Chapter 7

EMT1150
Introduction to Circuit Analysis

Department of Computer
Engineering Technology

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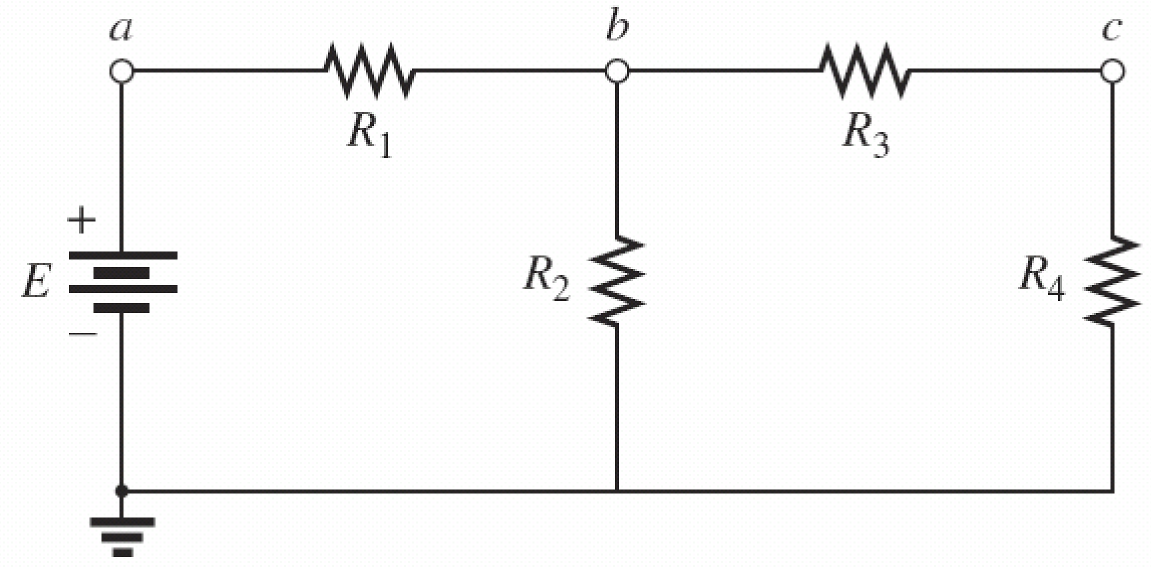
Chapter7 Series and Parallel Networks

- Identify Series-Parallel Networks
- Understand Reduce and Return Approach
- Use Block Diagram Approach
- Descriptive Examples
- Solve Ladder Networks

Series and Parallel Networks

- What is the relationship for circuit elements?
- How to find voltage and current for each element?

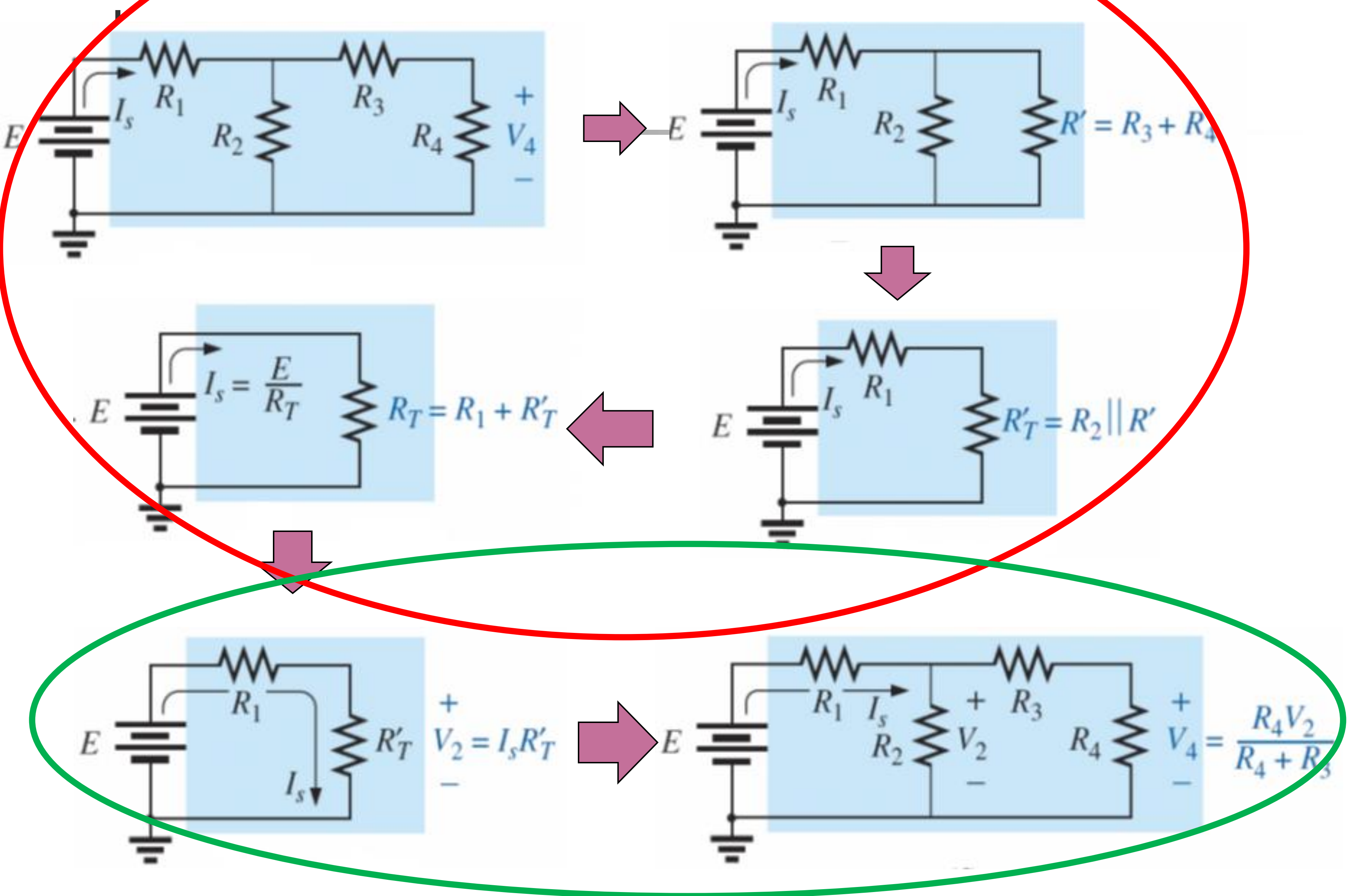


Series-parallel networks are networks that contain both series and parallel circuit configurations

Basic principle

- Take a moment to study the problem "in total", find the relationship.
- Examine each region of the network independently, then combine them together.
- Redraw the network as often as possible with reduced branches.
- When you have a solution, check whether it is reasonable or not.

