Chapter 9

EMT1150
Introduction to Circuit Analysis

Department of Computer Engineering Technology

Fall 2018

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Review

- Series-parallel circuit
  - Reduce and Return Approach
  - Block Diagram Approach
  - Ladder Network
- Basic principle
  - Have big picture first
  - Redraw the circuit if needed

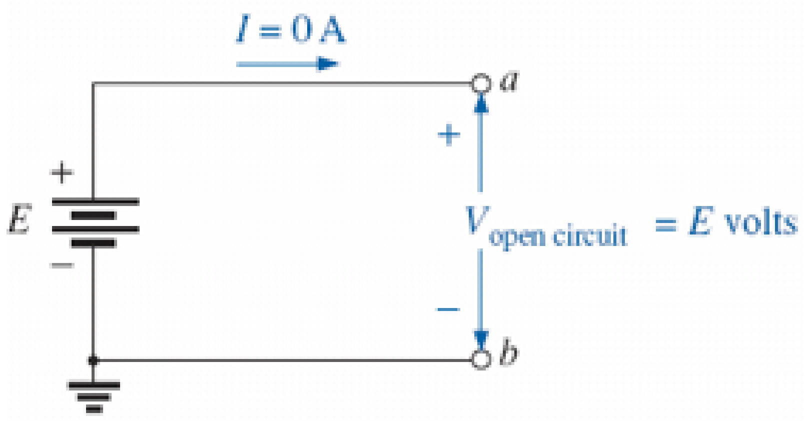
When circuit has multiple sources, hard to solve directly

Chapter9 Network Theorems

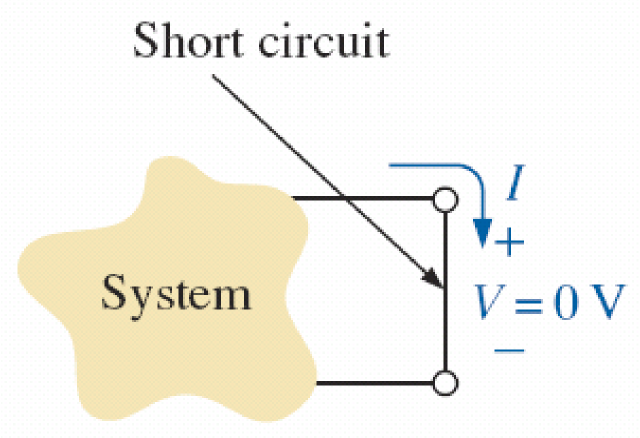
- Superposition theorem
- Thévenin’s Theorem

Open circuit and short circuit

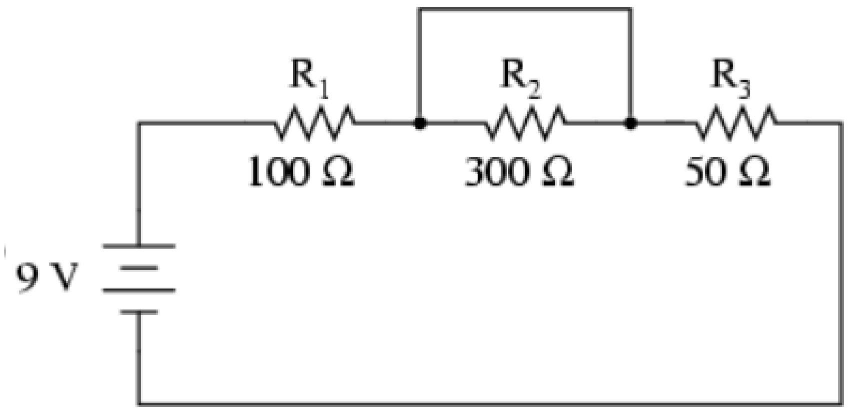
- Open circuit: two isolated terminals are not connected by an element of any kind.
- Short circuit: a wire is direct connected between two terminals of a network.



$I = 0A,\ R = \infty\ \Omega$



$V = 0V,\ R = 0\ \Omega$



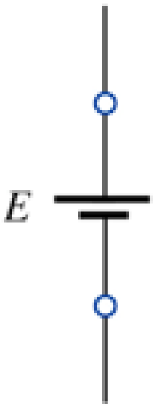
Superposition theorem

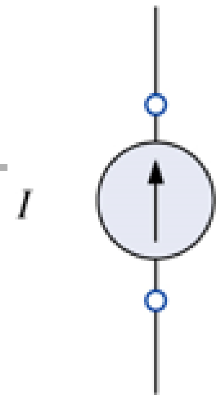
- The **superposition theorem** can be used to find the solution to networks with two or more sources that are not in series or parallel.
- The current through, or voltage across, an element in a linear bilateral network is equal to the algebraic sum of the current or voltages produced independently by each source.

Procedure

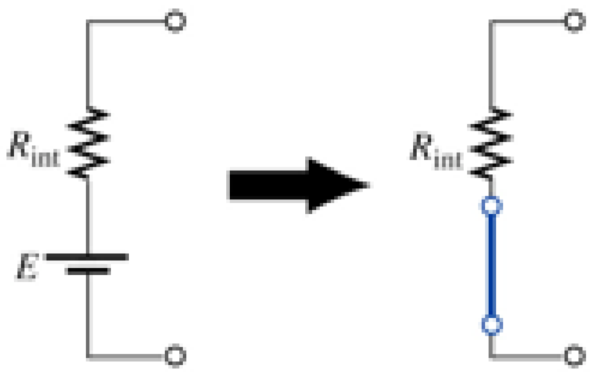
- Identify each voltage source and current source
- Redraw the circuit with only one selected source, kill other sources, calculate the current or voltage generated from this single source
- Repeat this process for each source.
- Sum of the current or voltages produced independently by each source.

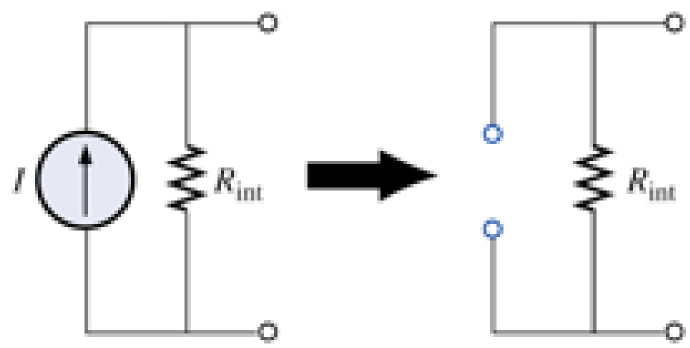
How to kill a source?



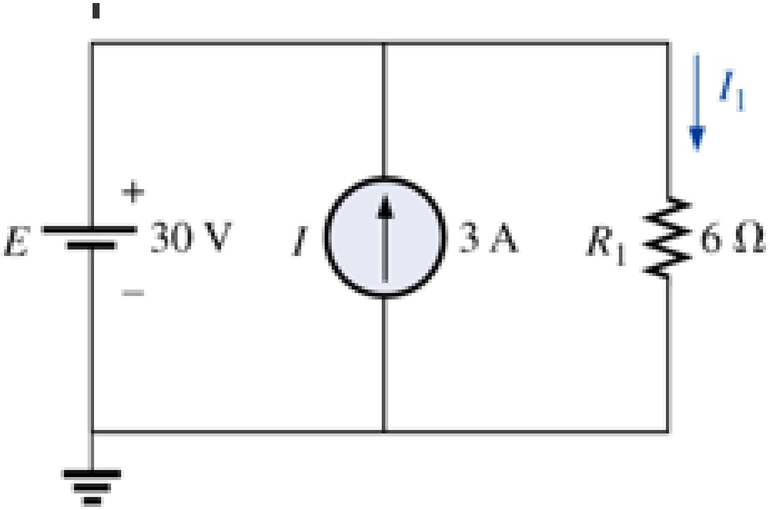


If there are internal resistances associate with the source, they must remain in the network.



