Chapter 10

EMT1150
Introduction to Circuit Analysis

Department of Computer Engineering Technology

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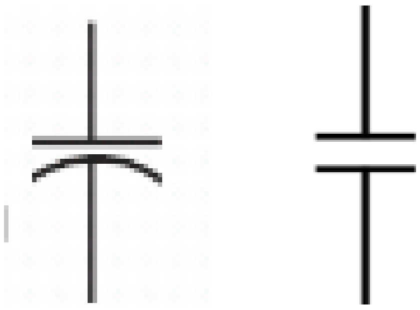
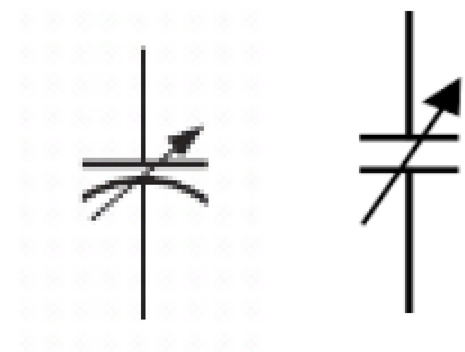
Chapter10 Capacitors

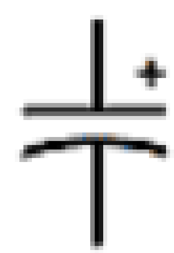
- Introduction to Capacitors
- The Electric Field
- Capacitance
- Capacitors in Series and Parallel
- Transients in Capacitive Networks

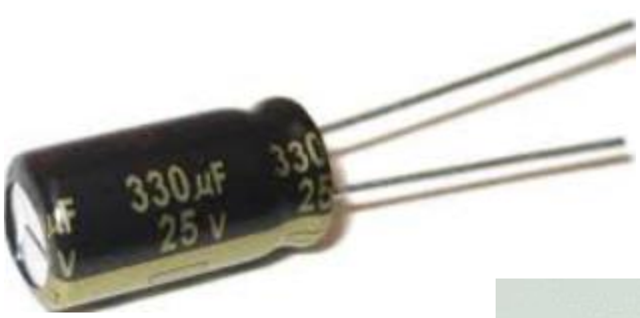
Introduction

- Always compare with resistors
- Two-terminal device
- Symbol





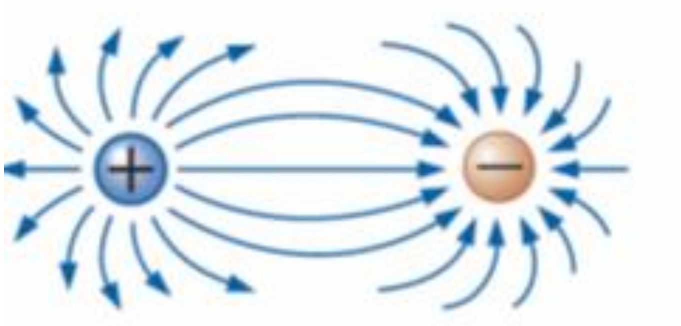




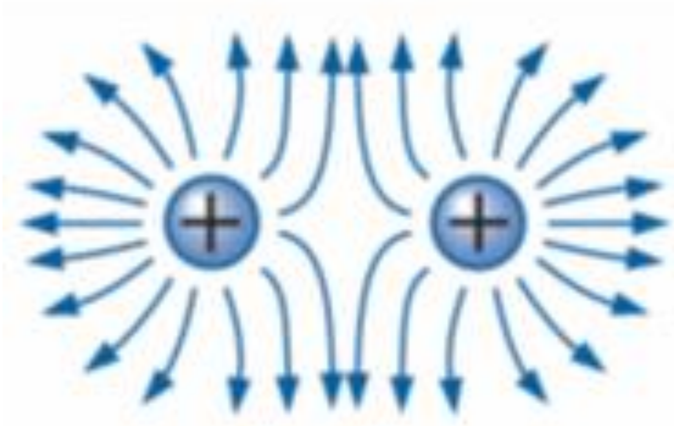
Intro CNTD..

- Capacitor **does not dissipate energy** as does the resistor but **store it.**
- Capacitor displays its true characteristics only when a change in the voltage or current is made in the network.

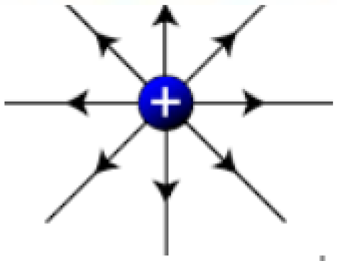
Electric field

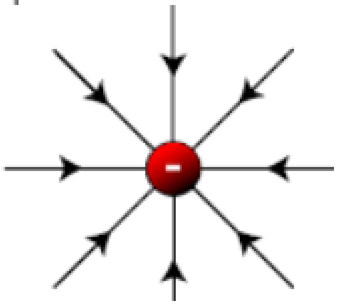


- Electric filed/electrostatic field is a field of force generated by the electrical charges.



- It is represented by electric flux lines, which indicate the strength of the electric field at any point around any charged body.

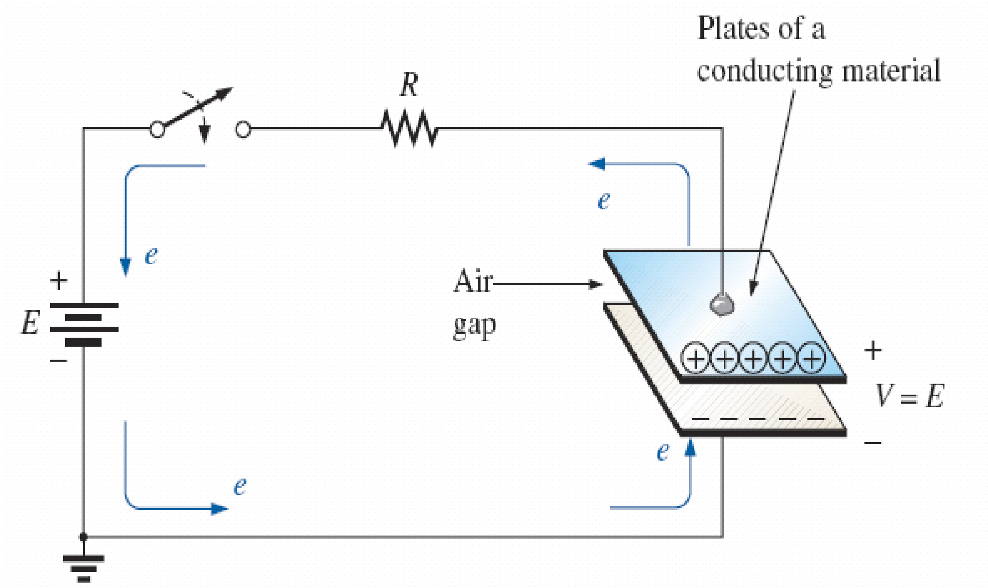




- The denser the lines of flux, the stronger is the electric field.

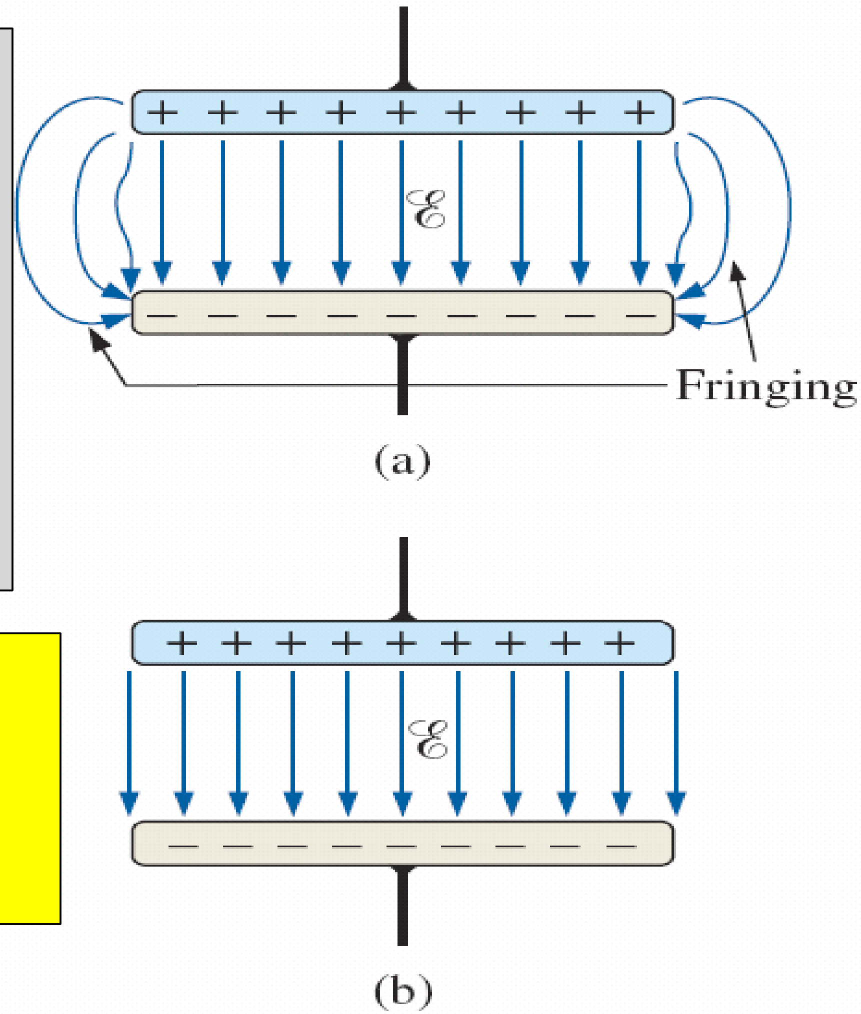
Capacitor

- A ***capacitor*** is constructed simply of two parallel conducting plates separated by insulating material.
- Insulating material is called dielectric material.



Electric field in capacitor

Note the fringing that occurs at the edges as the flux lines originating from the points farthest away from the negative plate strive to complete the connection



$$\mathcal{E} = \frac{V}{d}$$

$\mathcal{E}$: Volt/m

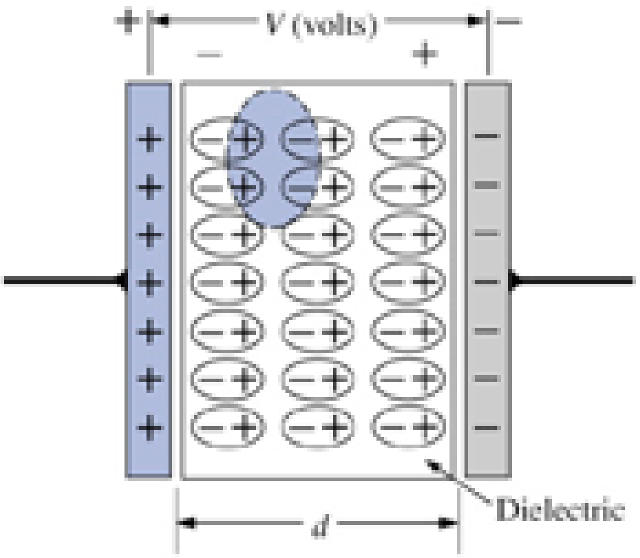
V: Volt

d: m

Capacitance is a measure of a capacitor's ability to store charge on its plates.