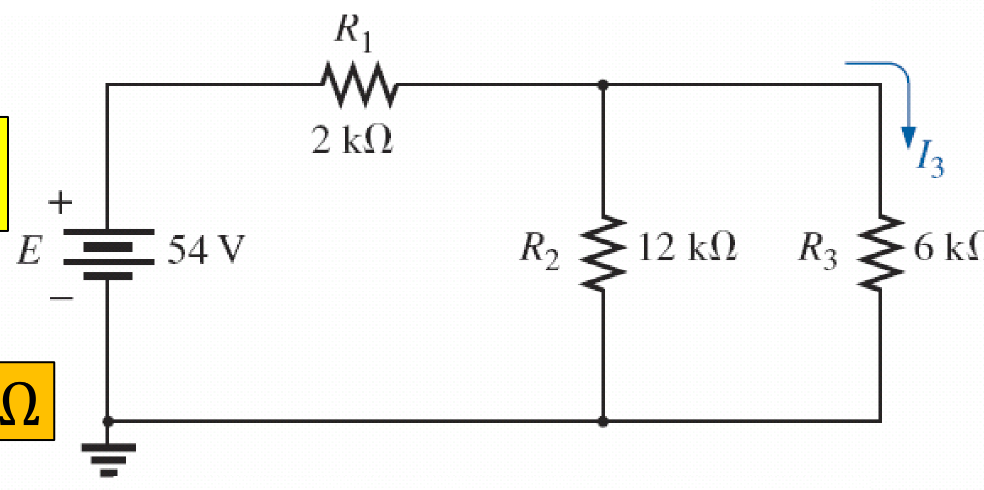
Reduce and Return Approach



Reduce and Return Approach

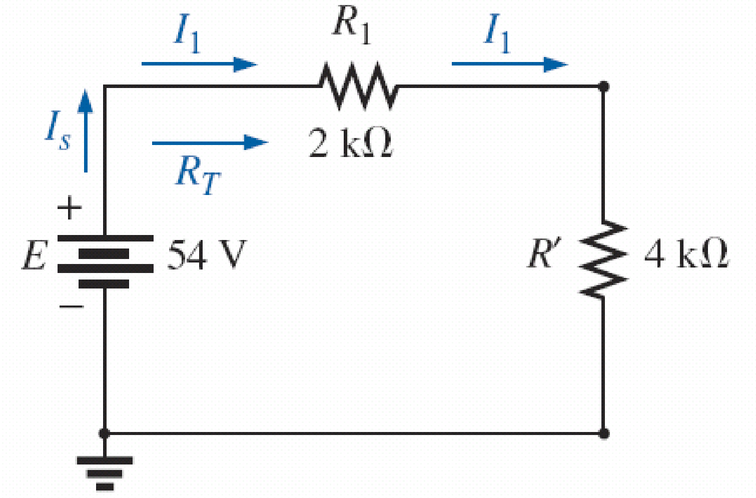
- Reduce phase: reduce the network to its simplest form across the source, then determine the source current.
- Return phase: use the resulting source current to work back to the desired unknowns.

Exp1: Find current I_3 for the series-parallel network.



$$R' = R_2 // R_3 = \frac{(12k\Omega)(6k\Omega)}{12k\Omega + 6k\Omega} = 4k\Omega$$

$$R_T = R' + R_1 = 4k\Omega + 2k\Omega = 6k\Omega$$



$$I_s = \frac{E}{R_T} = \frac{54V}{6k\Omega} = 9(mA)$$

Current Divider Rule:

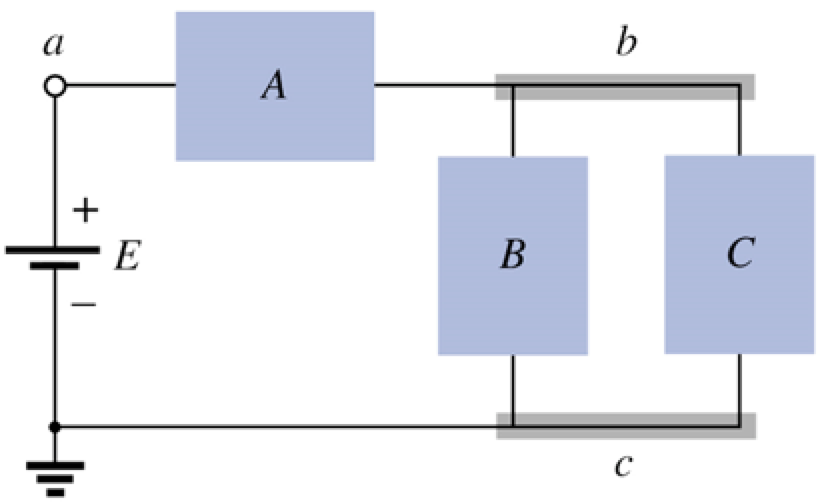
$$I_3 = Is\frac{R'}{R_3} = 9mA\frac{4k\Omega}{6k\Omega} = 6(mA)$$

Common mistake:

$$I_3 = Is\frac{R_T}{R_3} = 9mA\frac{6k\Omega}{6k\Omega} = 9(mA)$$

Block diagram approach

- Group elements together according to their relationships.
- Then reveal the voltage and current relation between groups.



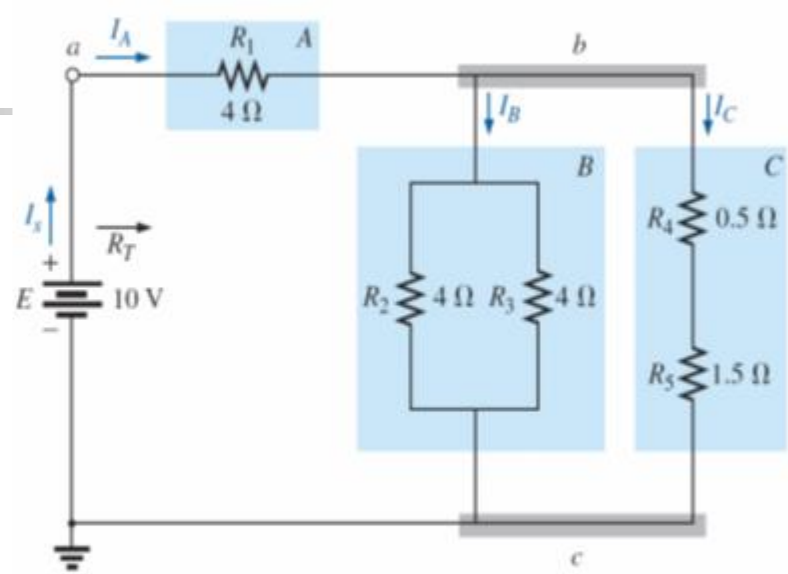
- Lastly, examine the impact of the individual component in each group.

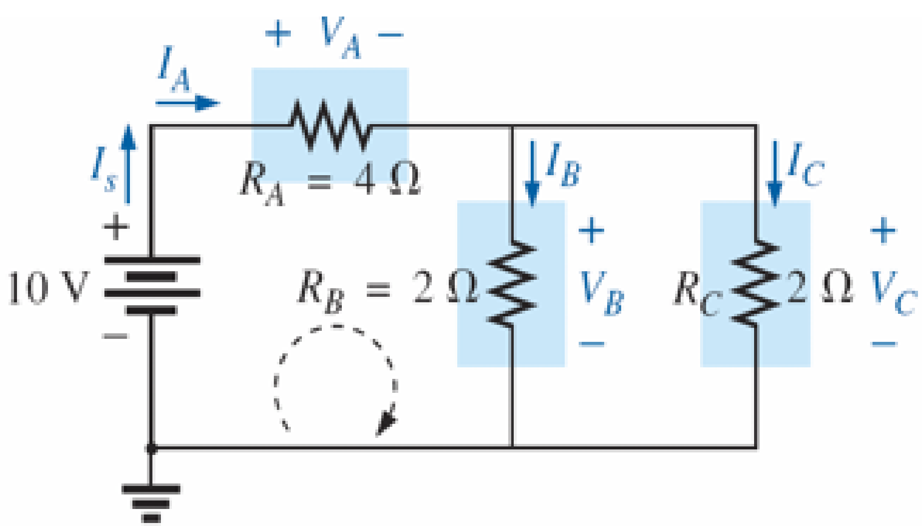
Exp2: Determine all currents and voltage of the network.