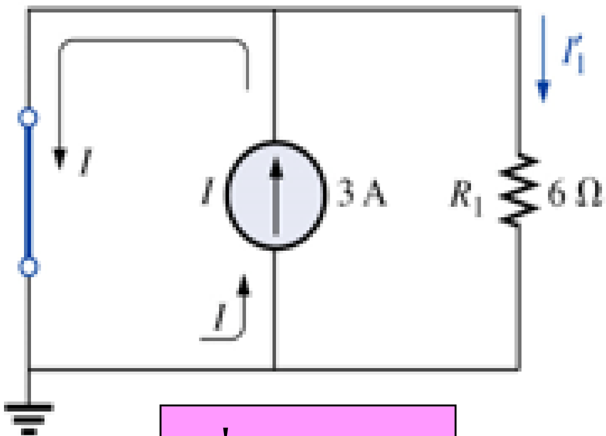


Example1: Determine I_1 for the network

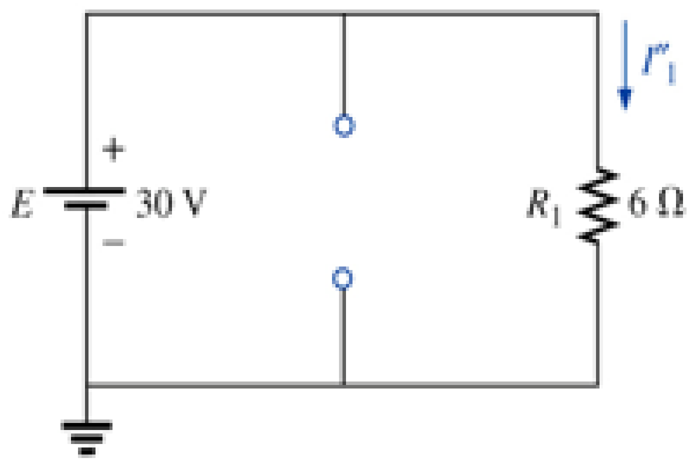


$$I_1'' = \frac{E}{R_1} = \frac{30V}{6\Omega} = 5A$$

$$I_1 = I_1' + I_1''$$
$$= 0A + 5A$$
$$= 5A$$

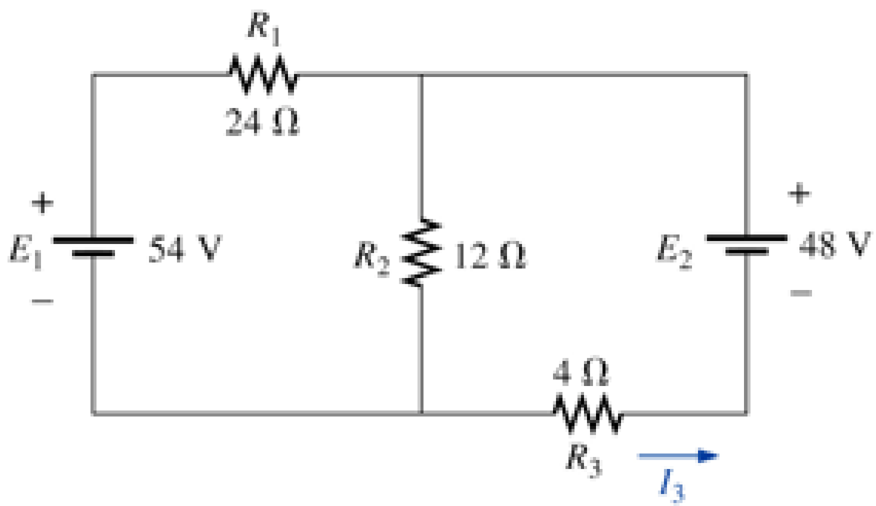


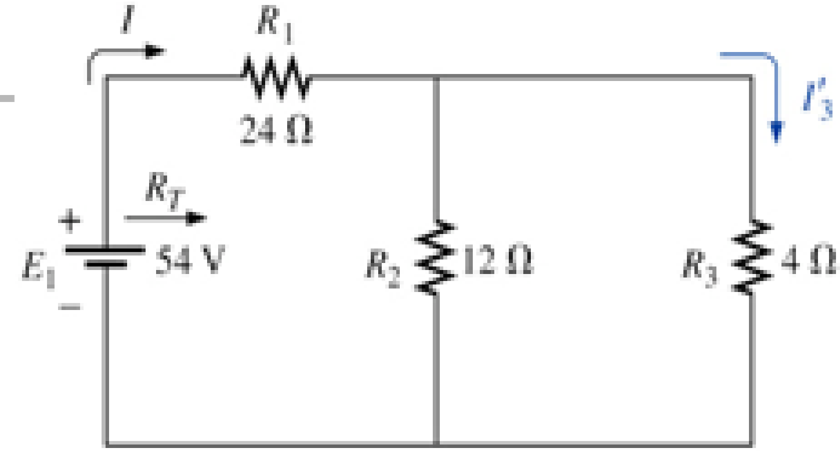
$$I_1' = 0A$$

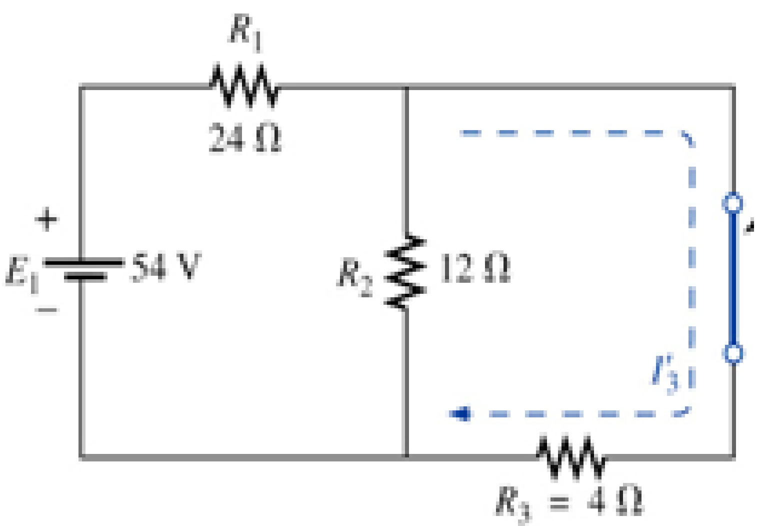


(b)

Example 2: Using superposition, determine the current through the 4-Ω resistor, I_3.

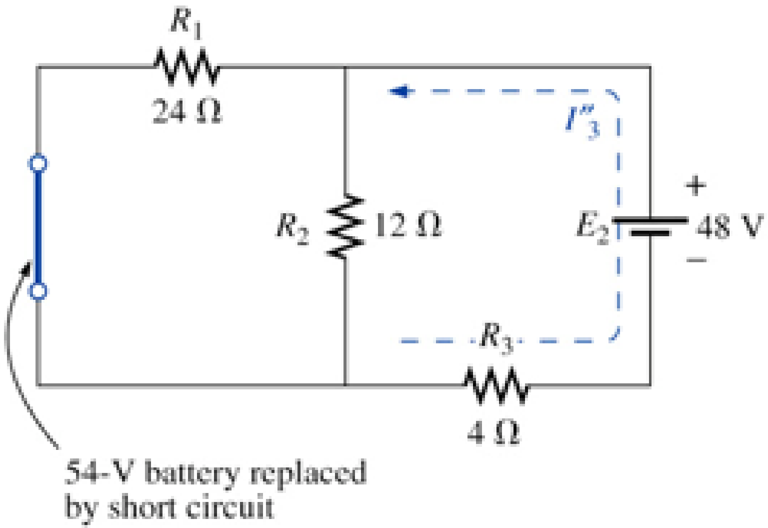


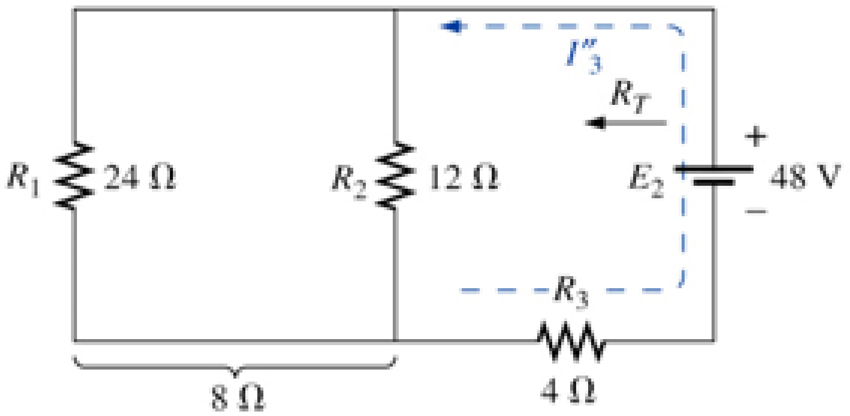




$$R_T = R_1 + R_2 \,//\, R_3$$
$$= 24\Omega + 12\Omega \,//\, 4\Omega$$
$$= 24\Omega + 3\Omega = 27\Omega$$
$$I = \frac{E_1}{R_T} = \frac{54V}{27\Omega} = 2\,A$$

$$I_3^{'} = \frac{R_2\,I}{R_2 + R_3} = \frac{(12\,A)(2\,A)}{12\Omega + 4\Omega} = 1.5\,A$$



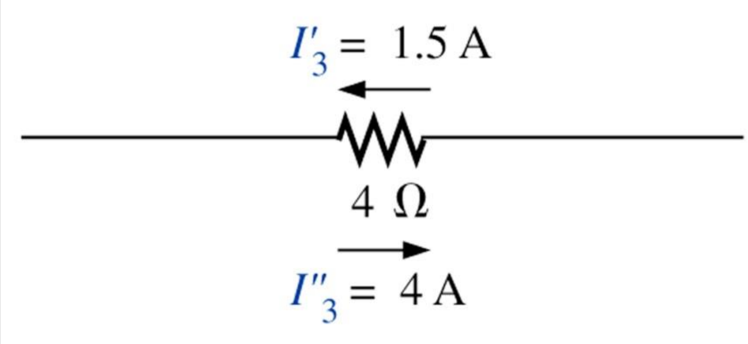


$$R_T = R_3 + R_1 \,//\, R_2 = 4\Omega + 24\Omega \,//\, 12\Omega = 4\Omega + 8\Omega = 12\Omega$$

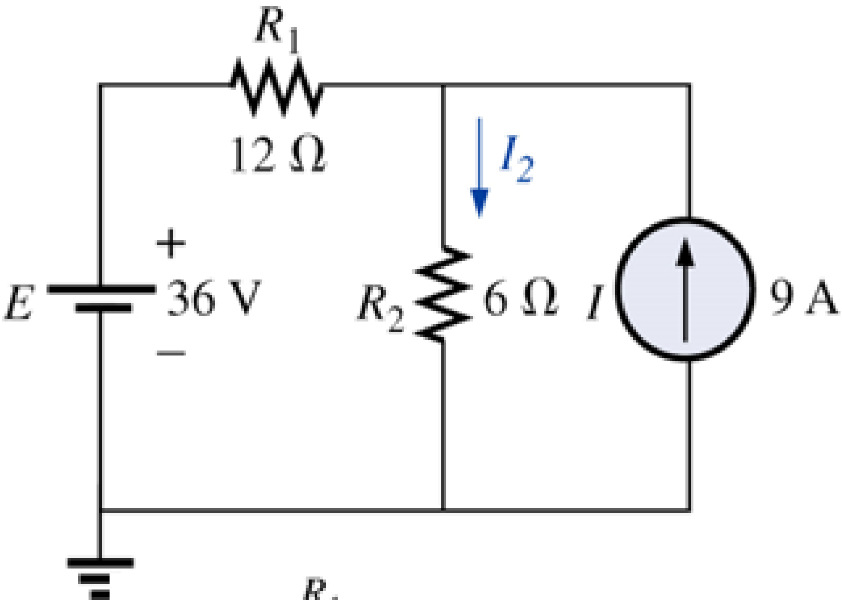
$$I_3'' = \frac{E_2}{R_T} = \frac{48V}{12\Omega} = 4\,A$$