Unit: Farad (F)

A capacitor has a capacitance of 1 farad if 1 coulomb of charge is deposited on the plates by a potential difference of 1 volt cross the plates.



$$C = \frac{Q}{V}$$

C = farad (F)
Q = coulombs (C)
V = volts (V)

Example1: If 40V are applied across a 470μF capacitor, find the charge on the plates.

$$\boxed{Q = CV = (470\mu F)(40V) = 18.8\ (mC)}$$

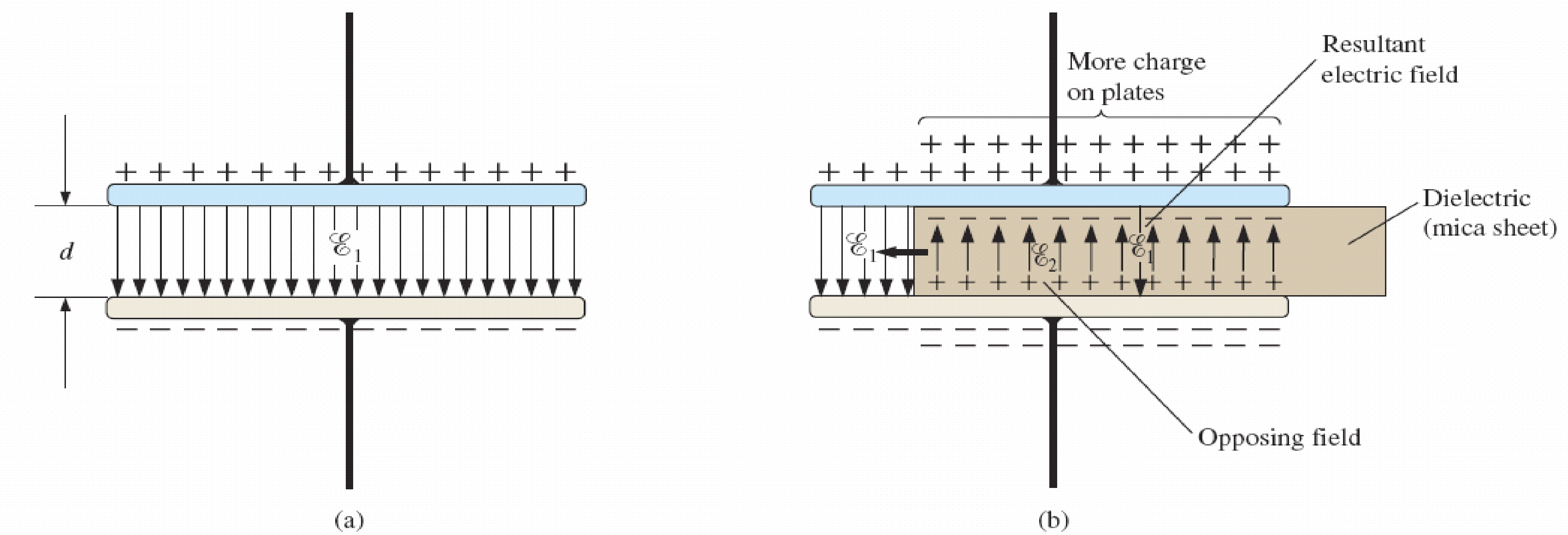
Capacitance

- The capacitance of any capacitor is due primarily to three factors:
  - Permittivity (Dielectric material)
  - Distance
  - Area

$$C = \varepsilon \frac{A}{d}$$

C: Farad (F)
ε : Permittivity (F/m)
A: m^2
d: m

- **Permittivity** is the measure of the resistance that is encountered when forming an electric field in a medium.



- In general, permittivity of other materials can compare to the permittivity of vacuum.

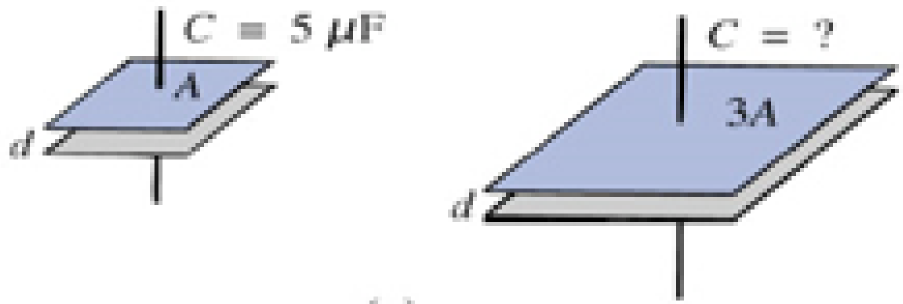
$$\boxed{\epsilon_r = \frac{\varepsilon}{\epsilon_0}}$$

$\varepsilon_0 = 8.854 \times 10^{-12}$ F/m

Dielectric	ε_r (Average value)
Vacuum	1.0
Air	1.0006
Teflon	2.0
Rubber	3.0
Oil	4.0
Mica	5.0
Ceramics	20-7500

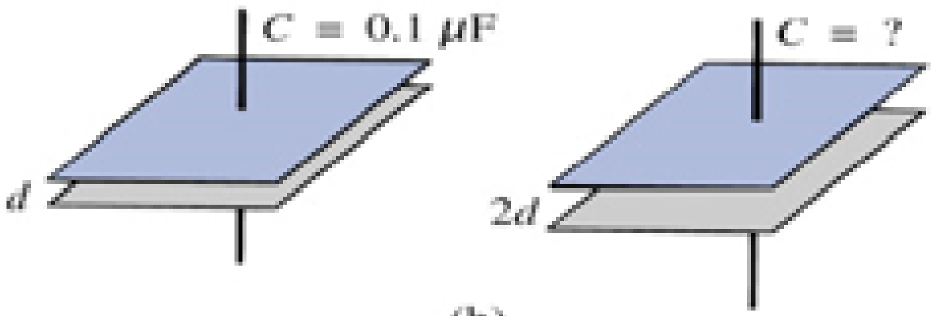
$$C = \varepsilon_0 \varepsilon_r \frac{A}{d}$$

Example2: Determine the capacitance of each capacitor on the right side.



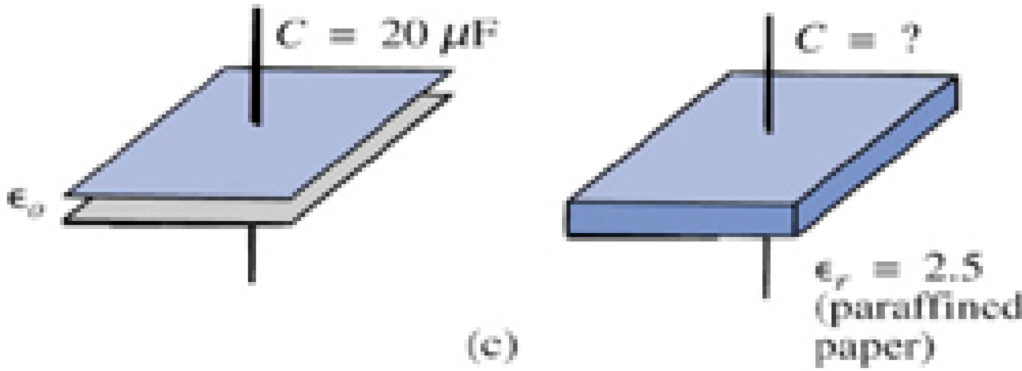
(a)

(a) C = 3(5μF) =15 (μF)

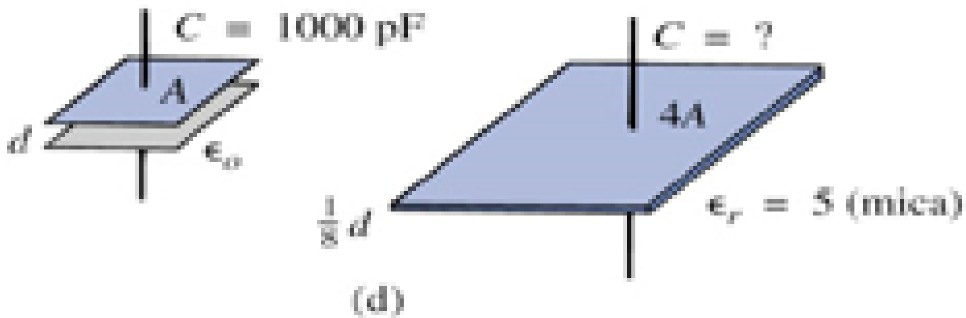


(b)

(b) C = (0.1μF)/2 =0.05 (μF)



(c) C = 2.5 × (20μF)
=50 (μF)



(d) C = 4 × 5 ×
(1000pF) /(1/8)
=0.16 (μF)

Example3: For the capacitor in the figure:

a. Determine the capacitance.

b. Determine the electric field strength between the plates if 450 V are applied across the plates.

c. Find the resulting charge on each plate.

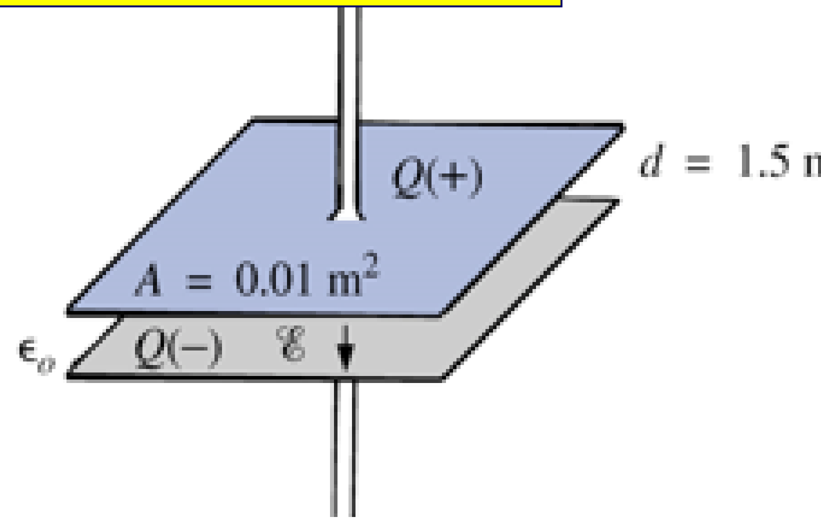
$$(a).C_o = \frac{\varepsilon_o A}{d} = \frac{\left(8.85\times10^{-12}F/m\right)\left(0.01m^2\right)}{1.5\times10^{-3}m} = 59.0\times10^{-12}F$$

$$(b).\varepsilon = \frac{V}{d} = \frac{450V}{1.5\times10^{-3}m} \cong 300\times10^{3}V/m$$

$$(d).C = \frac{Q}{V}$$

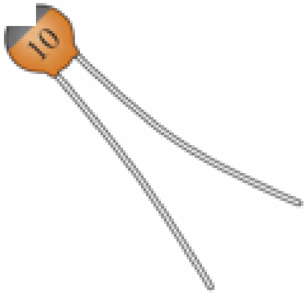
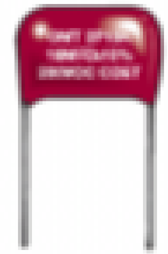
$$Q = CV = \left(59.0\times10^{-12}\right)\left(450V\right)$$

$$= 26.550\times10^{-9}C = 26.55nC$$



Type of capacitors

- Fixed capacitor
  - *Electrolytic capacitors*
  - *Ceramic (disc) capacitor*
  - *Film/foil polyester capacitor*
  - *Mica capacitors*
  - *Oil-filled capacitor*
- Variable capacitor