$$A: \quad R_A = 4\,\Omega$$

$$B: \quad R_B = R_2 // R_3 = \frac{R}{N} = \frac{4\,\Omega}{2} = 2\,\Omega$$

$$C: \quad R_C = R_4 + R_5 = 2\,\Omega$$

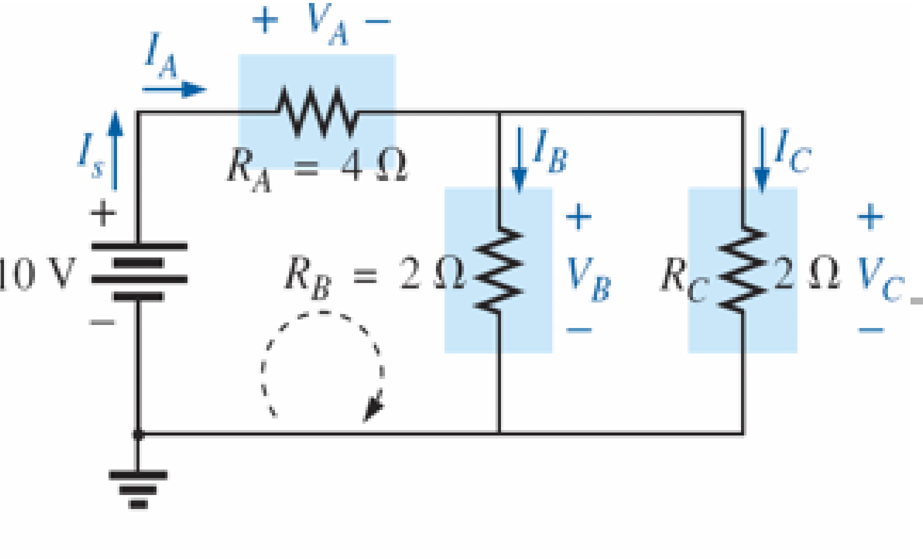




$$R_{B//C} = \frac{R}{N} = \frac{2\,\Omega}{2} = 1\,\Omega$$

$$R_T = R_A + R_{B//C} = 4\Omega + 1\Omega = 5\,\Omega$$

$$I_s = \frac{E}{R_T} = \frac{10V}{5\Omega} = 2\,A$$



$$I_A = I_s = 2\,A$$

$$I_B = I_C = \frac{I_A}{2} = \frac{I_s}{2} = \frac{2\,A}{2} = 1\,A$$

$$I_{R_2} = I_{R_3} = \frac{I_B}{2} = 0.5\,A$$

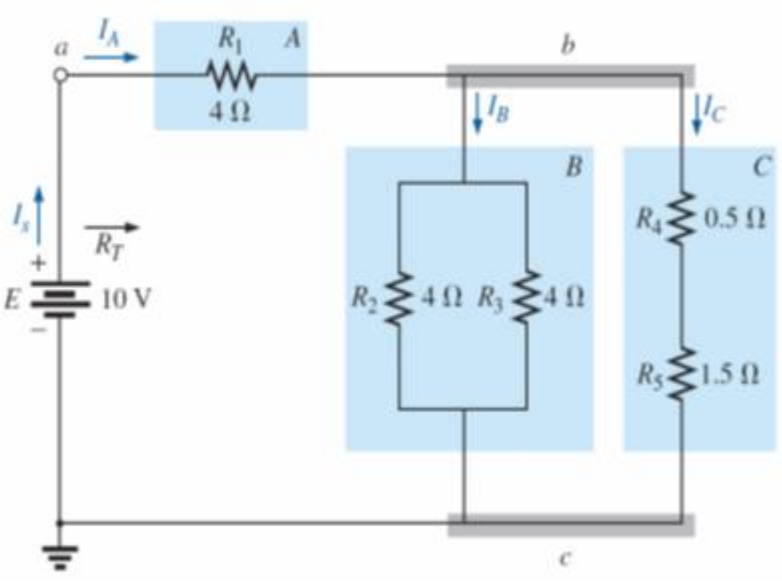
$$V_A = I_A R_A = (2\,A)(4\,\Omega) = 8\,V$$

$$V_B = I_B R_B = (1\,A)(2\,\Omega) = 2\,V$$

$$V_C = V_B = 2\,V$$

$$VDR: V_{R_4} = V_C \frac{R_4}{R_C} = 2V \frac{0.5\Omega}{2\Omega} = 0.5\,V$$

$$V_{R_5} = V_C \frac{R_5}{R_C} = 2V \frac{1.5\Omega}{2\Omega} = 1.5\,V$$



Exp3: Determine all currents and voltage.

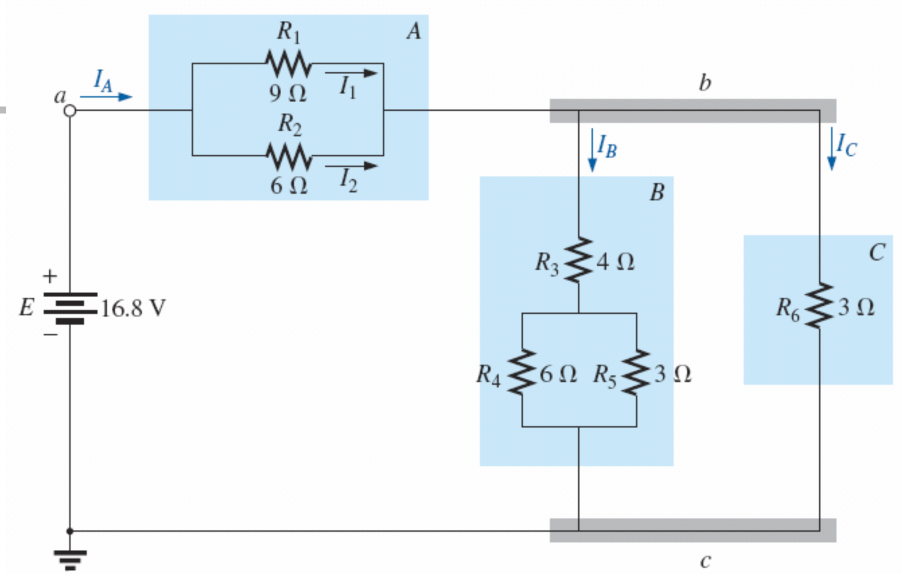
$$R_A = R_{1//2} = \frac{(9\,\Omega)(6\,\Omega)}{9\,\Omega + 6\,\Omega}$$

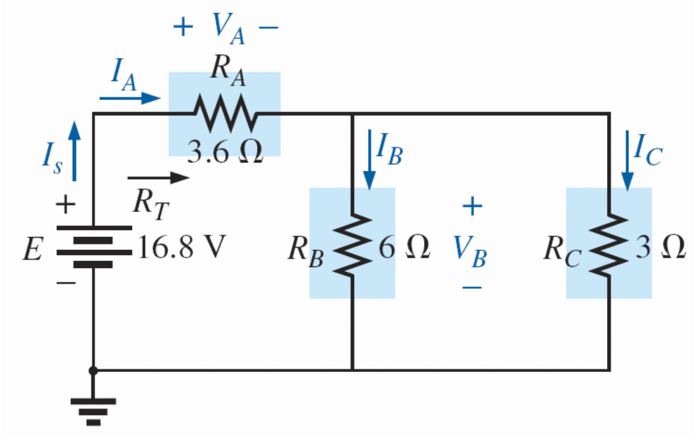
$$= \frac{54\,\Omega}{15\,\Omega} = 3.6\,\Omega$$

$$R_B = R_3 + R_{4//5} = 4\,\Omega + \frac{(6\,\Omega)(3\,\Omega)}{6\,\Omega + 3\,\Omega}$$

$$= 4\,\Omega + 2\,\Omega = 6\,\Omega$$

$$R_C = 3\,\Omega$$





$$R_T = R_A + R_{B//C} = 3.6\,\Omega + \frac{(6\,\Omega)(3\,\Omega)}{6\,\Omega + 3\,\Omega}$$

$$= 3.6\,\Omega + 2\,\Omega = 5.6\,\Omega$$

$$I_s = \frac{E}{R_T} = \frac{16.8V}{5.6\,\Omega} = 3\,A$$

$$I_A = I_s = 3\,A$$

Applying the current divider rule yields

$$I_B = \frac{R_C I_A}{R_C + R_B} = \frac{(3\Omega)(3A)}{3\Omega + 6\Omega} = 1\,A$$