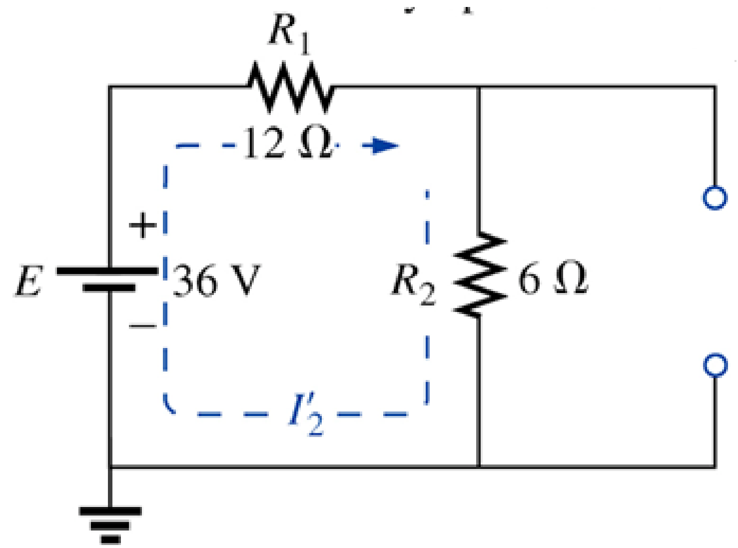
$$I_3 = I_3'' - I_3' = 4\,A - 1.5\,A = 2.5\,A$$

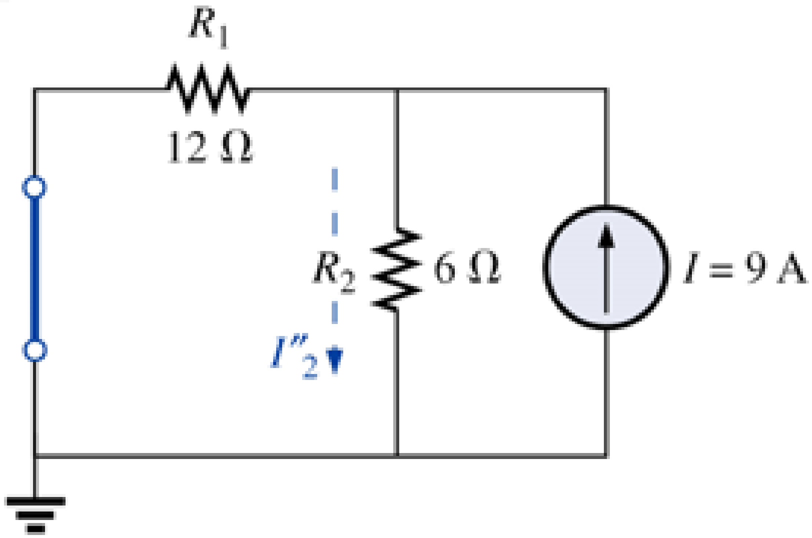
$(direction\ of\ I_3'')$



Example 3: (a). Using superposition, find the current through the 6-Ω resistor. (b). Determine the power of 6Ω resistor.





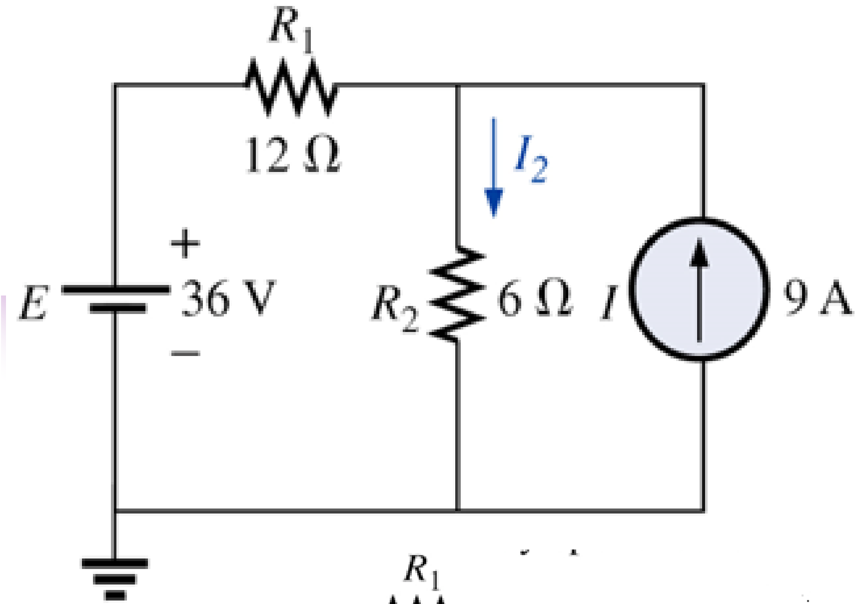


(a) Consider the effect of voltage source:

$$I_2' = \frac{E}{R_1+R_2} = \frac{36V}{12\Omega+6\Omega} = 2(A)$$

$$I_2'' = \frac{IR_p}{R_2} = \frac{(9A)(4\Omega)}{6\Omega} = 6(A)$$

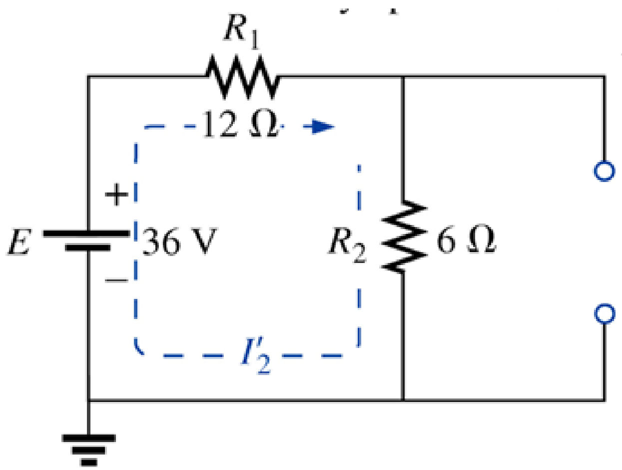
$$I_2 = I_2' + I_2'' = 2A + 6A = 8A$$



(b) The power of 6Ω resistor is
$P = I^2R = (8A)^2(6\Omega) = 384(W)$

Superposition theorem is not applicable to power.

$P_1 = I_2'^2R = (2A)^2(6\Omega)$
$= 24(W)$



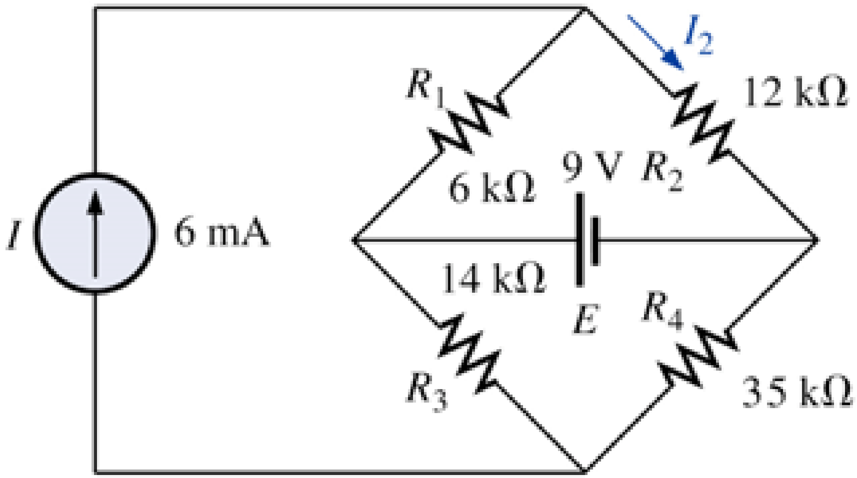
$P_1 = I_2'^2R = (2A)^2(6\Omega)$
$= 24(W)$

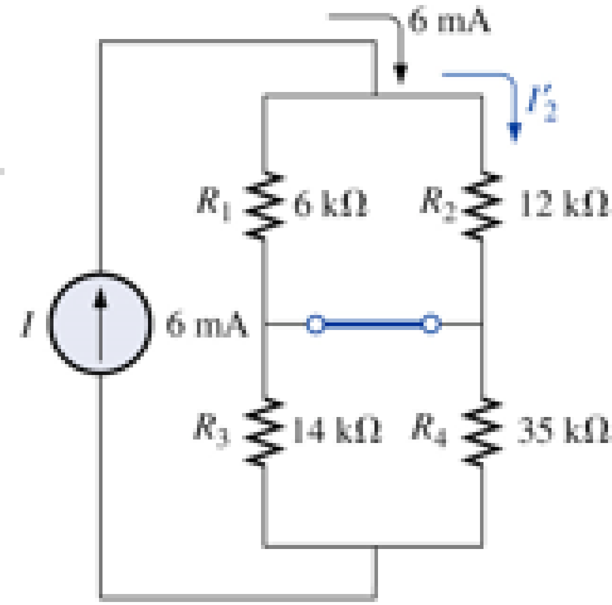
$P_2 = I_2''^2R = (6A)^2(6\Omega)$
$= 216(W)$

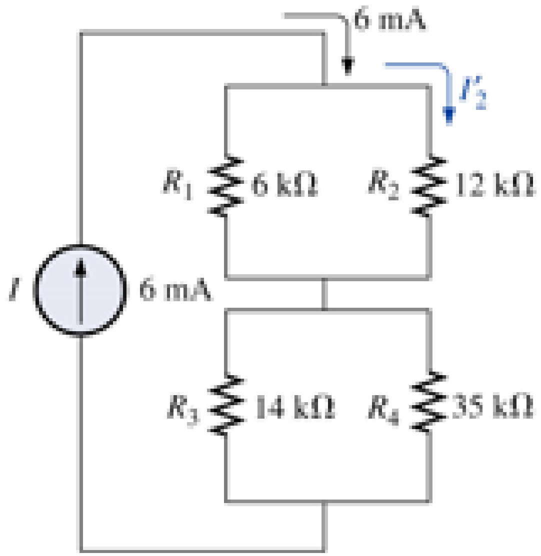
$P_1 + P_2 = 24W + 216W = 240W \neq 384W$

$Because\ (2A)^2 + (6A)^2 \neq (8A)^2$

Example 4: Find the current through the 12-kΩ resistor.