Capacitors in Series

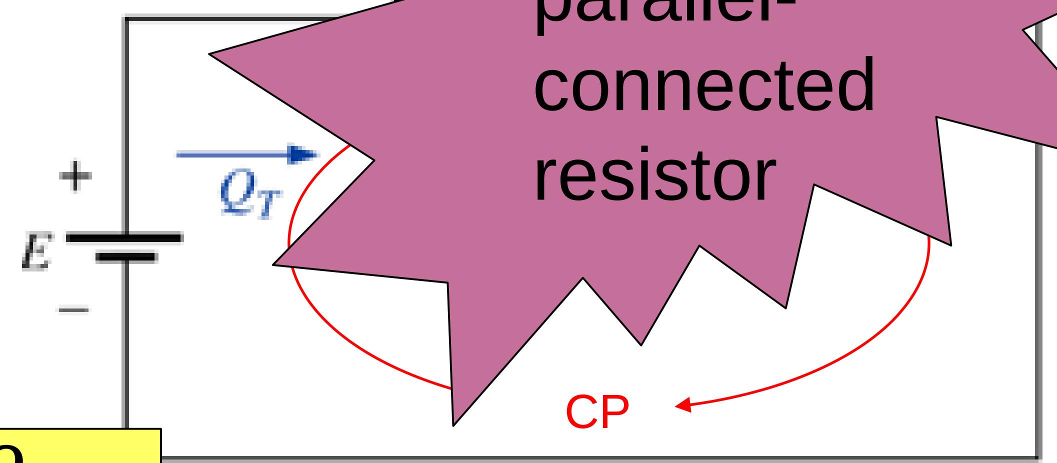
Apply KVL to CP

$$E = V_1 + V_2 + V_3$$

$$V = \frac{Q}{C}$$

$$\frac{Q_T}{C_T} = \frac{Q_1}{C_1} + \frac{Q_2}{C_2} + \frac{Q_3}{C_3}$$

$$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$



Similar format to parallel-connected resistor

$$Q_T = Q_1 = Q_2 = Q_3$$

$$C_T = \frac{C_1 C_2}{C_1 + C_2}$$

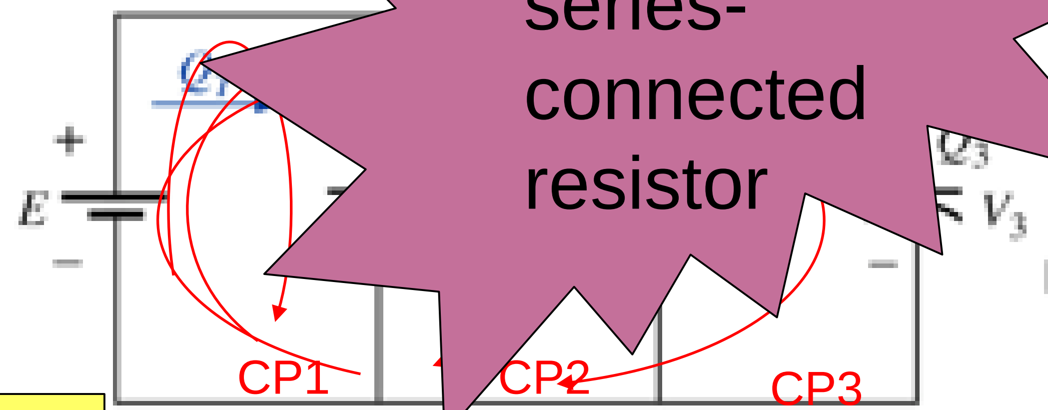
Capacitors in parallel

Similar format to series-connected resistor

Apply KVL to CPs

$$E = V_1 = V_2 = V_3$$

$$V = \frac{Q}{C}$$



$$\frac{Q_T}{C_T} = \frac{Q_1}{C_1} = \frac{Q_2}{C_2} = \frac{Q_3}{C_3}$$

$$Q_T = Q_1 + Q_2 + Q_3$$

$$Q = CV$$

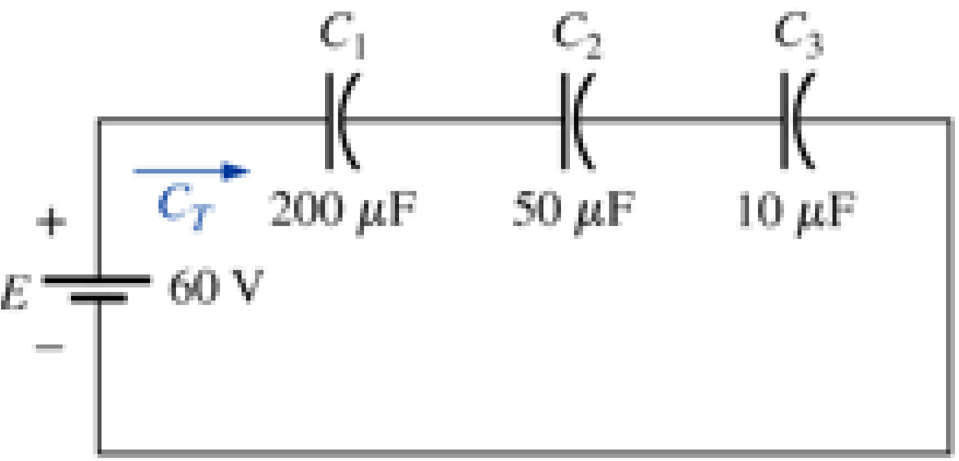
$$C_T E = C_1 V_1 + C_2 V_2 + C_3 V_3$$

$$C_T = C_1 + C_2 + C_3$$

Example4: a. Find the total capacitance.

b. Determine the charge on each plate.

c. Find the voltage across each capacitor.



$$b.\quad Q_T = Q_1 = Q_2 = Q_3 = C_T E = (8\times 10^{-6}\,F)(60V) = 480\ \mu C$$

$$a.\quad \frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{200\times 10^{-6}\,F} + \frac{1}{50\times 10^{-6}\,F} + \frac{1}{10\times 10^{-6}\,F} = 0.125\times 10^{6}$$

$$C_T = \frac{1}{0.125\times 10^{6}} = 8\ \mu F$$

$$c.\quad V_1 = \frac{Q_1}{C_1} = \frac{480\times 10^{-6}\,C}{200\times 10^{-6}\,F} = 2.4\,V$$

$$V_2 = \frac{Q_2}{C_2} = \frac{480\times 10^{-6}\,C}{50\times 10^{-6}\,F} = 9.6\,V$$

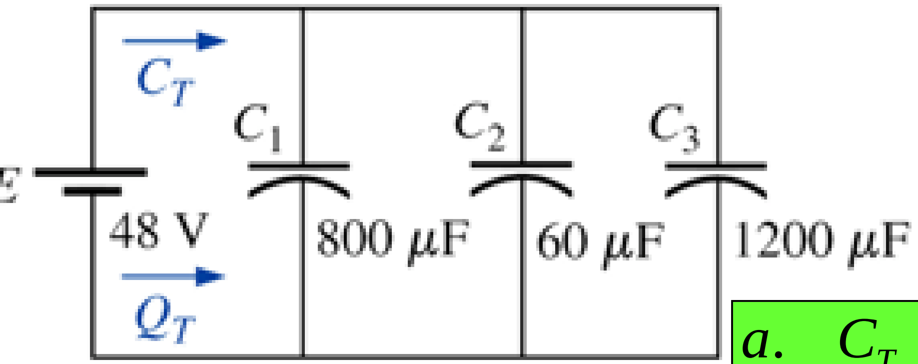
$$V_3 = \frac{Q_3}{C_3} = \frac{480\times 10^{-6}\,C}{100\times 10^{-6}\,F} = 48.0\,V$$

$$and\quad E = V_1 + V_2 + V_3 = 60\,V$$

Example5: a. Find the total capacitance.

b. Determine the charge on each plate.

c. Find the total charges.

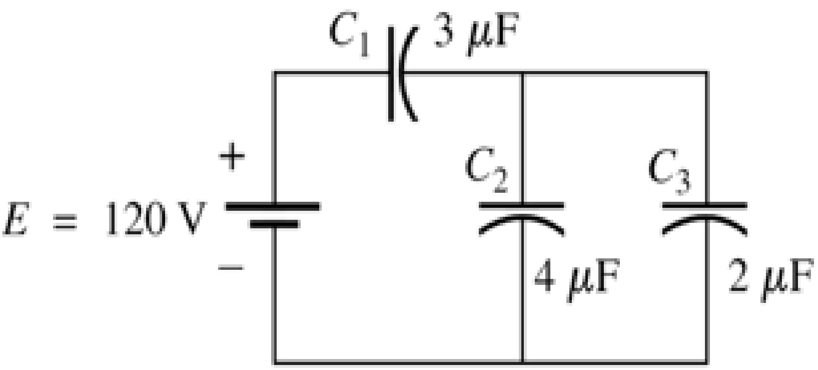


$$
\begin{aligned}
a.\quad C_T &= C_1 + C_2 + C_3 \\
&= 800\,\mu F + 60\,\mu F + 1200\,\mu F \\
&= 2060\,\mu F
\end{aligned}
$$

$$
\begin{aligned}
b.\quad Q_1 &= C_1 E = (800\times 10^{-6}\,F)(48V) = 38.4\,mC \\
Q_2 &= C_2 E = (60\times 10^{-6}\,F)(48V) = 2.88\,mC \\
Q_3 &= C_1 E = (1200\times 10^{-6}\,F)(48V) = 57.6\,mC
\end{aligned}
$$

$$
c.\quad Q_T = Q_1 + Q_2 + Q_3 = 98.88\,mC
$$

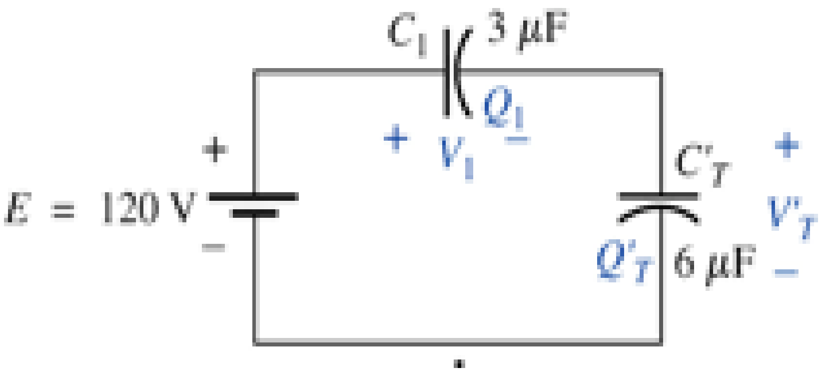
Example6: Find the voltage across and charge on each capacitor for the network



$$C_T^{'} = C_2 + C_3 = 4\,\mu F + 2\,\mu F = 6\,\mu F$$

$$C_T = \frac{C_1 C_T^{'}}{C_1 + C_T^{'}} = \frac{(3\,\mu F)(6\,\mu F)}{3\,\mu F + 6\,\mu F} = 2\,\mu F$$

$$Q_T = C_T E = (2\times10^{-6}\,F)(120V) = 240\,\mu C$$

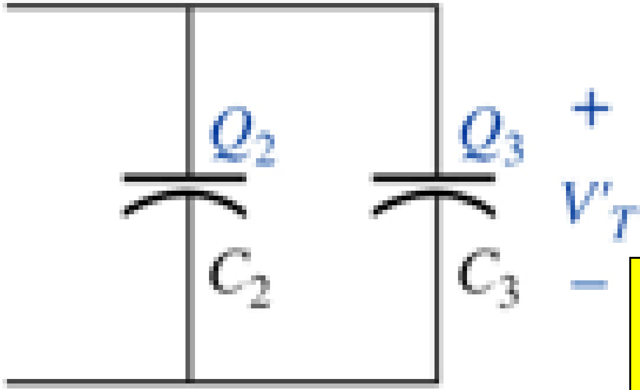


$$Q_T = Q_1 = Q_T^{'}$$

$$Q_1 = 240\,\mu C$$

$$V_1 = \frac{Q_1}{C_1} = \frac{240\times10^{-6}\,C}{3\times10^{-6}\,F} = 80\,V$$