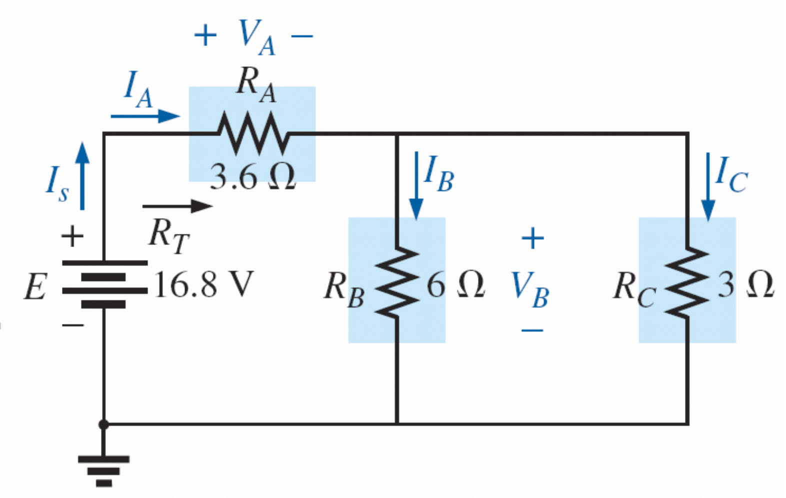
By Kirchhoff's current law,

$$I_C = I_A - I_B = 3A - 1A = 2\,A$$

By Ohm's law,

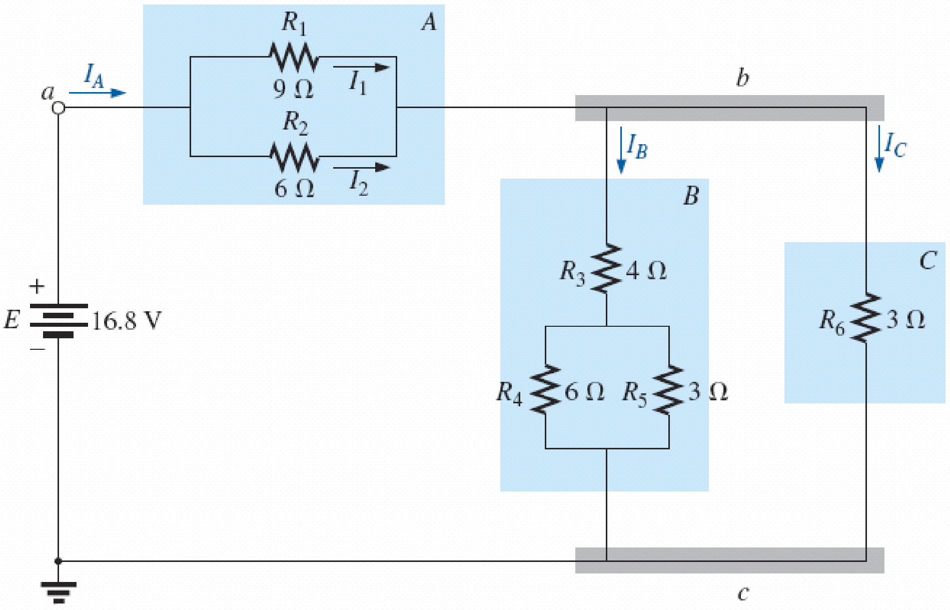
$$V_A = I_A R_A = (3A)(3.6\Omega) = 10.8\,V$$

$$V_B = I_B R_B = V_C = I_C R_C = (2A)(3\Omega) = 6\,V$$



$$I_1 = \frac{R_2 I_A}{R_2 + R_1} = \frac{(6\Omega)(3A)}{6\Omega + 9\Omega} = 1.2\,A$$

$$I_2 = I_A - I_1 = 3A - 1.2A = 1.8\,A$$



$$I_{R_4} = \frac{R_5 I_B}{R_4 + R_5} = \frac{(3\Omega)(1A)}{6\Omega + 3\Omega} = 0.33\,A$$

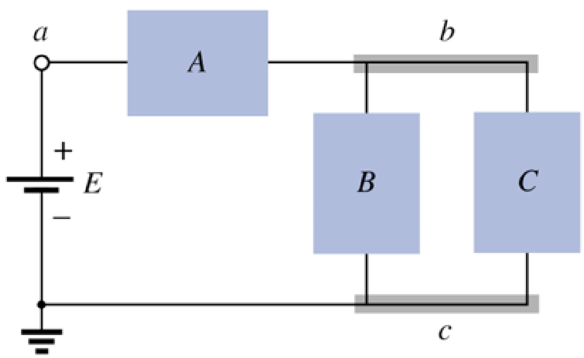
$$I_{R_5} = I_B - I_{R_4} = 1A - 0.33A = 0.67\,A$$

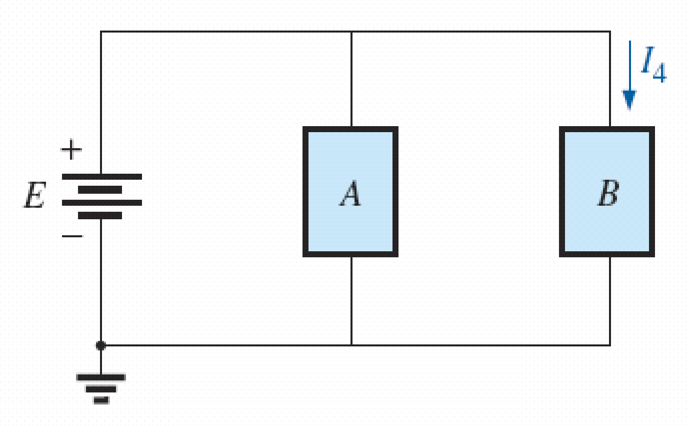
$$V_{R_3} = \frac{R_3 V_B}{R_3 + R_{4//5}} = \frac{(4\Omega)(6V)}{4\Omega + 2\Omega} = 4\,(V)$$

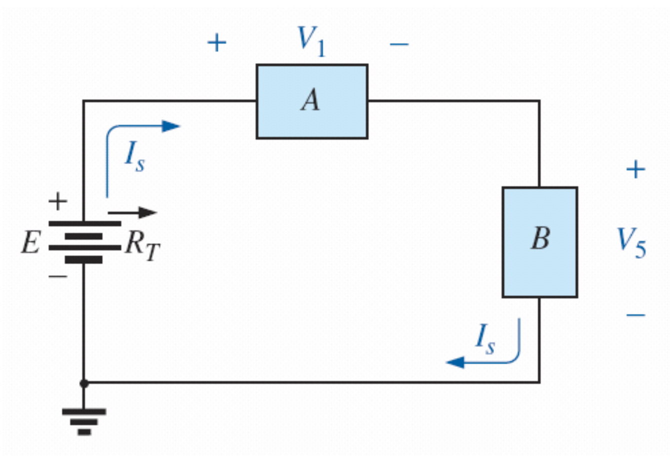
$$V_{R_{4//5}} = \frac{R_{4//5} V_B}{R_3 + R_{4//5}} = \frac{(2\Omega)(6V)}{4\Omega + 2\Omega} = 2(V)$$