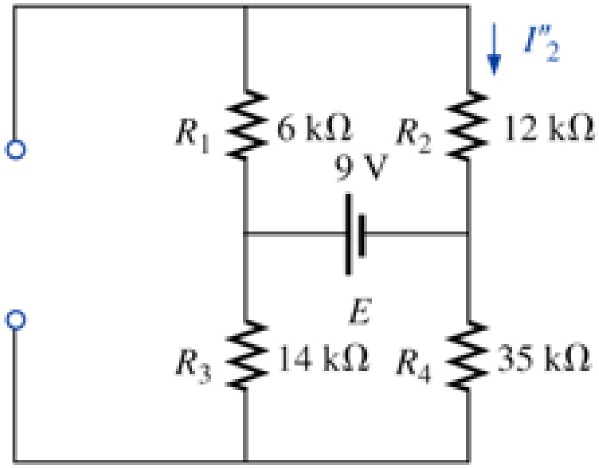


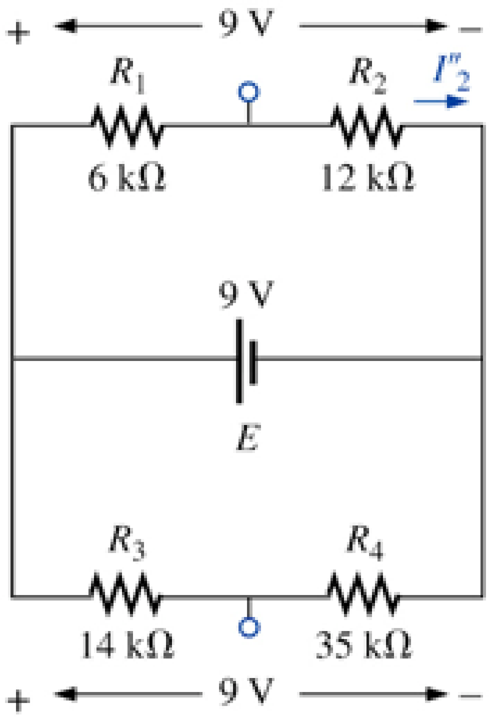




$$I_2' = \frac{R_T}{R_2} I = \frac{R_1 R_2}{(R_1+R_2)R_2} I = \frac{R_1 I}{(R_1+R_2)}$$

$$= \frac{(6k\Omega)(6mA)}{(6k\Omega + 12k\Omega)} = 2mA$$



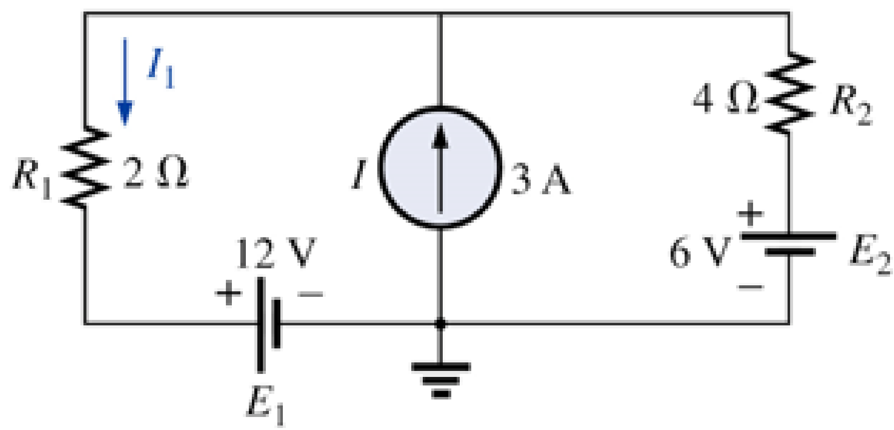


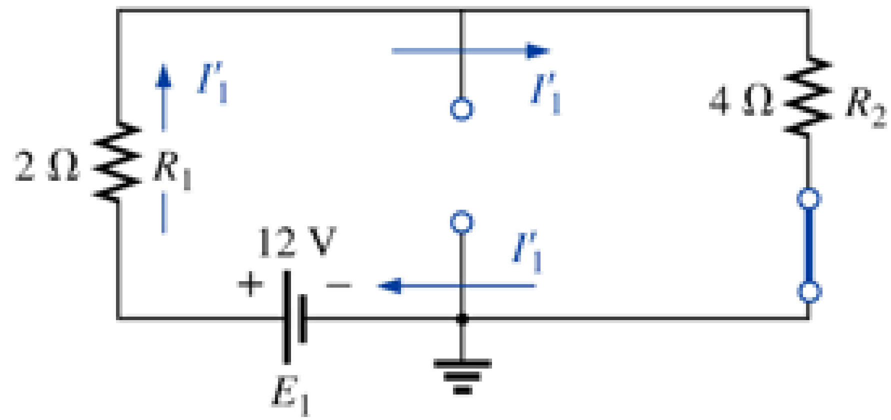
$$I_2'' = \frac{E}{R_1 + R_2} = \frac{9V}{6k\Omega + 12k\Omega} = 0.5(mA)$$

Since $I_2^{'}$ *and* $I_2^{''}$ *have the same direction through* R_2*,*
the desired current is the sum of the two:

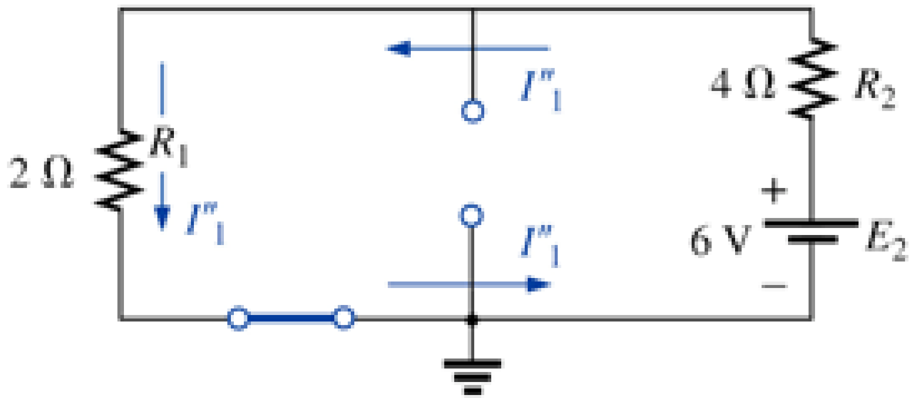
$$I_2 = I_2^{'} + I_2^{''} = 2\,mA + 0.5mA = 2.5mA$$

Example5: Find the current through the 2-Ω resistor of the network.

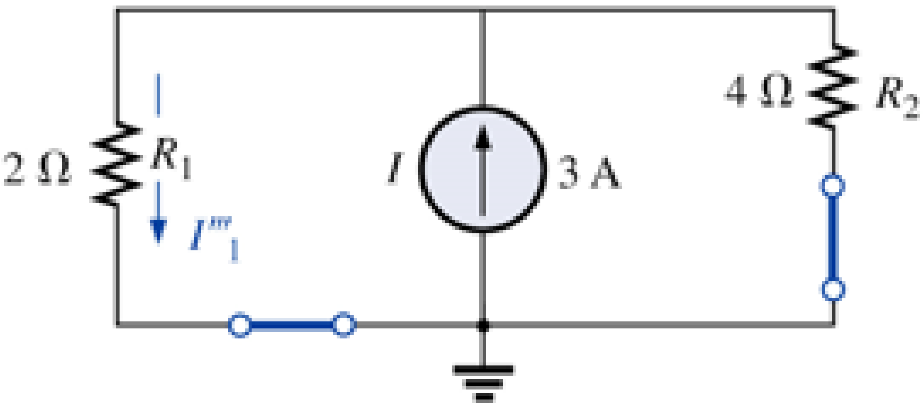




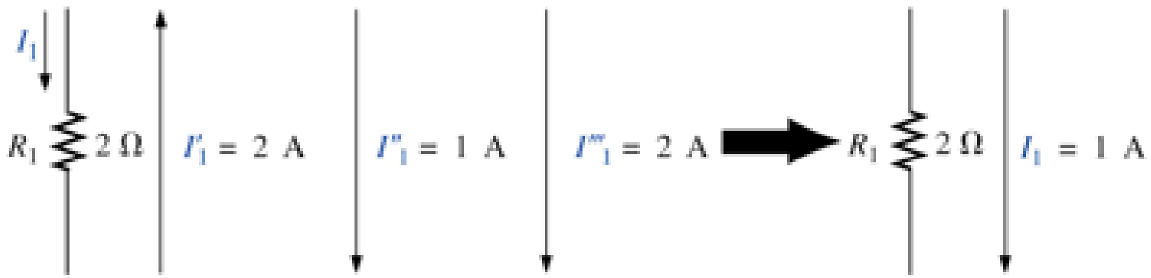
$$I_1' = \frac{E_1}{R_1 + R_2} = \frac{12V}{2\Omega + 4\Omega} = 2(A)$$



$$I_1'' = \frac{E_2}{R_1 + R_2} = \frac{6V}{2\Omega + 4\Omega} = 1(A)$$



$$I_1''' = \frac{R_2 I}{R_1 + R_2} = \frac{(4\Omega)(3A)}{2\Omega + 4\Omega} = 2(A)$$