$$Q_T^{'} = 240\ \mu C$$

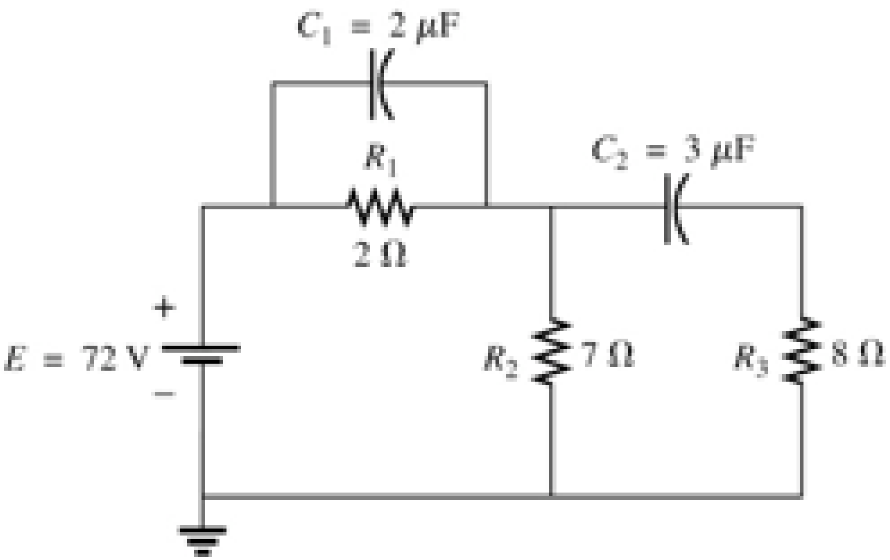


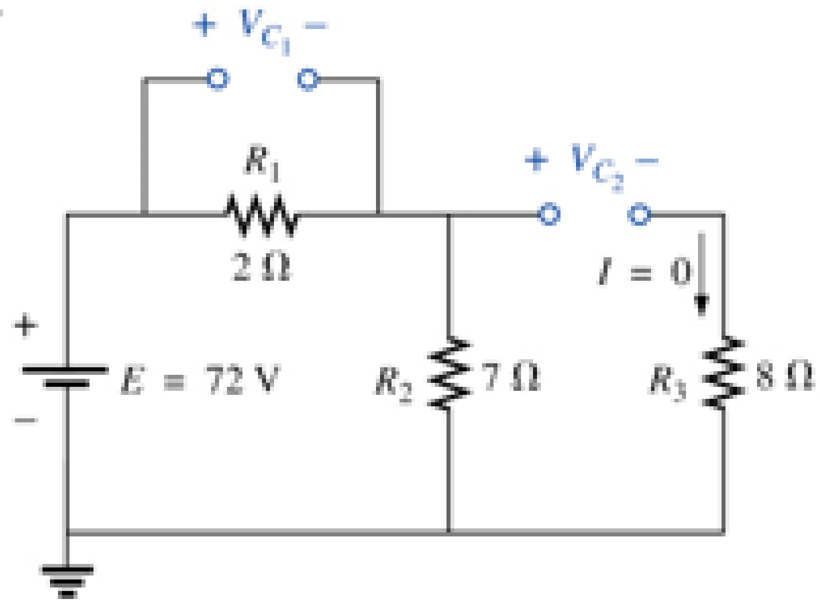
$$V_T^{'} = \frac{Q_T^{'}}{C_T^{'}} = \frac{240\times10^{-6}\,C}{6\times10^{-6}\,F} = 40\,V$$

$$Q_2 = C_2 V_T^{'} = (4\times10^{-6}\,F)(40V) = 160\ \mu C$$

$$Q_3 = C_3 V_T^{'} = (2\times10^{-6}\,F)(40V) = 80\ \mu C$$

Example7: Find the voltage across and charge on each capacitor for the network, after each has charged up to its final value





$$V_{C_2} = \frac{(7\Omega)(72V)}{7\Omega + 2\Omega} = 56(V)$$

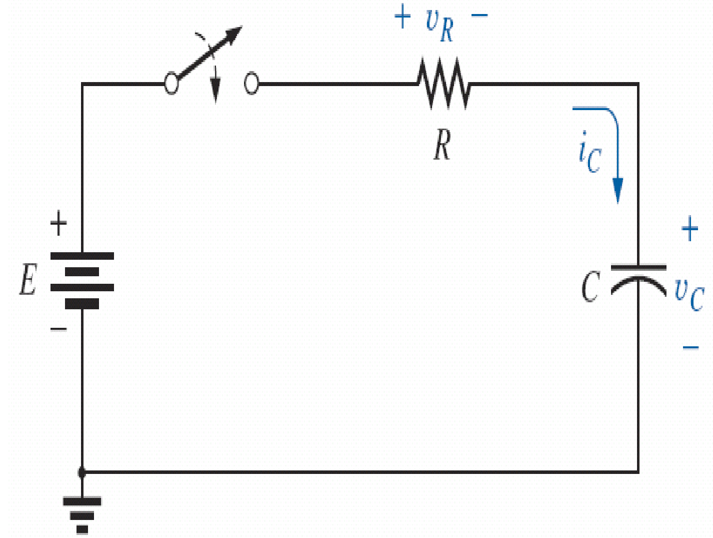
$$V_{C_1} = \frac{(2\Omega)(72V)}{7\Omega + 2\Omega} = 16(V)$$

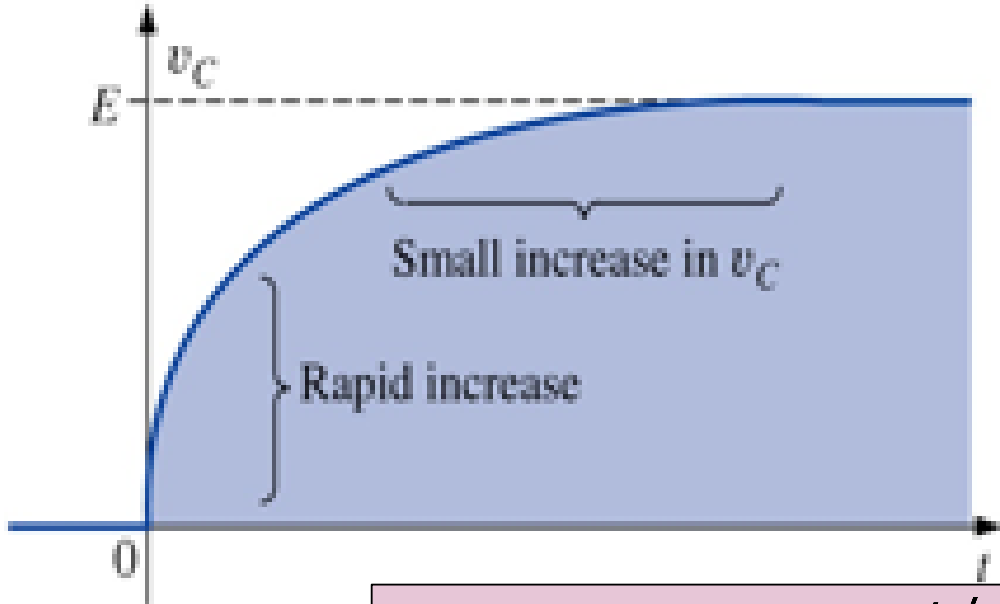
$$Q_1 = C_1 V_{C_1} = (2 \times 10^{-6} F)(16V) = 32\ \mu C$$

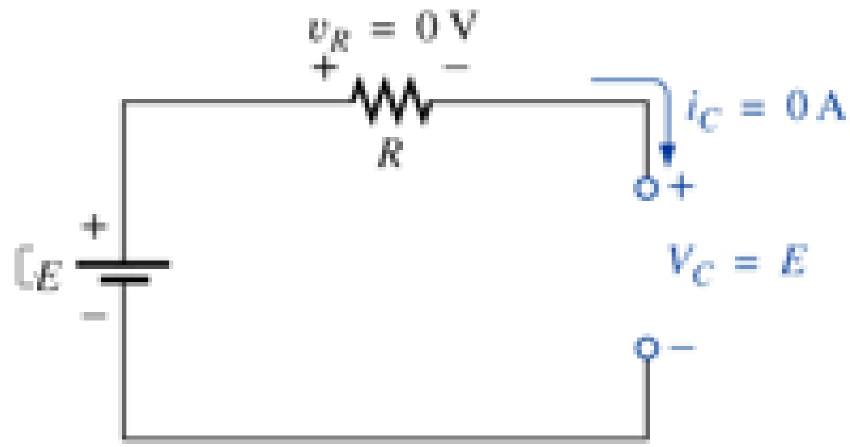
$$Q_1 = C_2 V_{C_2} = (3 \times 10^{-6} F)(56V) = 168\ \mu C$$

Transients in Capacitive Networks

- Charging phase
  - The placement of charge on the plates of a capacitor does not occur instantaneously.
  - Instead, it occurs over a period of time determined by the components of the network.