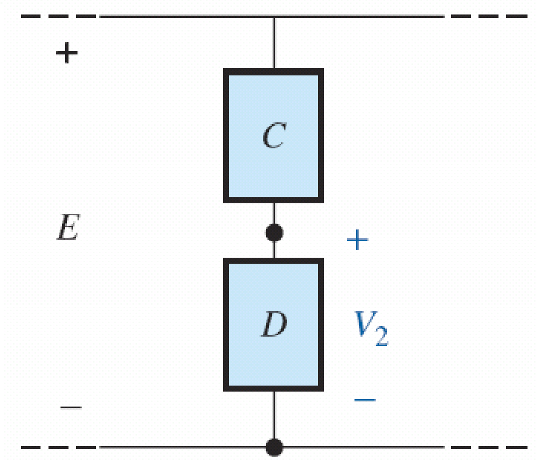
More descriptive examples

- Other possible block models







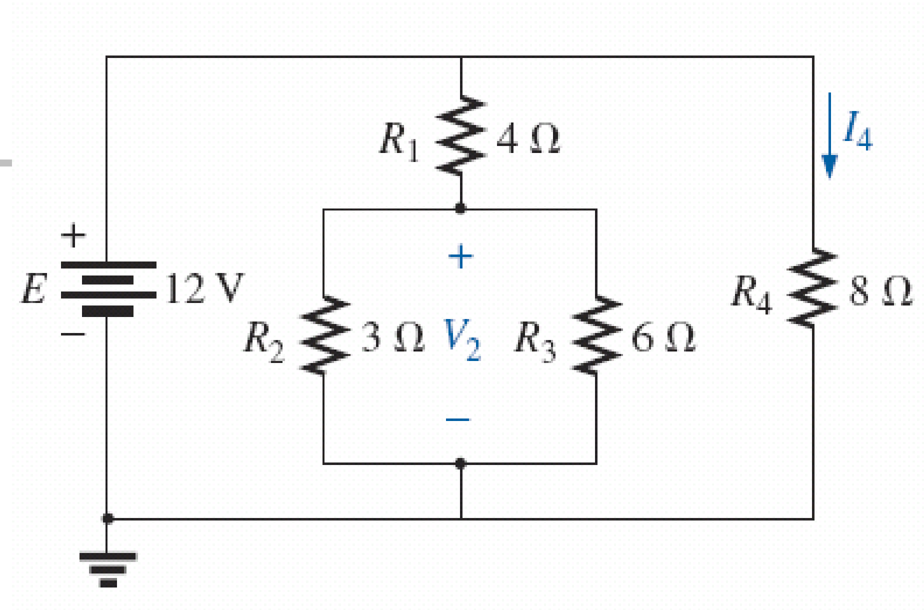


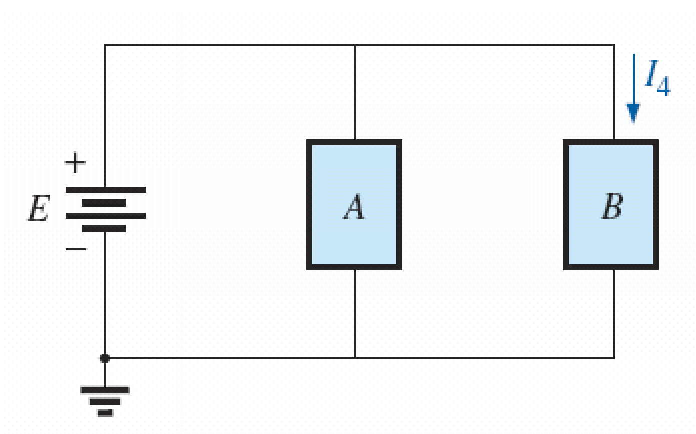
Exp4: Find the current I_4 and the voltage V_2 for the network.

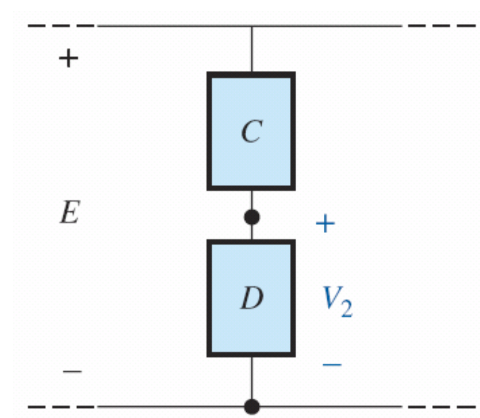
$$I_4 = \frac{E}{R_B} = \frac{E}{R_4} = \frac{12V}{8\Omega} = 1.5\,A$$

$$R_D = R_2 \,//\, R_3 = \frac{6\Omega \cdot 3\Omega}{6\Omega + 3\Omega} = 2\Omega$$

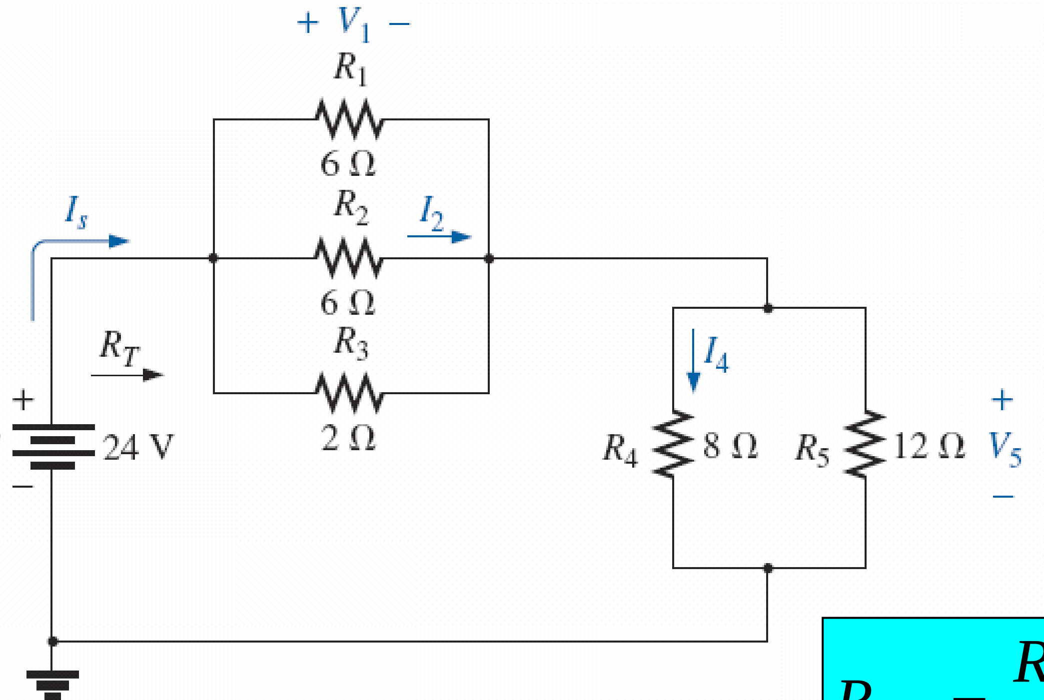
$$V_2 = \frac{R_D\,E}{R_D + R_C} = \frac{(2\Omega)(12V)}{2\Omega + 4\Omega} = 4\,V$$

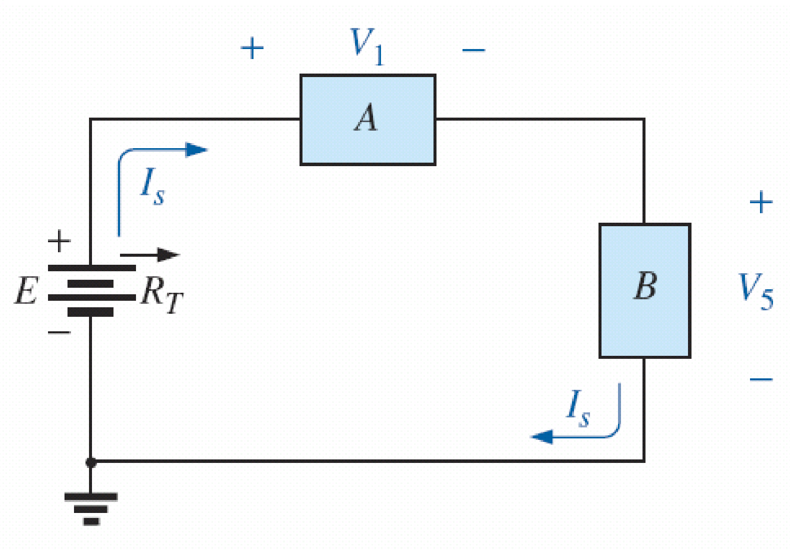






Exp5: Find the indicated currents and voltages for the network.