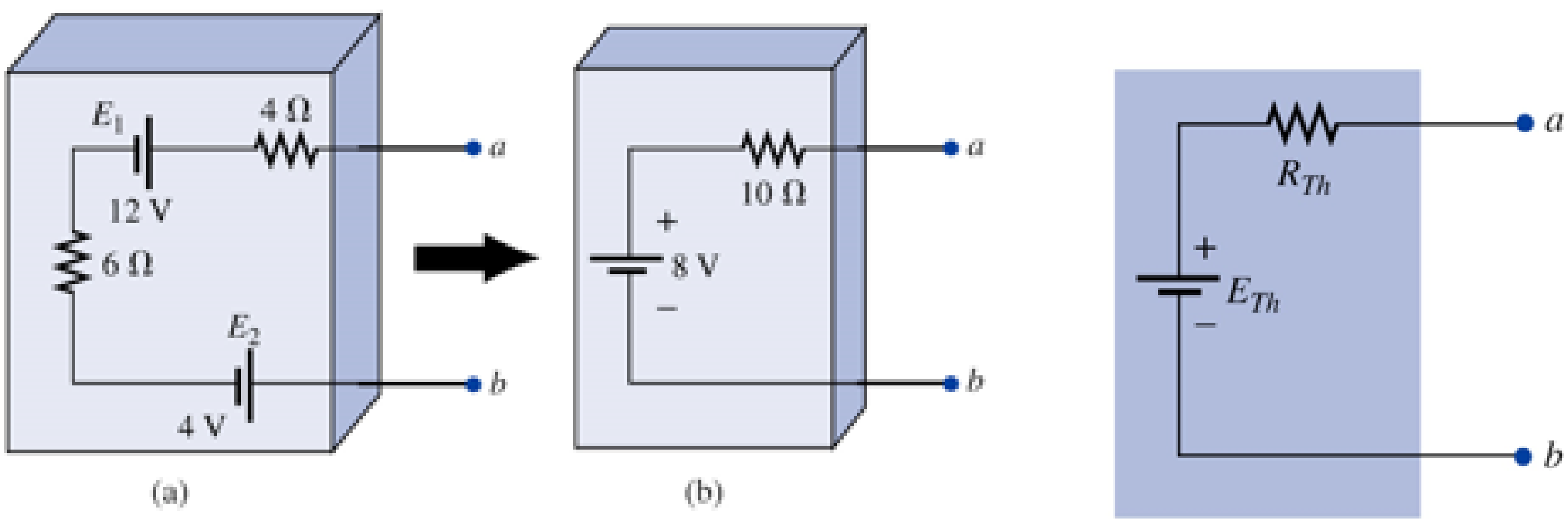


The total current through the 2Ω resistor

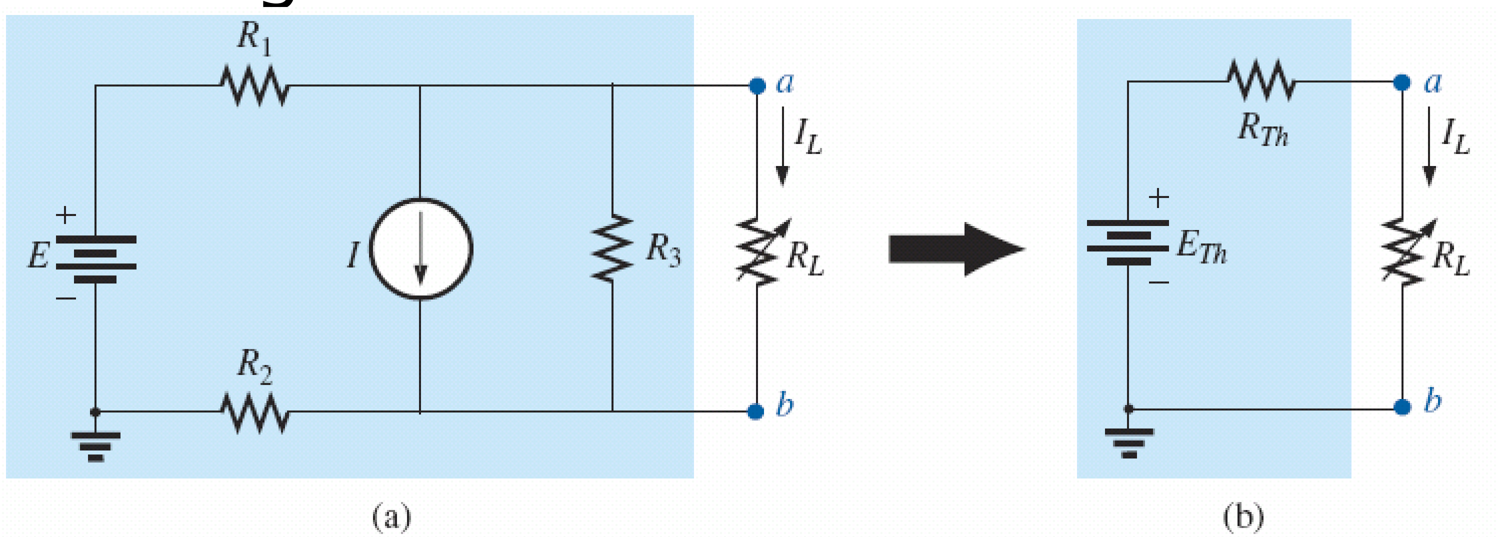
$$I_1 = -I_1' + I_1'' + I_1''' = -2A + 1A + 2A = 1(A)$$

Thévenin’s Theorem

- Thevenin's Theorem states that “Any **linear** circuit containing several voltages and resistances can be replaced by just one **single** voltage in series with a **single** resistance connected across the load“.



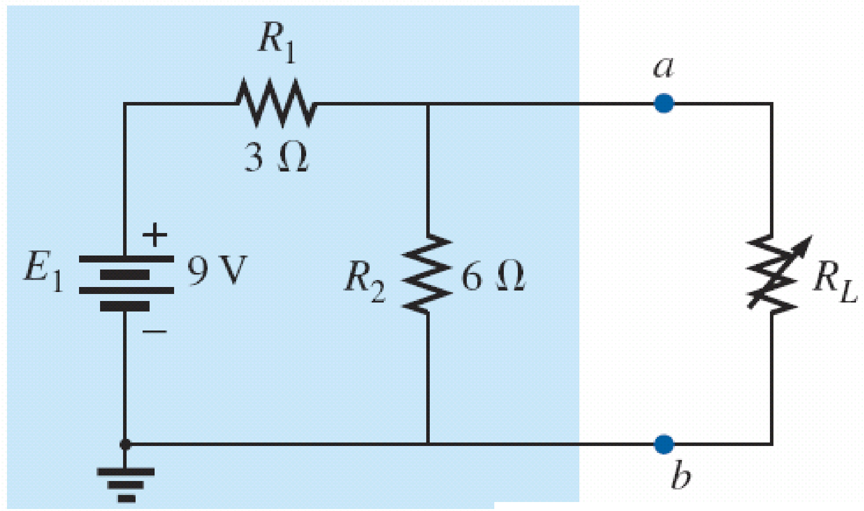
- Reduce the number of components required to establish the same characteristics at the output terminals.
- Investigate the effect of changing a particular component on the behavior of a network without having to analyze the entire network after each change.

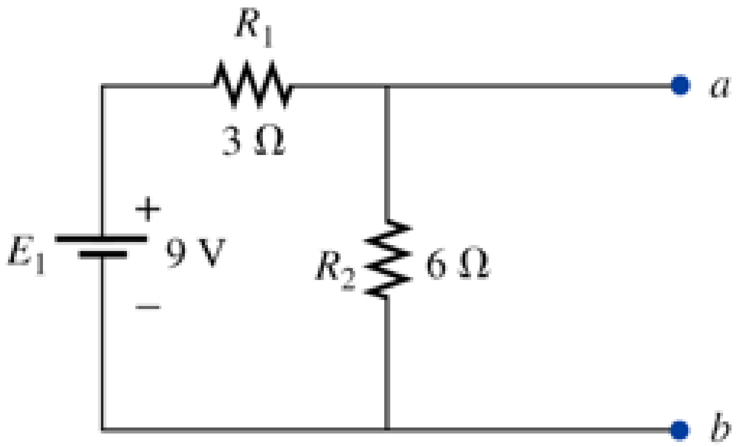


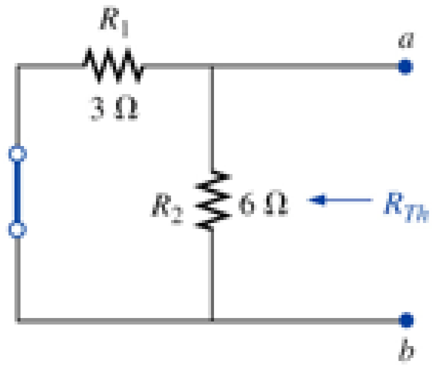
Procedures

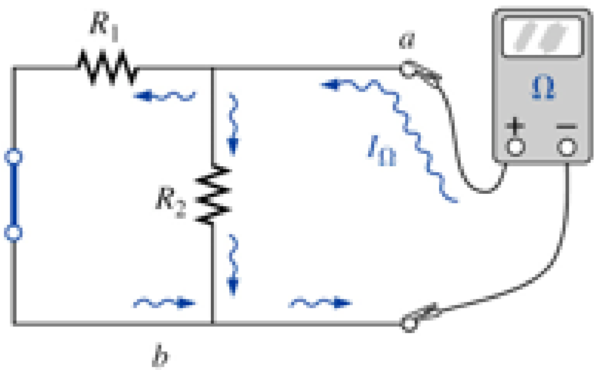
1. Remove that portion of the network where the Thévenin equivalent circuit is found.
2. Mark the terminals of the remaining two-terminal network.
3. Calculate R_{Th} by first setting all sources to zero and then finding the resultant resistance between the two marked terminals.
4. Calculate E_{Th} by first returning all sources to their original position and finding the open-circuit voltage between the marked terminals.
5. Draw the Thévenin equivalent circuit with the portion of the circuit previously removed replaced between the terminals of the equivalent circuit.

Example6: Find the Thevenin equivalent circuit for the network in the shaded area of the network. Then find the current through R_L for values of 2Ω, 10Ω, and 100Ω.