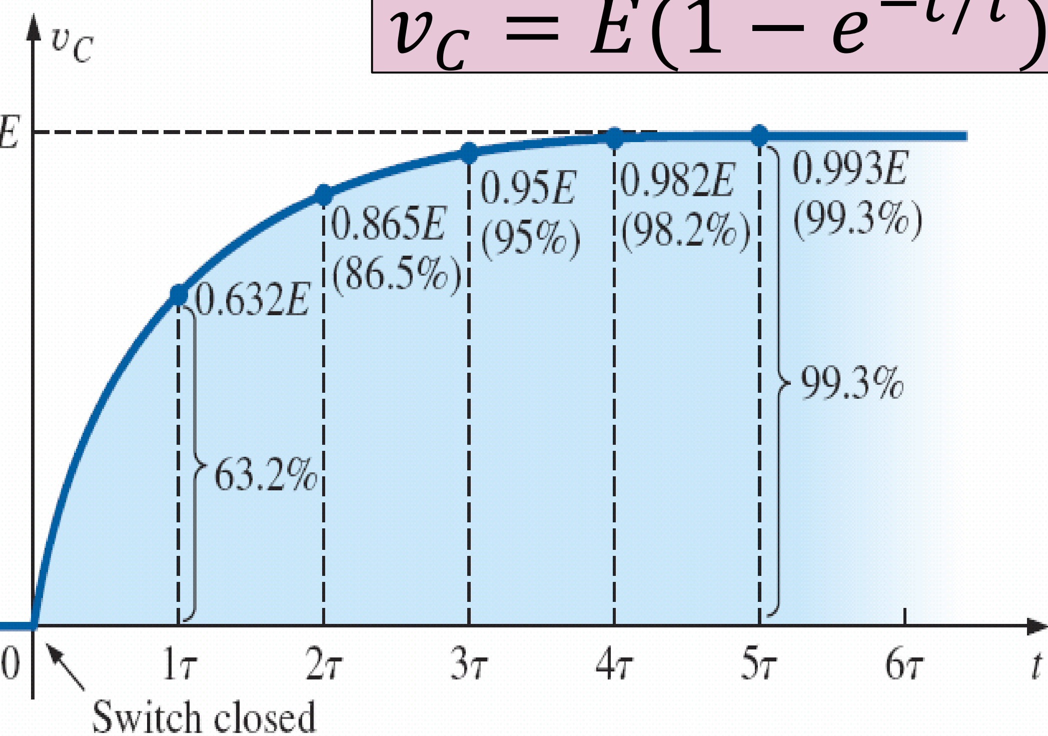


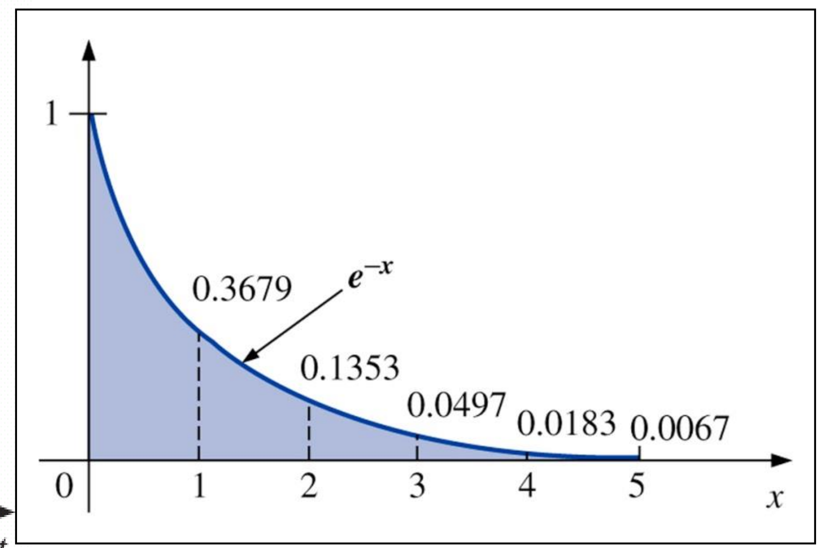


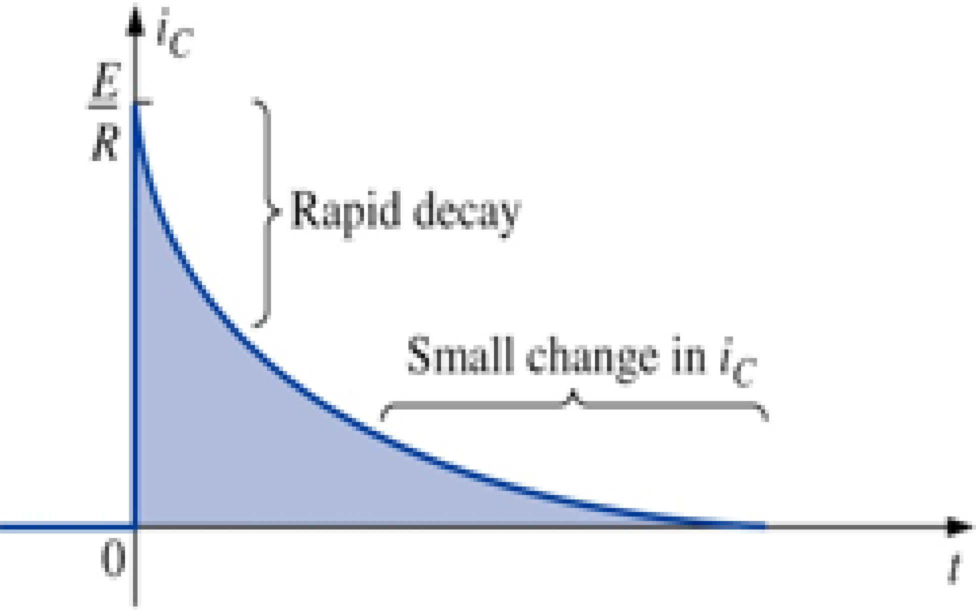


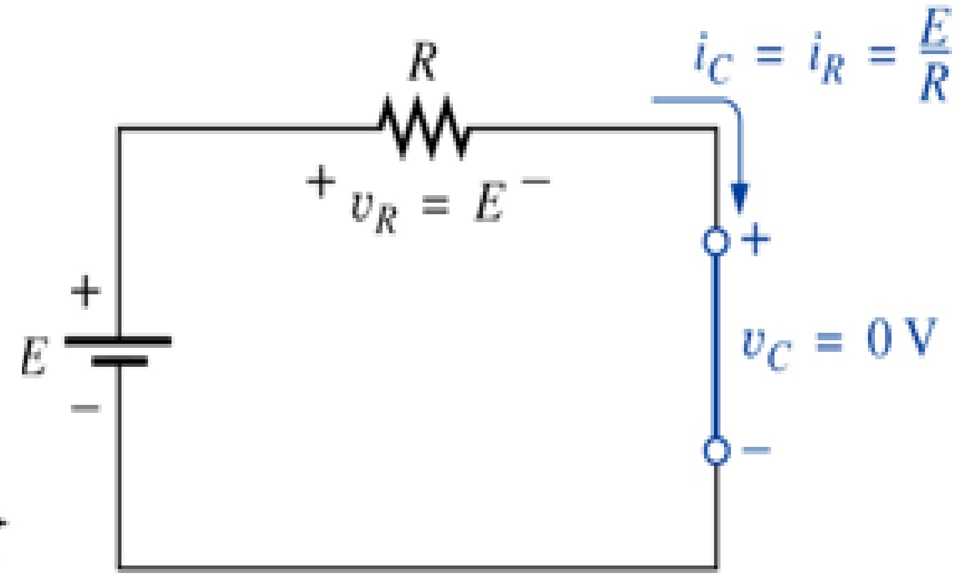
$$v_C = E(1 - e^{-t/\tau})$$

$$\tau = RC$$



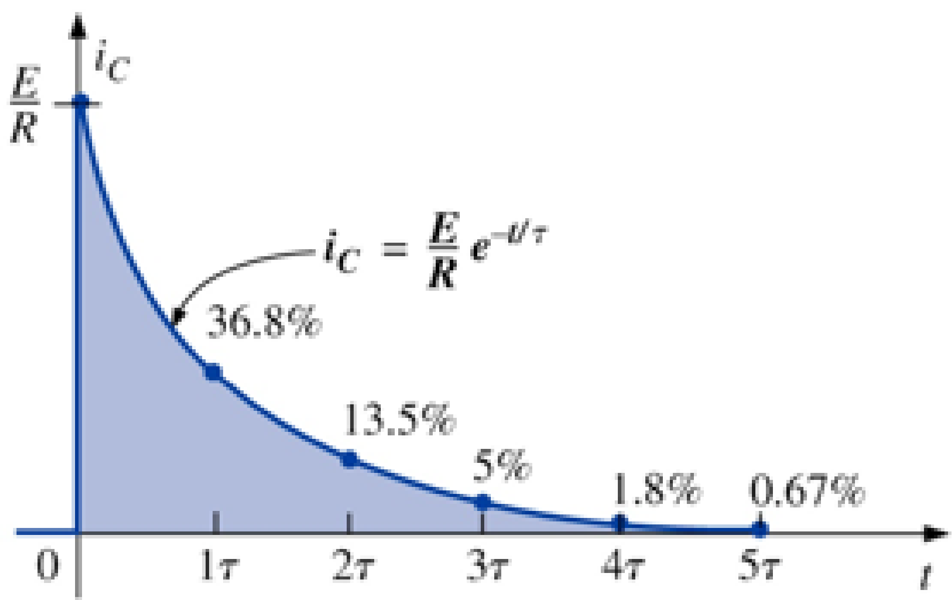






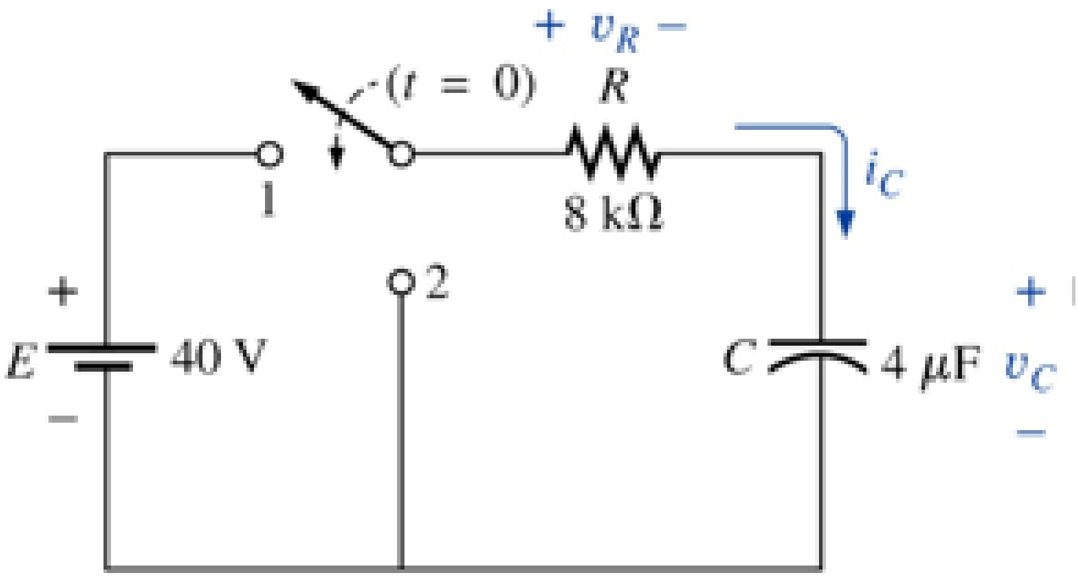
$$\boxed{i_C = \frac{E}{R} e^{-t/\tau}}$$

$$\boxed{\tau = RC}$$



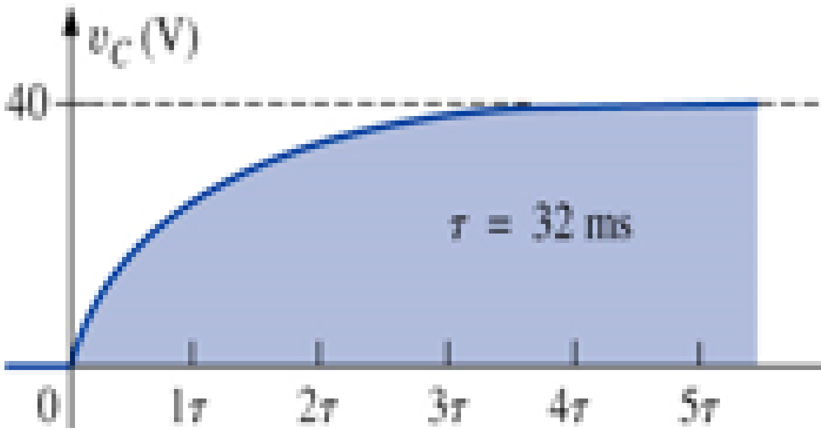
Example8: (a) Find the mathematical expressions for the transient behavior of v_C, i_C, and v_R for the circuit when the switch is moved to position 1. Plot the curves of v_C, i_C, and v_R.

b. How much time must pass before it can be assumed, for all practical purposes, that $i_C \approx 0$ A and $v_C \approx E$ volt?