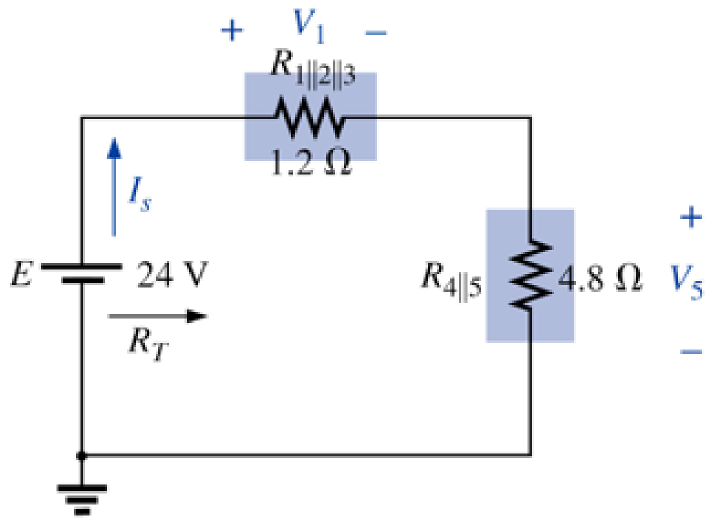




$$R_{1//2} = \frac{R}{N} = \frac{6\,\Omega}{2} = 3\,\Omega$$

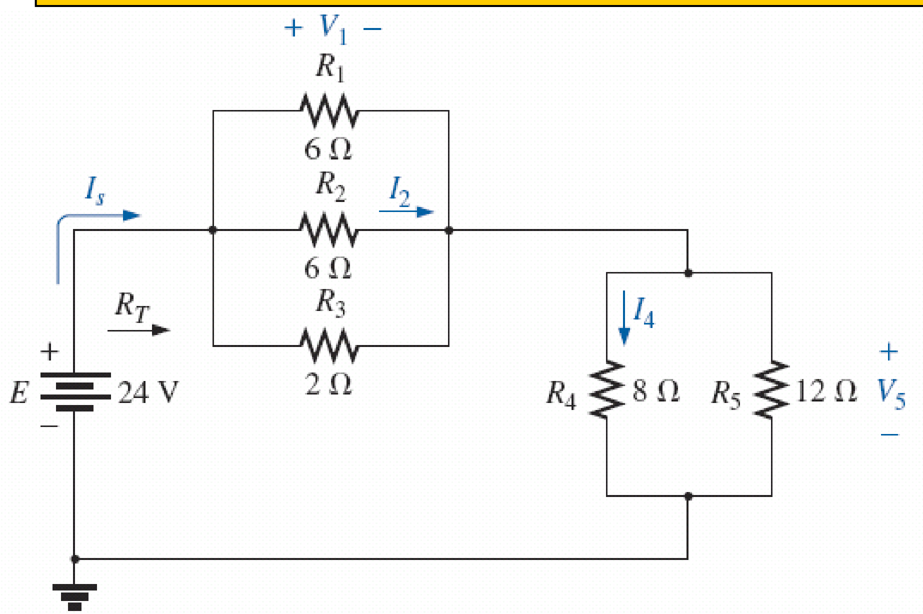
$$R_A = R_{1//2//3} = \frac{(3\,\Omega)(2\,\Omega)}{3\,\Omega + 2\,\Omega} = \frac{6\,\Omega}{5} = 1.2\,\Omega$$

$$R_B = R_{4//5} = \frac{(8\,\Omega)(12\,\Omega)}{8\,\Omega + 12\,\Omega} = \frac{96\,\Omega}{20} = 4.8\,\Omega$$



$$R_T = R_{1//2//3} + R_{4//5}$$
$$= 1.2\Omega + 4.8\Omega = 6\Omega$$
$$I_s = \frac{E}{R_T} = \frac{24V}{6\Omega} = 4\ A$$

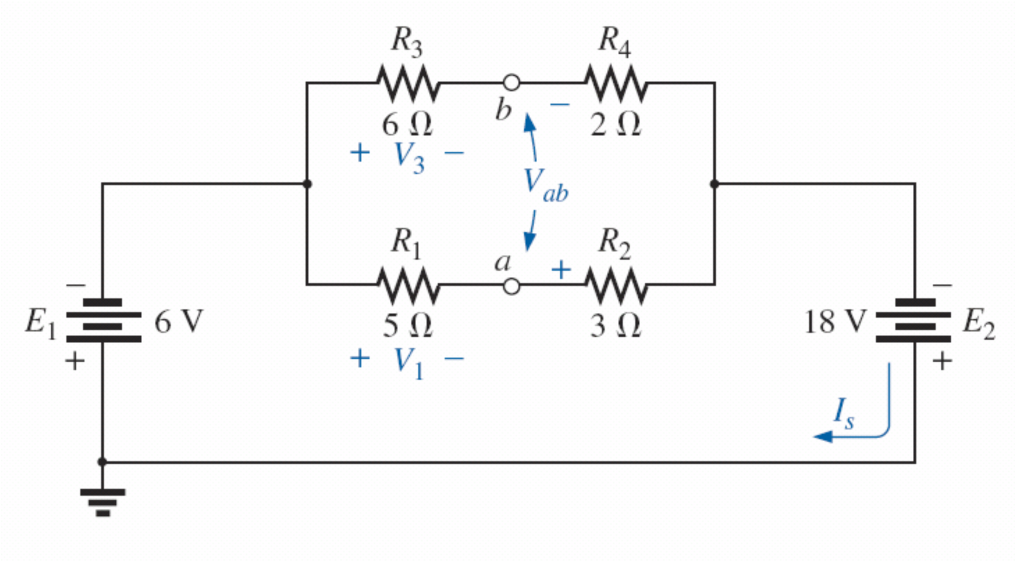
$$V_1 = I_s\, R_{1//2//3} = (4\ A)(1.2\,\Omega) = 4.8V$$
$$V_5 = I_s\, R_{4//5} = (4\ A)(4.8\,\Omega) = 19.2V$$

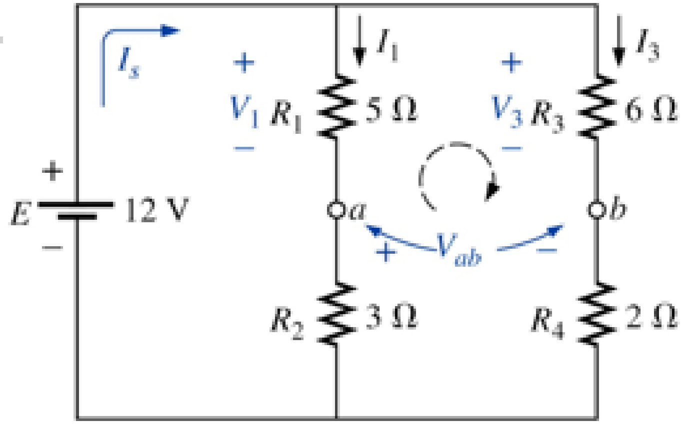


$$I_4 = \frac{V_5}{R_4} = \frac{19.2V}{8\Omega} = 2.4\ A$$
$$I_2 = \frac{V_2}{R_2} = \frac{V_1}{R_2} = \frac{4.8V}{6\Omega} = 0.8\ A$$

Exp6: a. Find the voltages V_1, V_3, and V_{ab} for the network.

b. Calculate the source current I_s





Applying the voltage divider rule yields

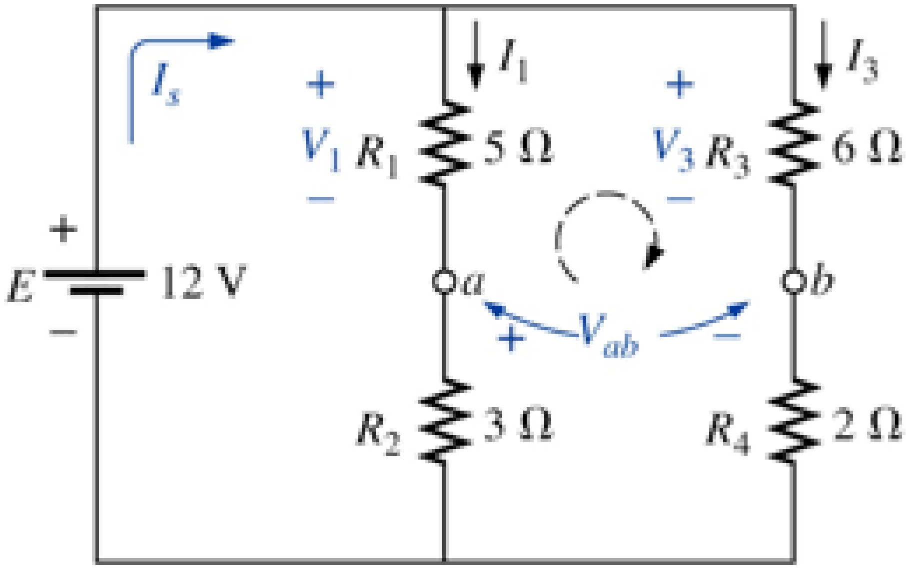
$$V_1 = \frac{R_1 E}{R_1 + R_2} = \frac{(5\Omega)(12V)}{5\Omega + 3\Omega} = 7.5\,V$$

$$V_3 = \frac{R_3 E}{R_3 + R_4} = \frac{(6\Omega)(12V)}{6\Omega + 2\Omega} = 9\,V$$

Applying Kirchhoff's voltage law around the indicated loop of Fig.

$$-V_1 + V_3 - V_{ab} = 0$$

$$V_{ab} = V_3 - V_1 = 9V - 7.5V = 1.5V$$



b.