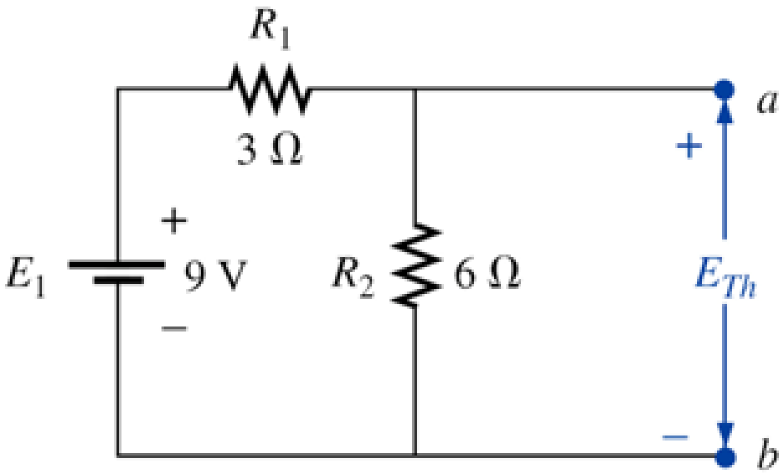


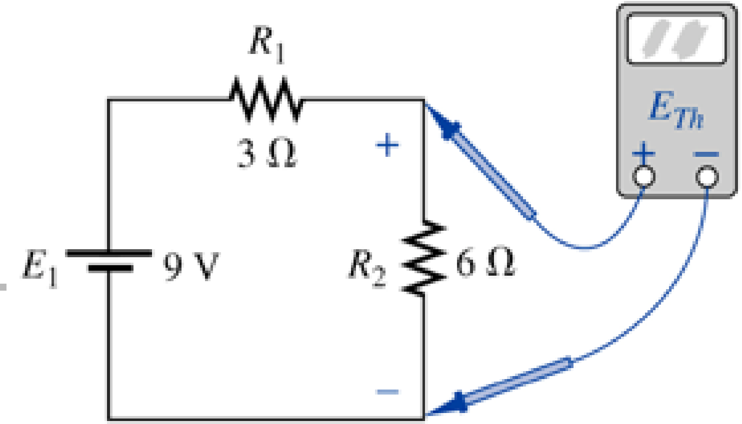




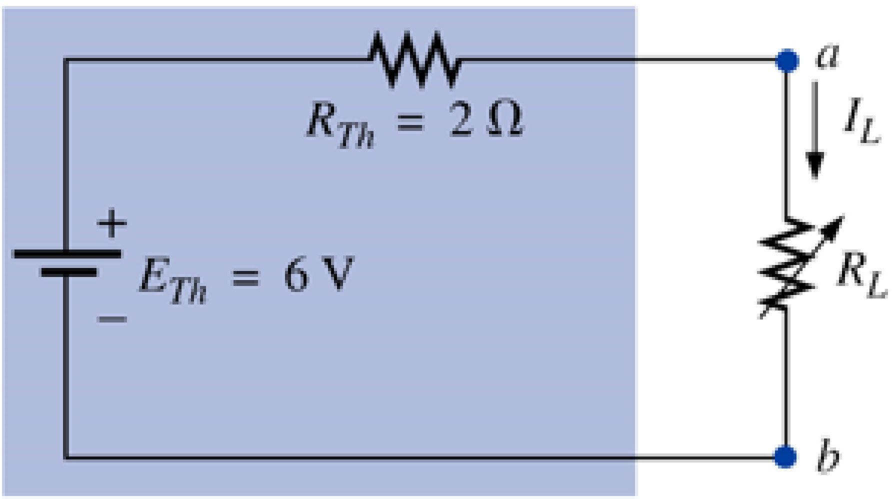


$$R_{TH} = R_1 \,//\, R_2 = \frac{(3\Omega)(6\Omega)}{3\Omega + 6\Omega} = 2\Omega$$





$$V_{TH} = \frac{R_2 E_1}{R_2 + R_1} = \frac{(6\Omega)(9V)}{6\Omega + 3\Omega} = 6(V)$$



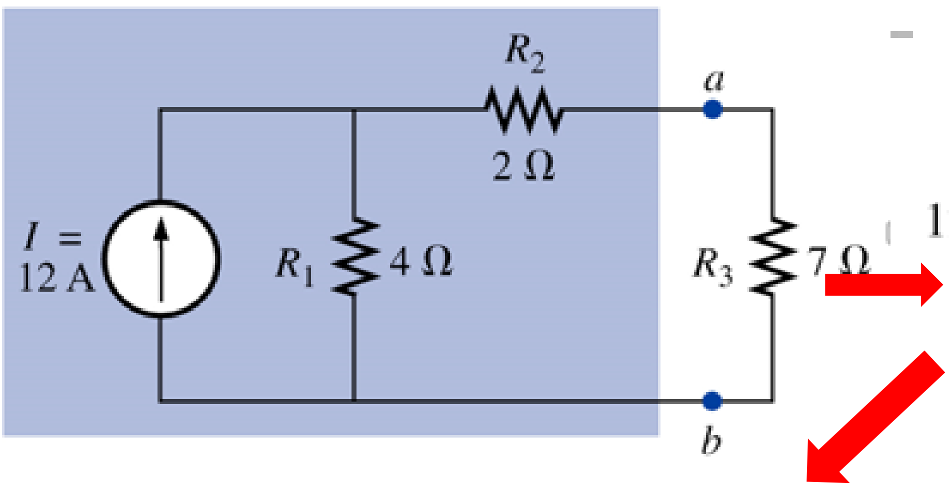
$$I_L = \frac{E_{TH}}{R_{TH} + R_L}$$

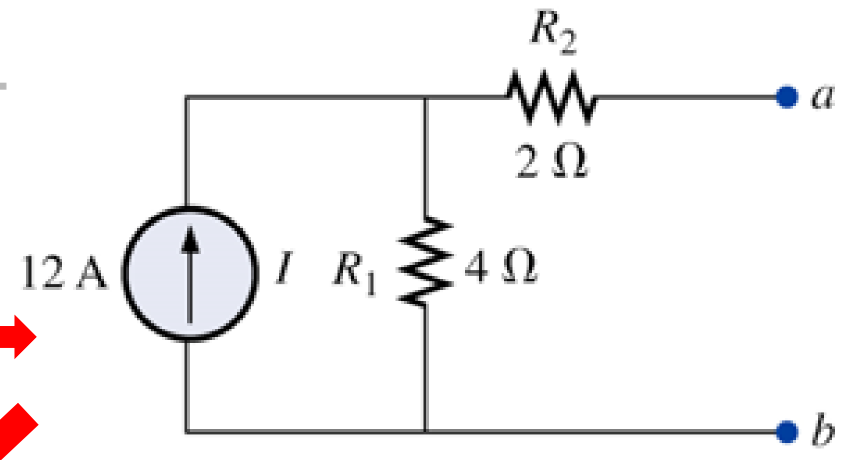
$$R_L = 2\Omega: \quad I_L = \frac{6V}{2\Omega + 2\Omega} = 1.5\ A$$

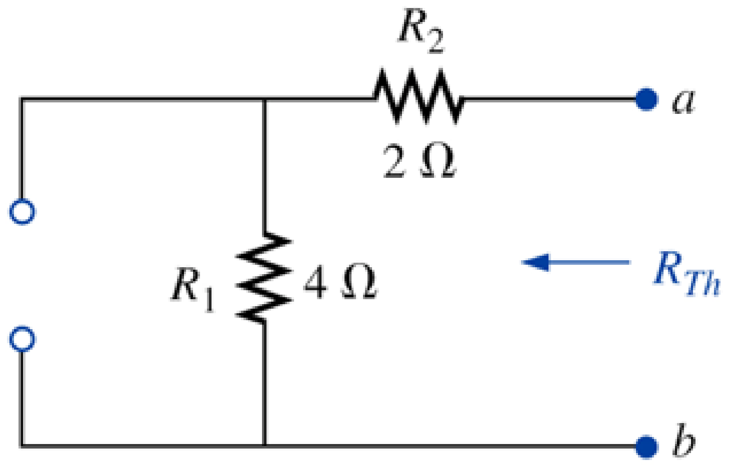
$$R_L = 10\Omega: \quad I_L = \frac{6V}{2\Omega + 10\Omega} = 0.5\ A$$

$$R_L = 100\Omega: \quad I_L = \frac{6V}{2\Omega + 100\Omega} = 0.059\ A$$

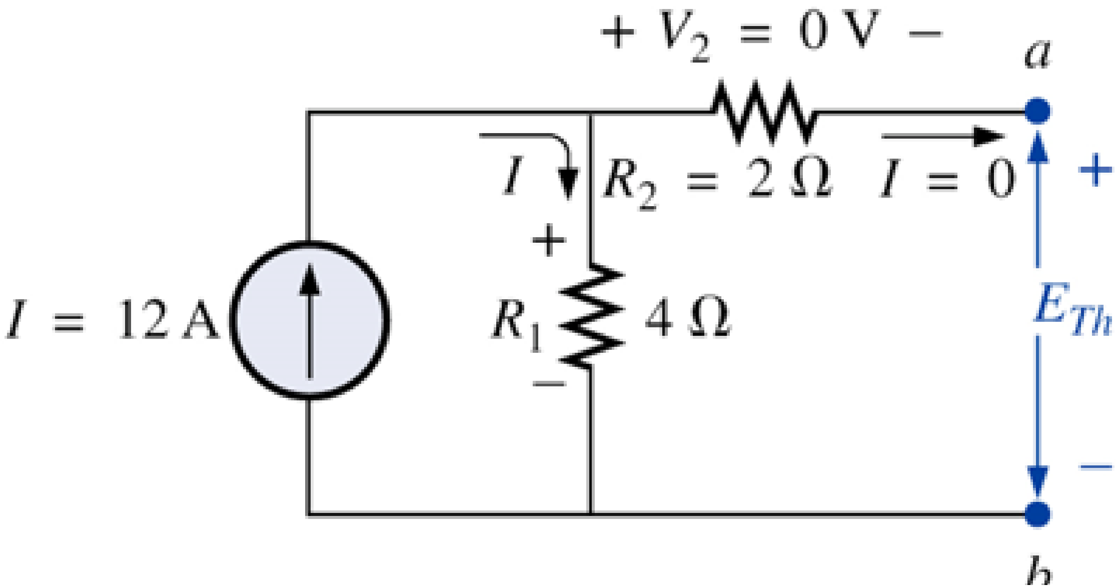
Example7: Find the Thevenin equivalent circuit for the network in the shaded area. Calculate the voltage on R_3.