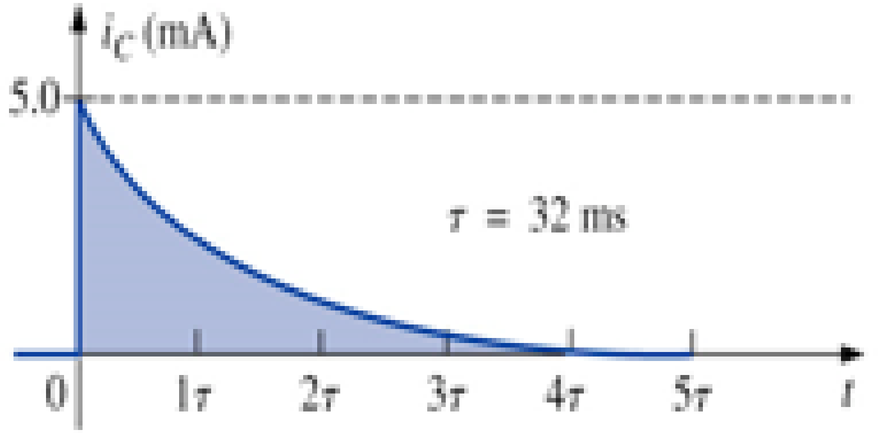


$$(a).\ \tau = RC = (8\times10^3\Omega)(4\times10^{-6}F)$$
$$= 32\times10^{-3}s = 32ms$$

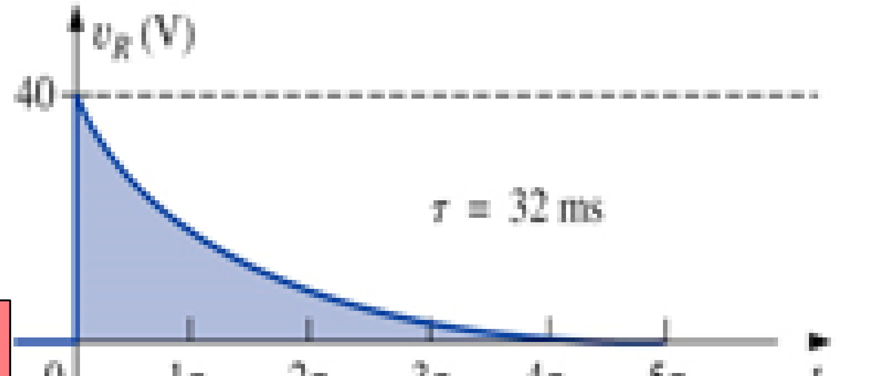
$$v_C = E\left(1-e^{-\frac{t}{\tau}}\right)$$
$$= 40\left(1-e^{-\frac{t}{32\times10^{-3}}}\right)\text{V}$$



$$i_C = \frac{E}{R}e^{-\frac{t}{\tau}}$$
$$= \frac{40V}{8k\Omega}e^{-\frac{t}{32\times10^{-3}}}$$
$$=5mAe^{-\frac{t}{32ms}}$$

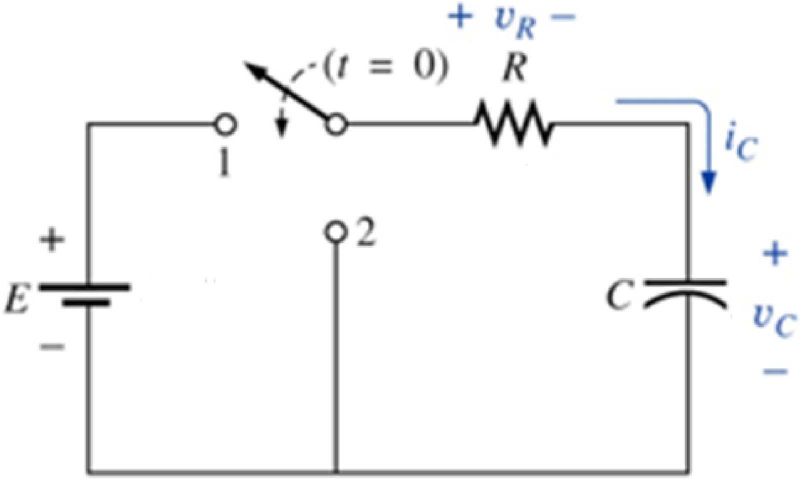


$$v_R = Ee^{-\frac{t}{\tau}} = 40e^{-\frac{t}{32ms}}\text{V}$$



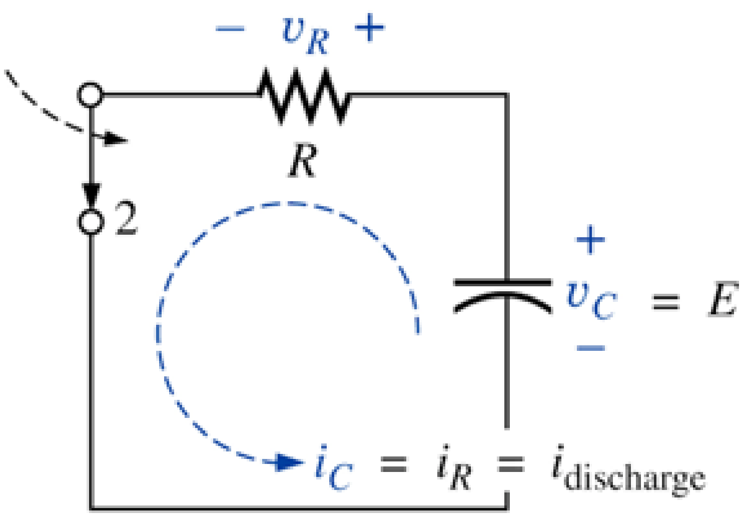
$$(b).\quad 5\tau = 5\times32ms = 160ms$$