By Ohm's law,

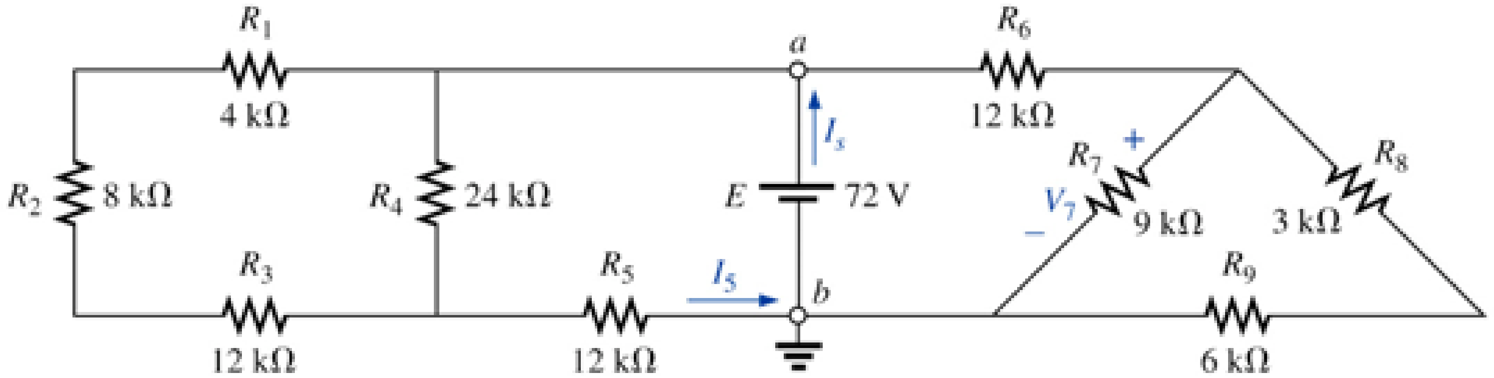
$$I_1 = \frac{V_1}{R_1} = \frac{7.5V}{5\Omega} = 1.5\,A$$

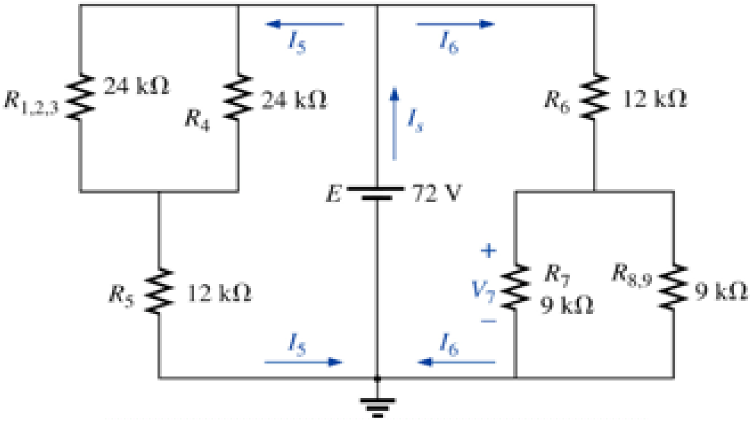
$$I_3 = \frac{V_3}{R_3} = \frac{9V}{6\Omega} = 1.5\,A$$

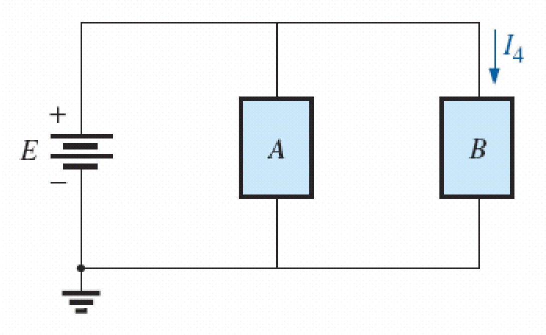
Applying Kirchhoff's current law,

$$I_s = I_1 + I_3 = 1.5A + 1.5A = 3A$$

Exp7: Calculate the indicated currents and voltage







$$I_5 = \frac{E}{R_{(1,2,3)//4} + R_5} = \frac{72V}{12k\Omega + 12k\Omega} = \frac{72V}{24k\Omega} = 3mA$$

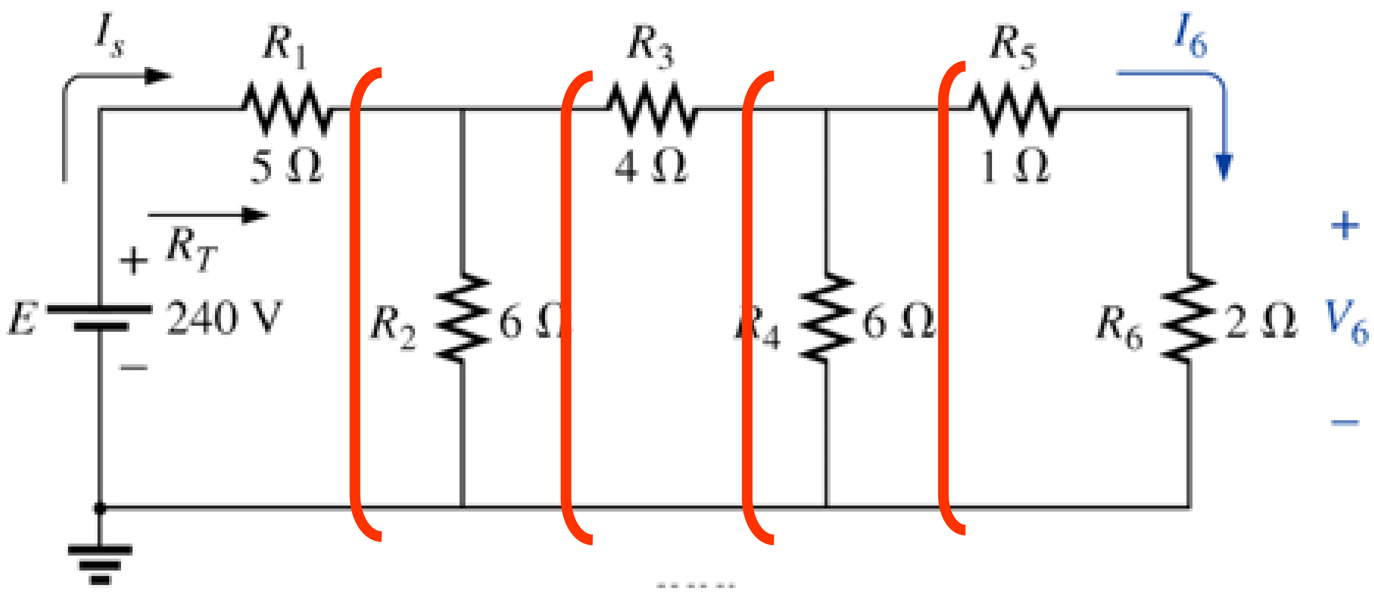
$$V_7 = \frac{R_{7//(8,9)}E}{R_{7//(8,9)} + R_6} = \frac{(4.5k\Omega)(72V)}{4.5k\Omega + 12k\Omega} = \frac{324V}{16.5} = 19.6V$$