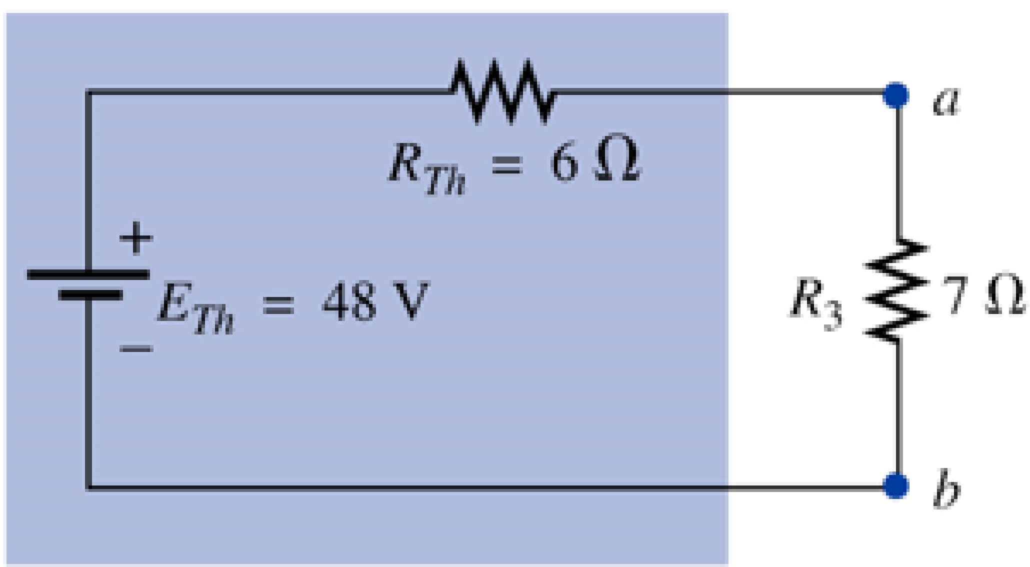


$$
\begin{aligned}
R_{TH} &= R_1 + R_2 \\
&= 4\Omega + 2\Omega \\
&= 6\Omega
\end{aligned}
$$



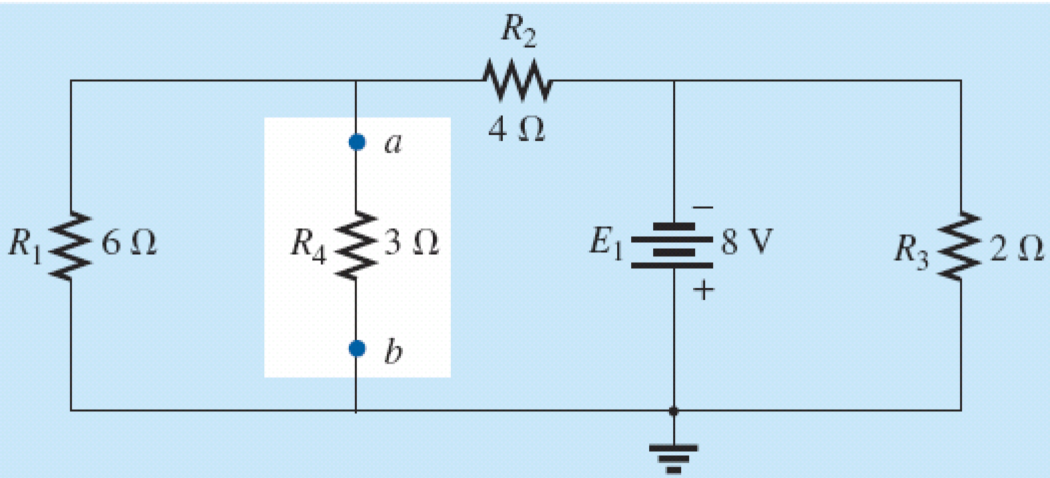
$$V_2 = I_2 R_2 = (0) R_2 = 0V$$

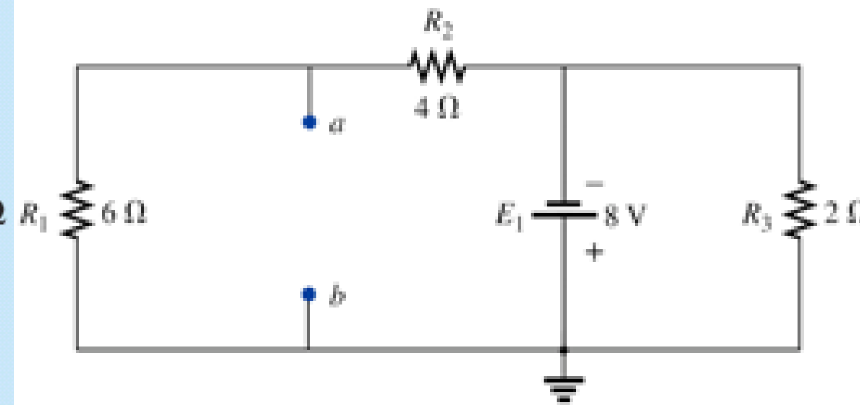
$$E_{Th} = V_1 = I_1 R_1 = (12A)(4\Omega) = 48(V)$$

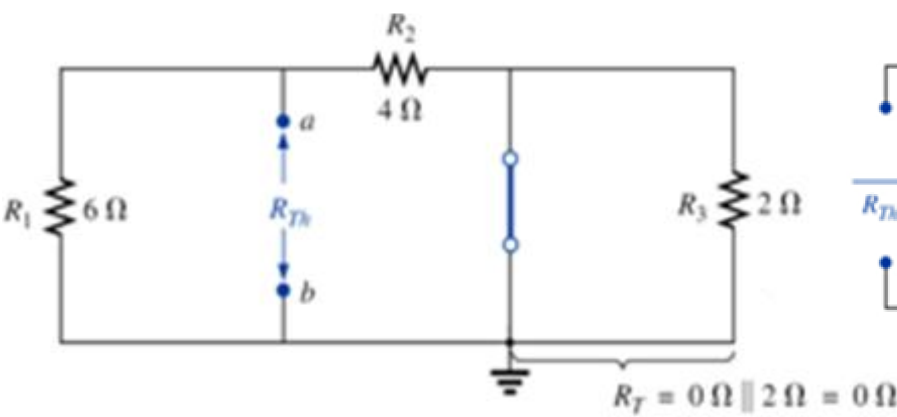


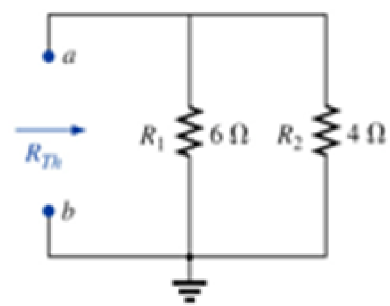
$$V_3 = E_{Th} \frac{R_3}{R_{Th} + R_3} = 48\text{V} \frac{7\Omega}{7\Omega + 6\Omega} = 25.85(V)$$

Example8: Find the Thevenin equivalent circuit for the network in the shaded area of the network

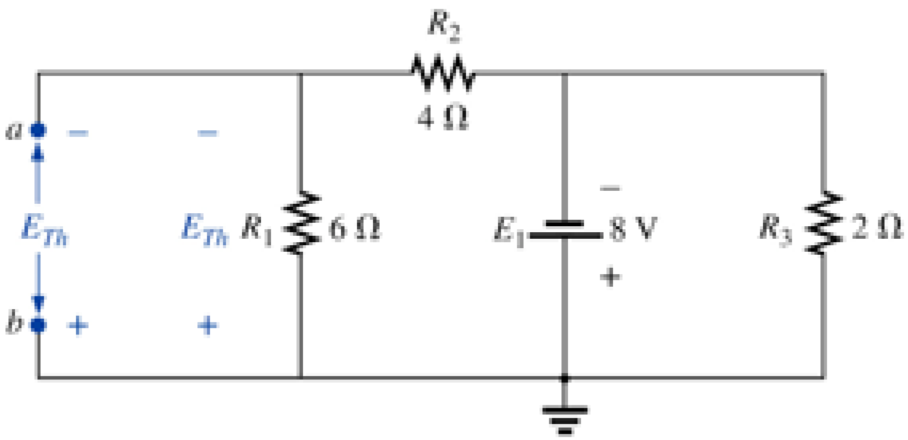


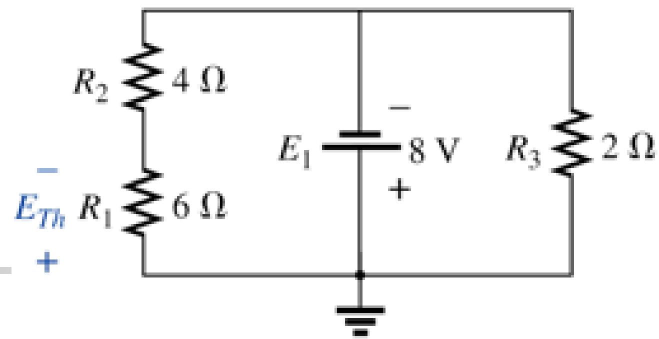


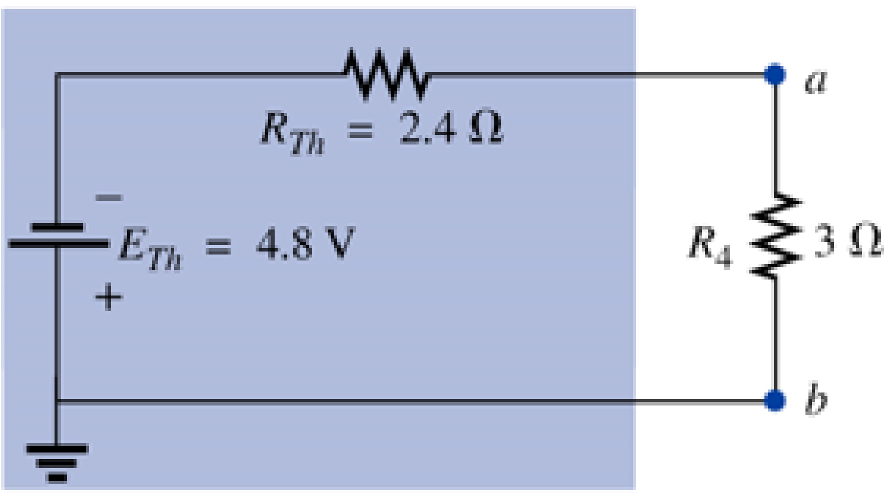




$$R_{TH} = R_1 // R_2 = \frac{(6\Omega)(4\Omega)}{6\Omega + 4\Omega} = 2.4\Omega$$





