}(R_1 + R_2) \\
&= -\left(1 + \frac{R_2}{R_1}\right)\mathrm{E}
\end{aligned}
$$

$$v_L = -\left(1 + \frac{R_2}{R_1}\right)\mathrm{E}e^{-\frac{t}{\tau\prime}}$$