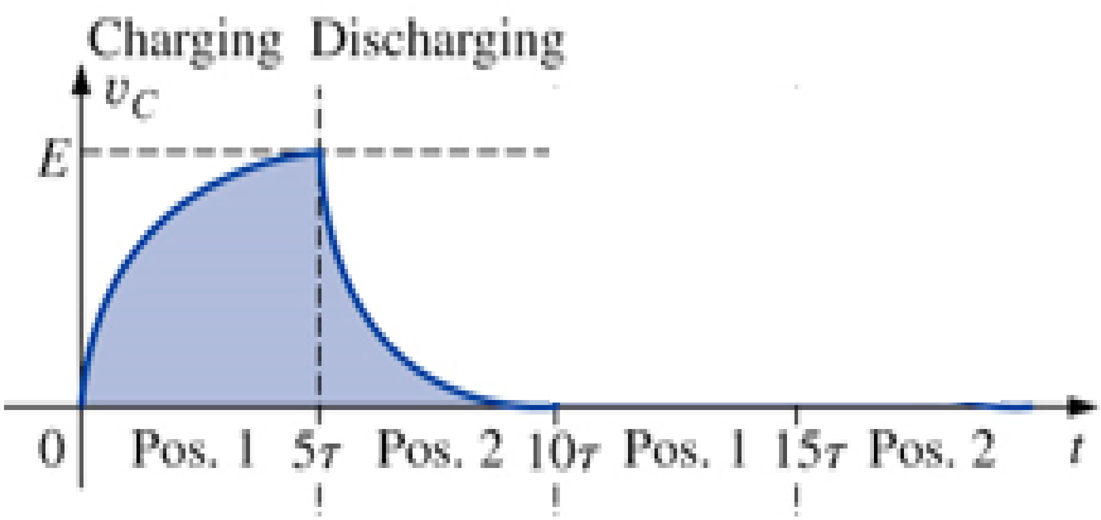
Discharging phase

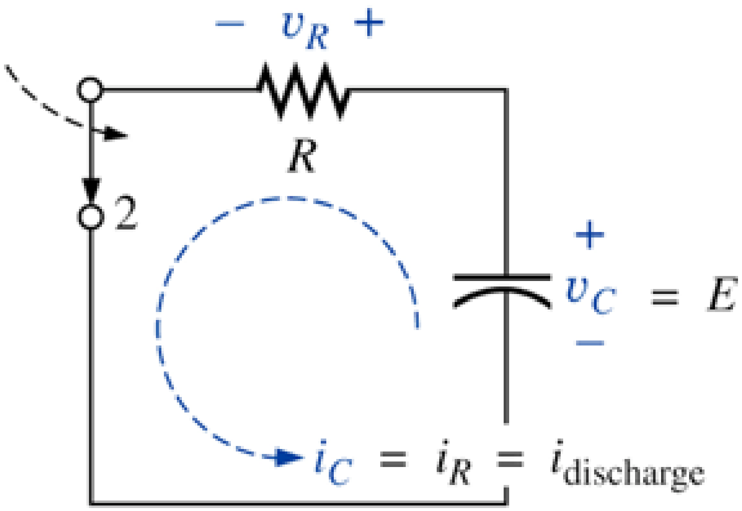


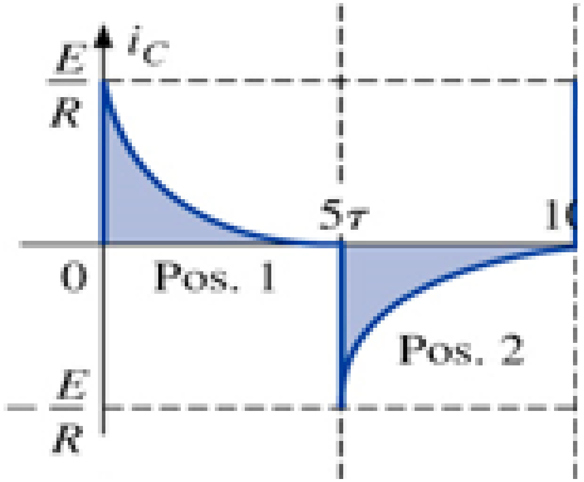
$$v_C = Ee^{-\frac{t}{\tau}}$$

$$\tau = RC$$





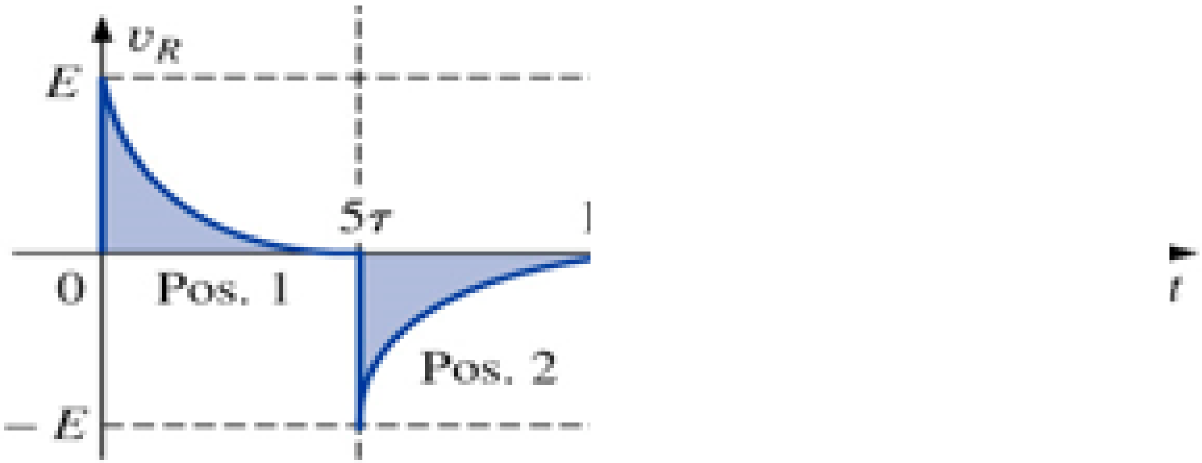


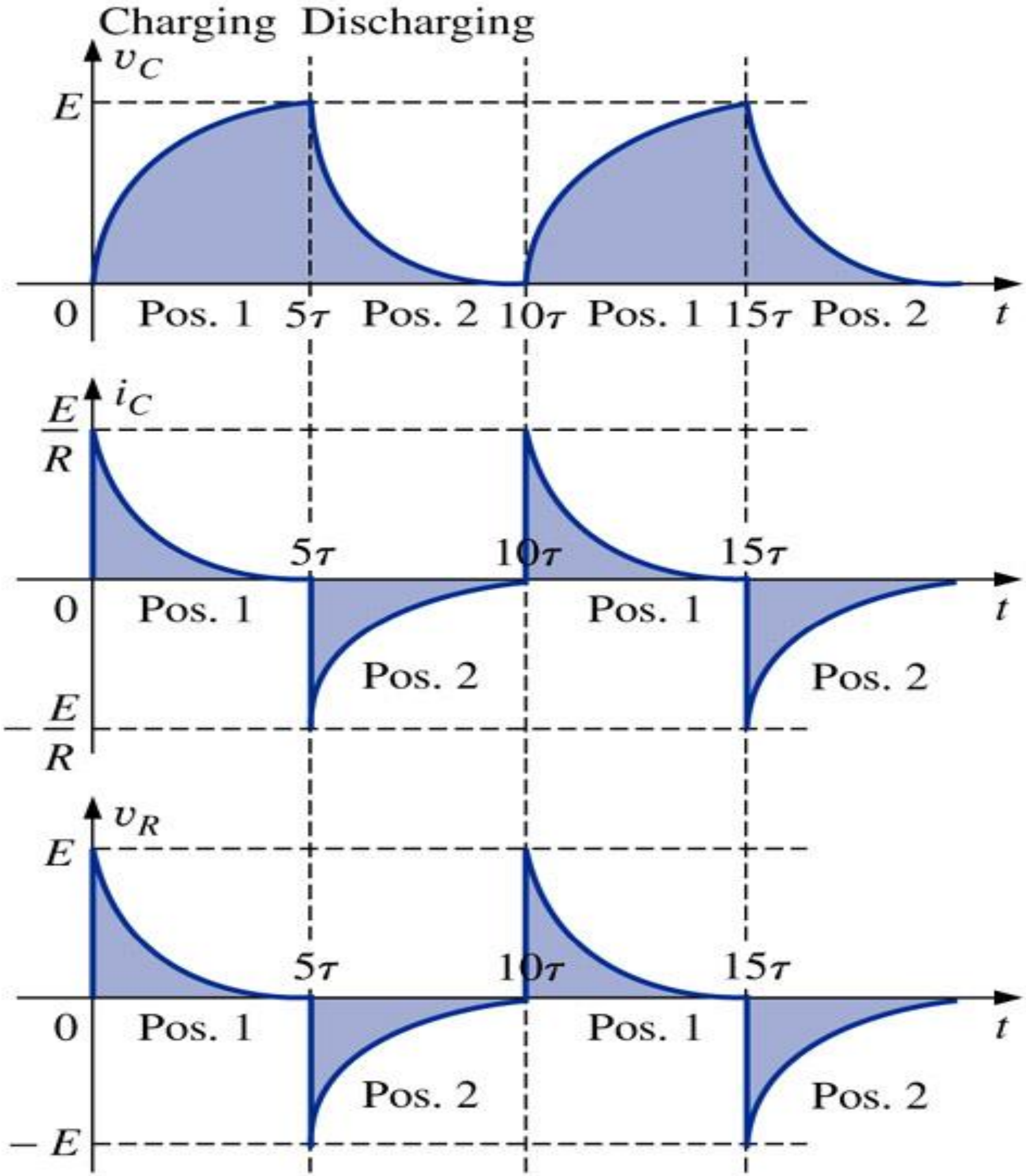


$$i_C = -\frac{E}{R}e^{-\frac{t}{\tau}}$$

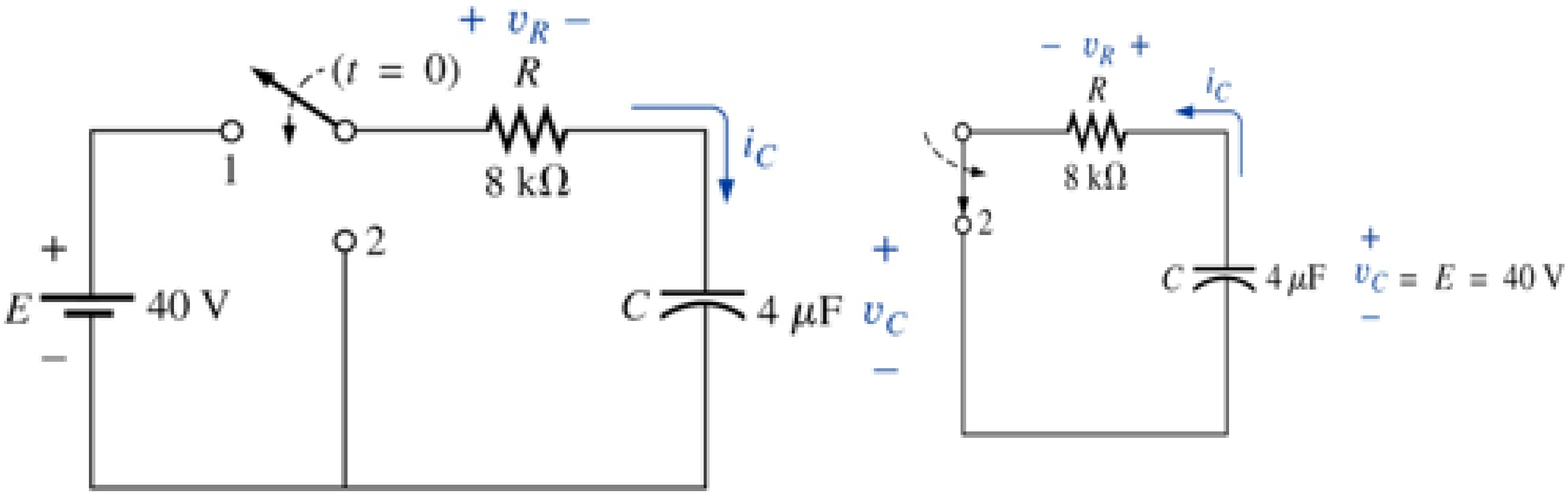
$$\tau = RC$$

$$v_R = -Ee^{-\frac{t}{\tau}}$$



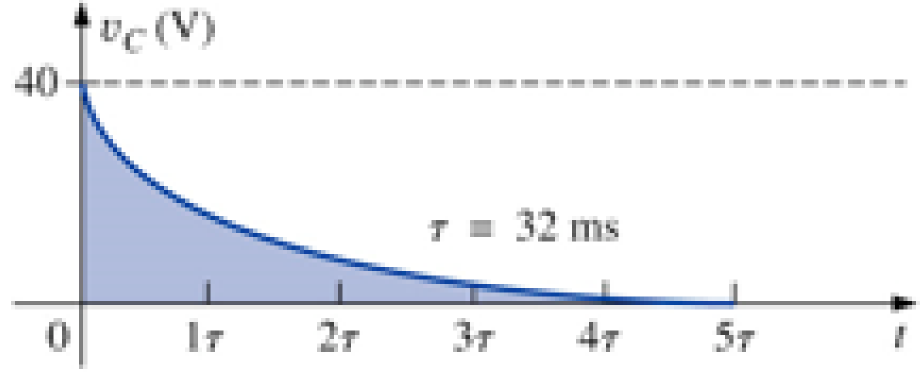

Charging Discharging
v_C
E
0 Pos. 1 5τ Pos. 2 10τ Pos. 1 15τ Pos. 2 t
i_C
$\frac{E}{R}$
5τ 10τ 15τ
0 Pos. 1 Pos. 1 t
Pos. 2 Pos. 2
$-\frac{E}{R}$
v_R
E
5τ 10τ 15τ
0 Pos. 1 Pos. 1 t
Pos. 2 Pos. 2
$-E$

Example9: After v_C in Example 8 has reached its final value of 40 V, the switch is shown into position 2. Find the mathematical expressions for the transient behavior of v_C, i_C, and v_R after the closing of the switch. Plot the curves for v_C, i_C, and v_R using the defined directions and polarities in example 8. Assume that t = 0 when the switch is moved to position 2.