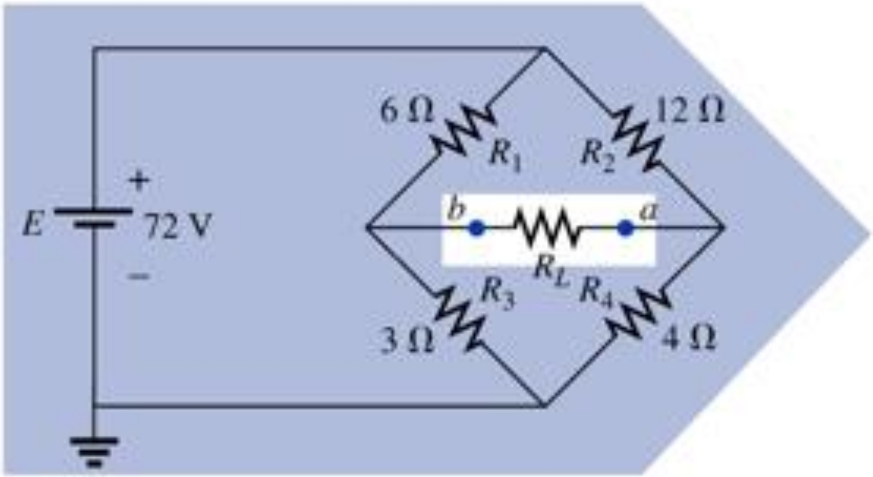


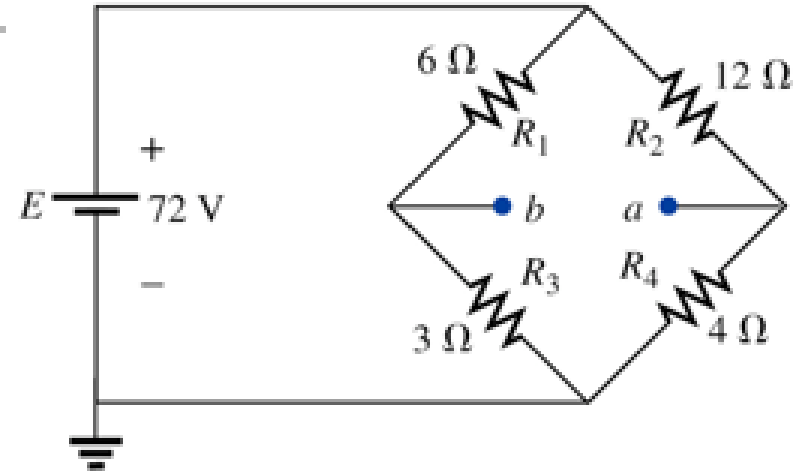
$$E_{TH} = \frac{R_1 E_1}{R_1 + R_2}$$

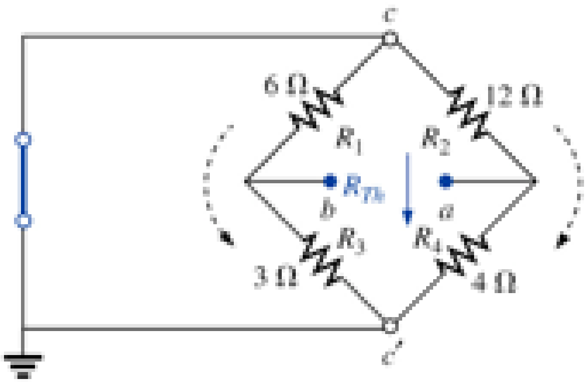
$$= \frac{(6\Omega)(8V)}{6\Omega + 4\Omega}$$

$$= 4.8V$$

Example9: Find the Thevenin equivalent circuit for the network in the shaded area of the network

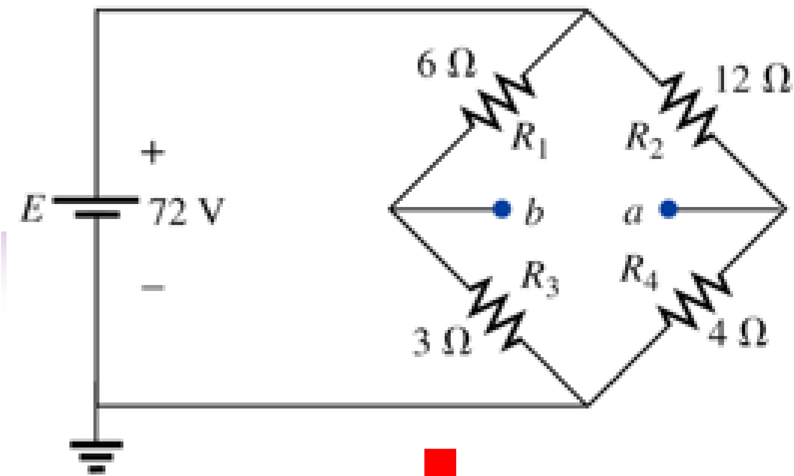


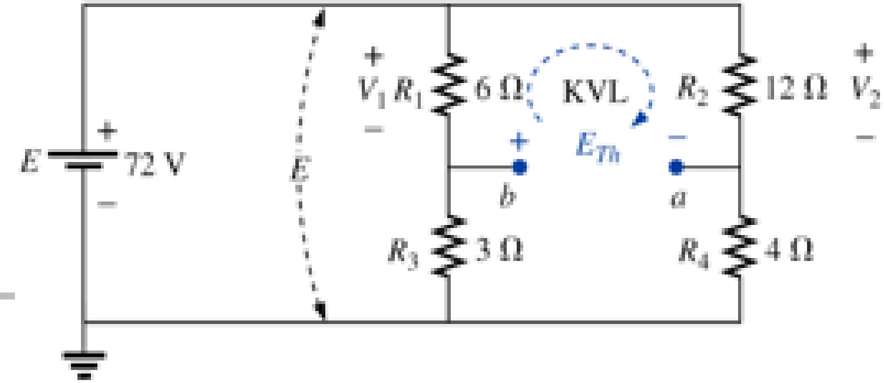


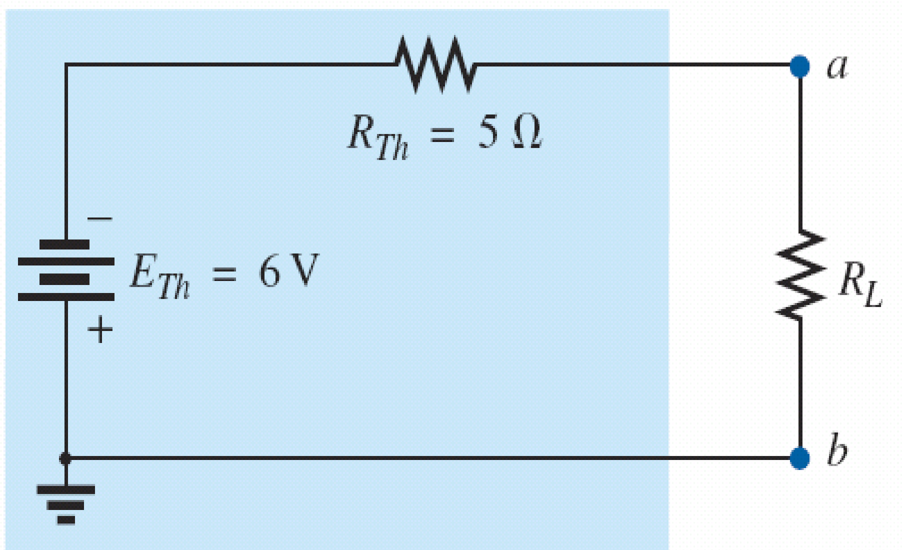


(a)

$$
\begin{aligned}
R_{TH} &= R_1 // R_3 + R_2 // R_4 \\
&= 6\Omega // 3\Omega + 4\Omega // 12\Omega \\
&= 2\Omega + 3\Omega = 5\Omega
\end{aligned}
$$







$$V_1 = \frac{R_1 E}{R_1 + R_3} = \frac{(6\Omega)(72V)}{6\Omega + 3\Omega} = 48V$$

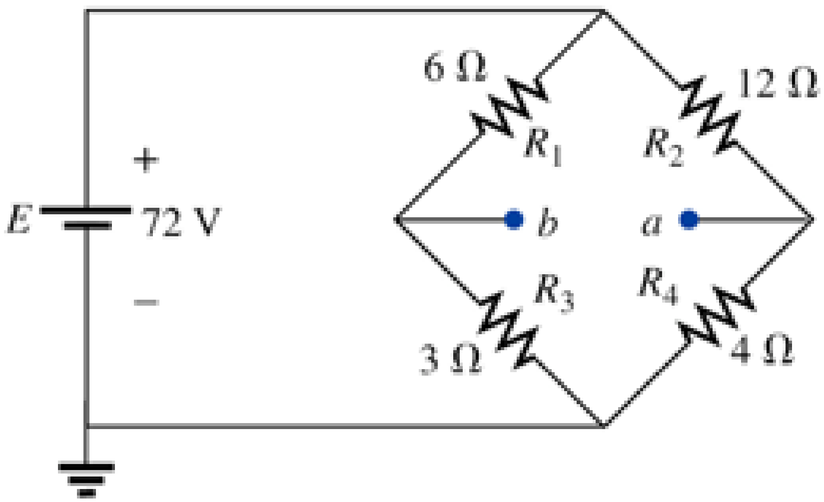
$$V_2 = \frac{R_2 E}{R_2 + R_4} = \frac{(12\Omega)(72V)}{12\Omega + 4\Omega} = 54V$$

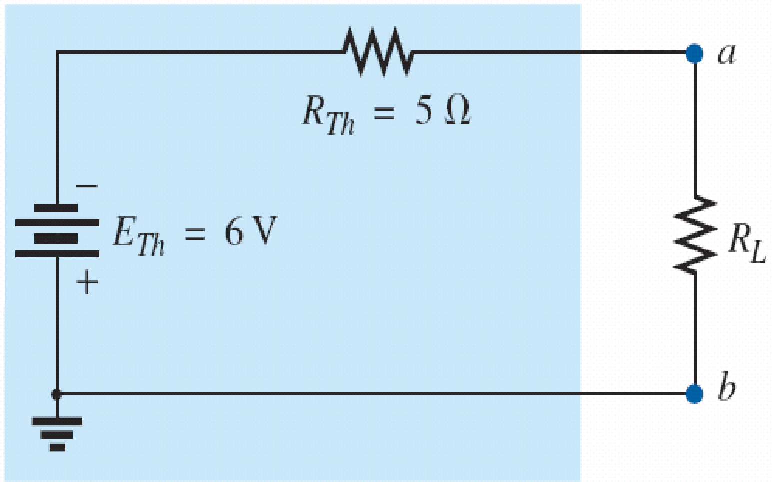
KVL@ CP

$$-V_1 + V_2 - E_{th} = 0$$

$$E_{th} = V_2 - V_1 = 54V - 48V = 6V$$

Extend to bridge circuit





$$E_{Th} = 0$$

$$V_a = V_b$$

$$\frac{ER_1}{R_1 + R_3} = \frac{ER_2}{R_2 + R_4}$$

$$R_1R_2 + R_1R_4 = R_1R_2 + R_2R_3$$

$$R_1R_4 = R_2R_3$$

$$\frac{R_1}{R_3} = \frac{R_2}{R_4}$$