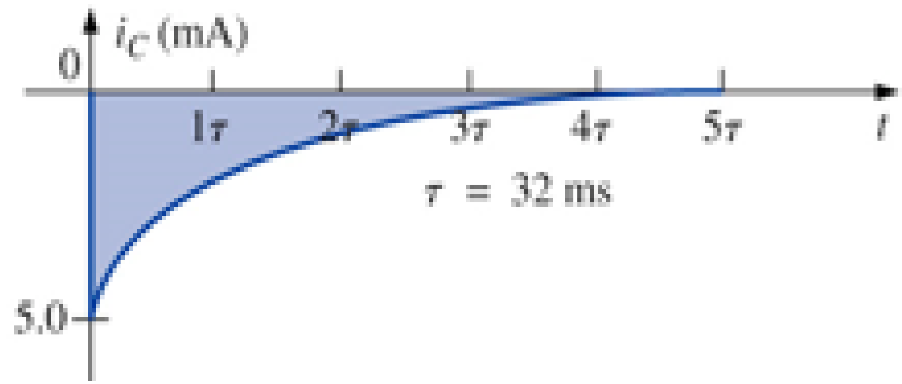
$$\tau = RC = 32ms$$

$$v_C = Ee^{-\frac{t}{\tau}} = 40e^{-\frac{t}{32ms}}\text{V}$$

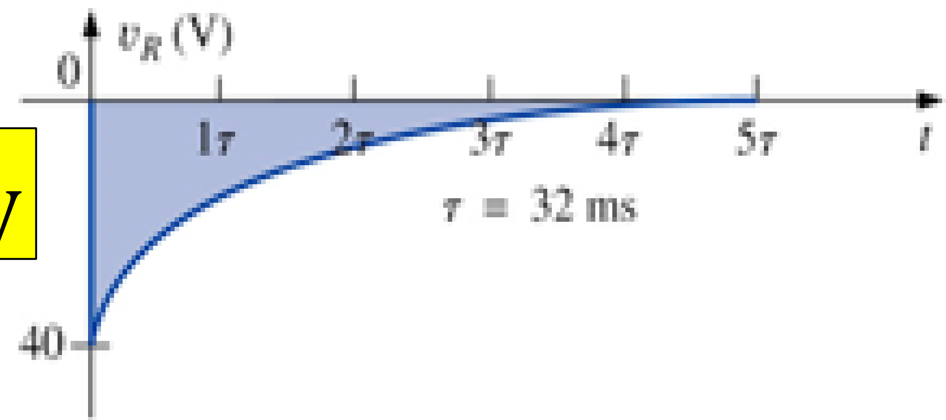


$$i_C = -\frac{E}{R}e^{-\frac{t}{\tau}}$$

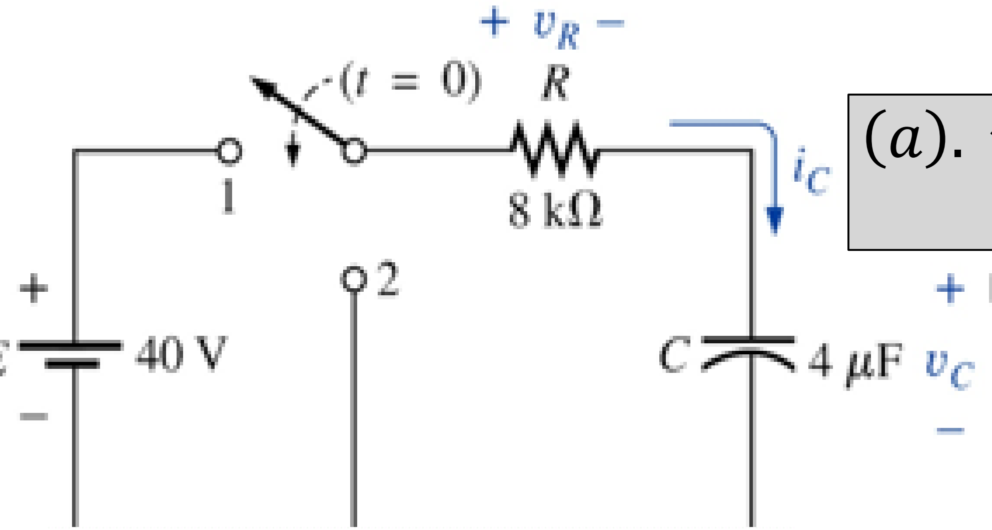
$$=5mAe^{-\frac{t}{32ms}}$$



$$v_R = -Ee^{-\frac{t}{\tau}} = -40e^{-\frac{t}{32ms}}\text{V}$$

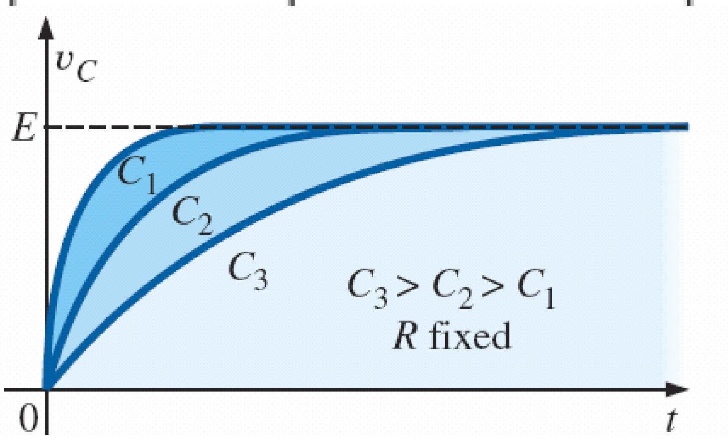


Example10: Find the mathematical expressions for the transient behavior of v_C for the circuit when the switch is moved to position 1 when C=4μF, 40μF, and 100μF. Plot the curve v_C



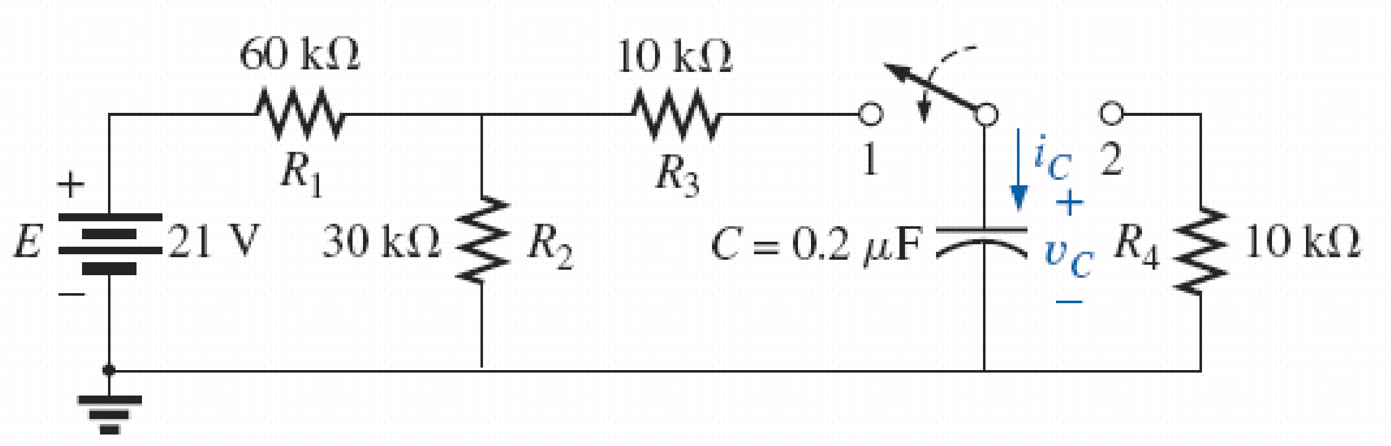
$$(a).\ \tau = RC_1 = (8\times10^{3}\Omega)(4\times10^{-6}F$$
$$= 32\times10^{-3}s = 32ms$$

$$\tau = RC_2$$
$$= (8\times10^{3}\Omega)(40\times10^{-6}F)$$
$$= 320\times10^{-3}s = 320ms$$

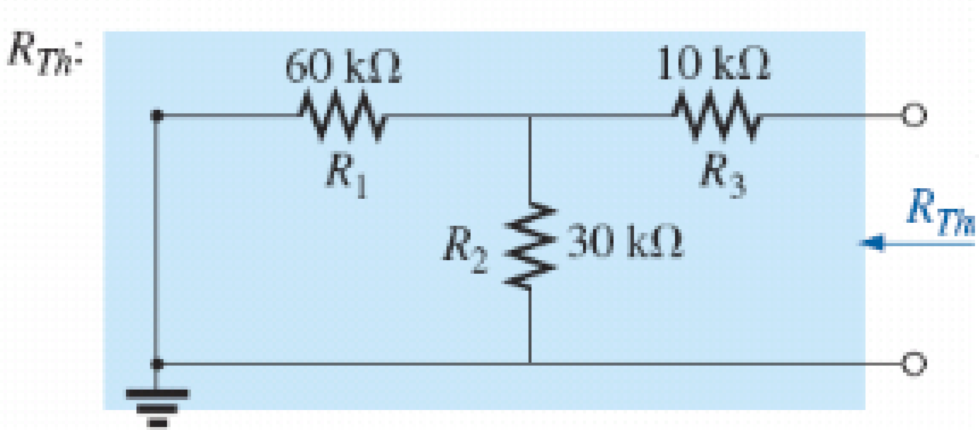


$$\tau = RC_3$$
$$= (8\times10^{3}\Omega)(100\times10^{-6}F)$$
$$= 0.8s$$

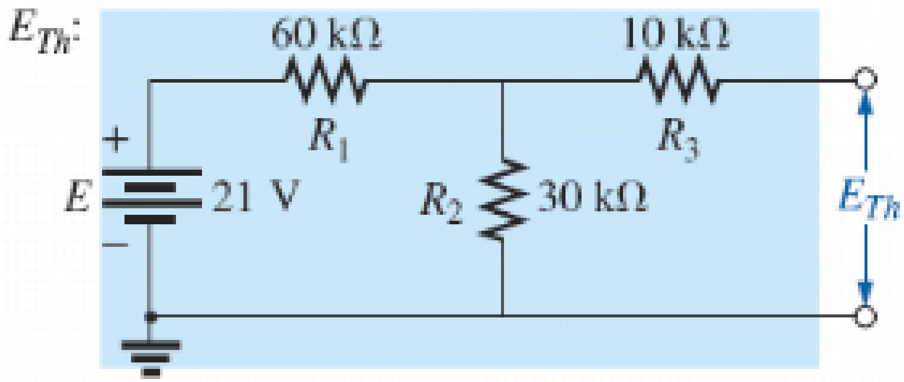
Example11: Find the mathematical expressions for the transient behavior of v_C and i_C for the circuit when the switch is closed position 1 at $t = 0s$.



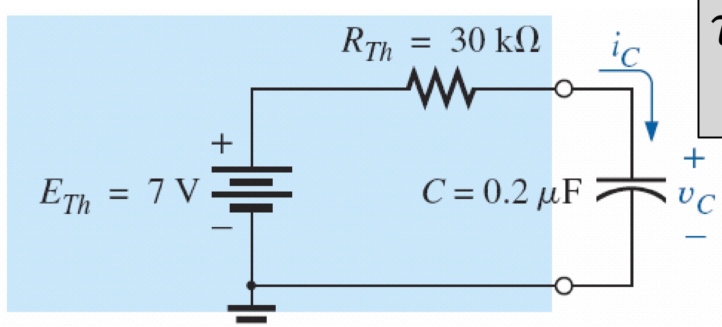
Find the Thevenin equivalent circuit without the capacitor.



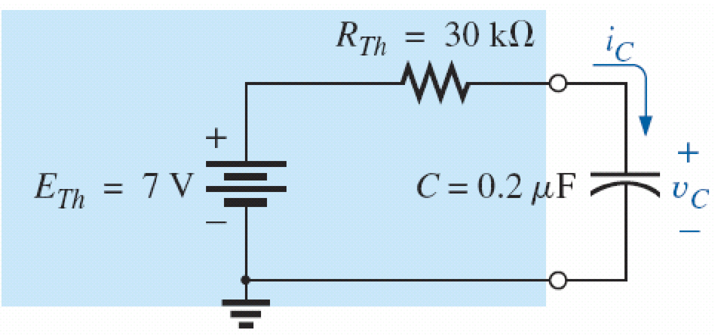
$$R_{Th} = (R_1//R_2) + R_3$$
$$= \frac{(60k\Omega)(30k\Omega)}{60k\Omega + 30k\Omega} = 30(k\Omega)$$



$$E_{Th} = \frac{ER_2}{R_1 + R_2}$$
$$= \frac{(21V)(30k\Omega)}{60k\Omega + 30k\Omega} = 7(V)$$

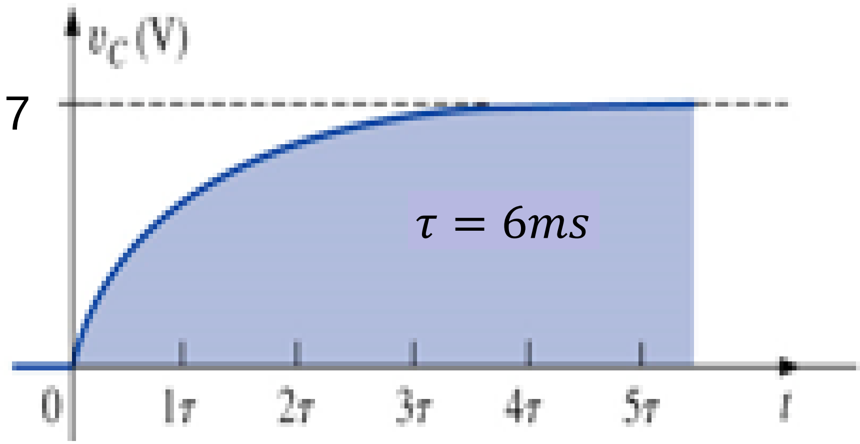


$$\tau = R_{Th}C \quad = (30k\Omega)(0.2 \times 10^{-6}F)$$
$$= 6 \times 10^{-3}s = 6ms$$



$$\tau = R_{Th}C = (30k\Omega)(0.2 \times 10^{-6}F) = 6 \times 10^{-3}s = 6ms$$

$$v_C = 7\left(1 - e^{-\frac{t}{\tau}}\right) = 7\left(1 - e^{-\frac{t}{6ms}}\right)\text{V}$$



$$i_C = \frac{E}{R}e^{-\frac{t}{\tau}} = \frac{7V}{30k\Omega}e^{-\frac{t}{6\times10^{-3}}} = 0.23mAe^{-\frac{t}{6ms}}$$