$$I_6 = \frac{E}{R_{6,7//(8,9)}} = \frac{72V}{12k\Omega + 4.5k\Omega} = 4.36mA$$

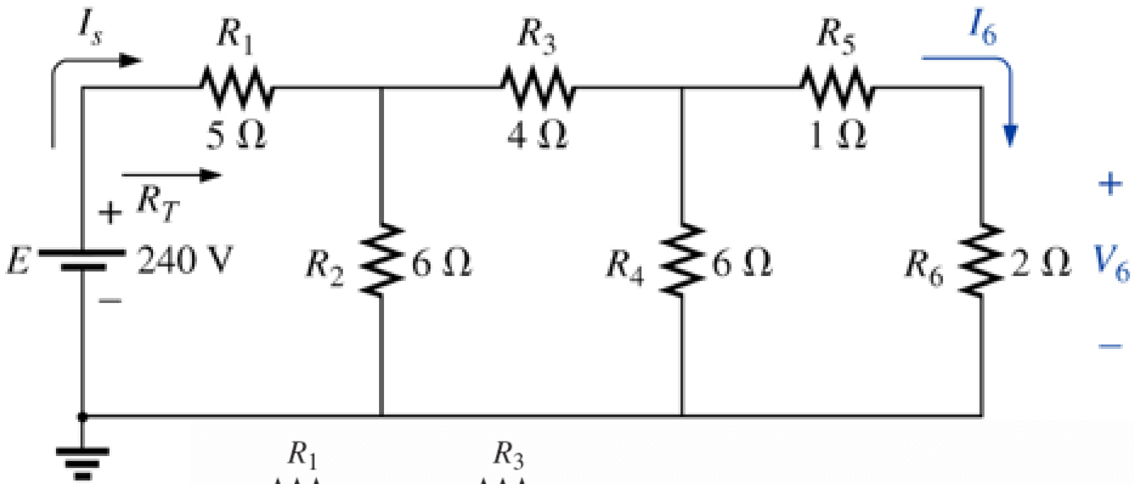
$$I_s = I_5 + I_6 = 3\ mA + 4.36\ mA = 7.36\ mA$$

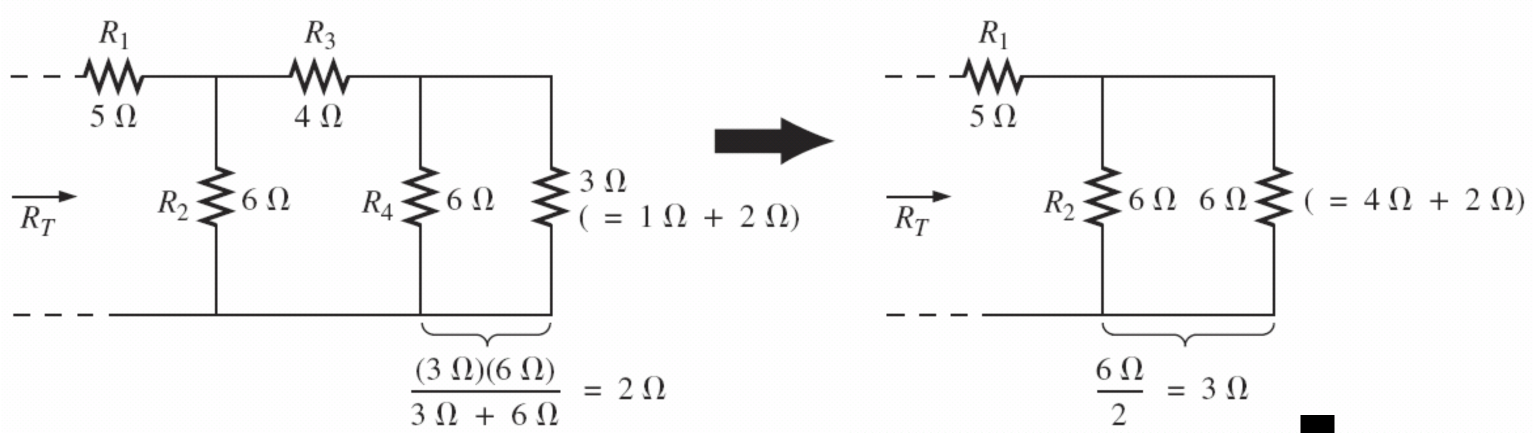
Ladder Network

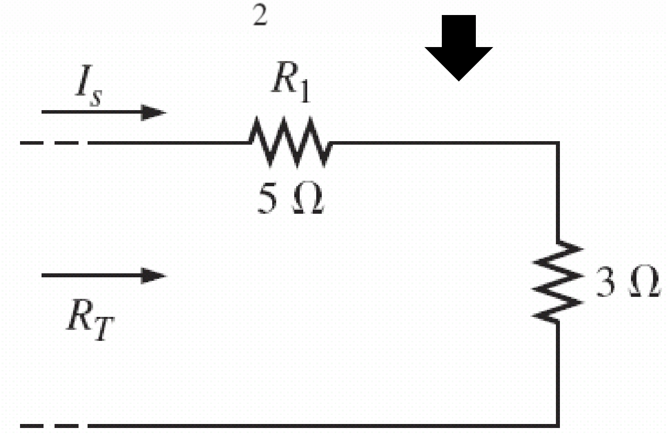
- The name comes from the repetitive structure.
- Use the reduce and return approach



Exp8: Calculate the indicated currents and voltage

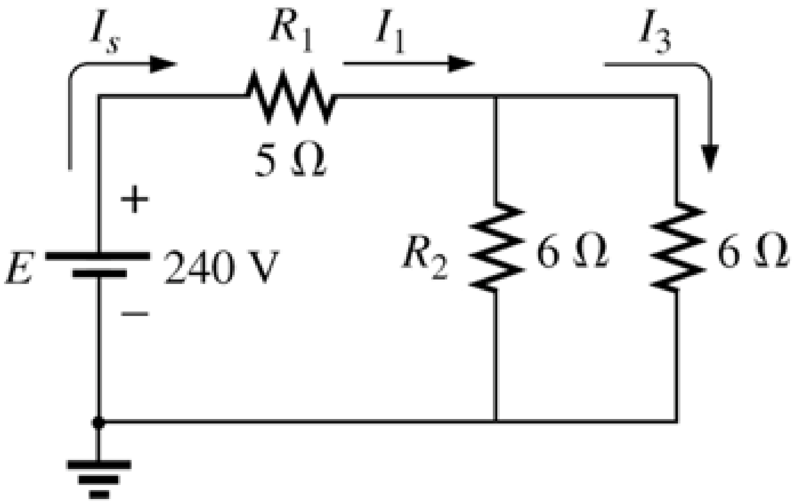






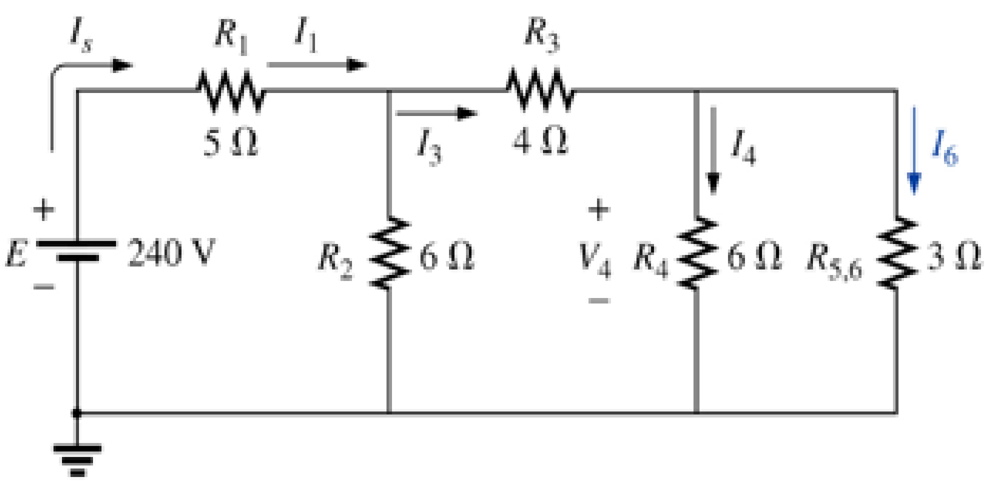
$$R_T = 5\Omega + 3\Omega = 8\Omega$$

$$I_s = \frac{E}{R_T} = \frac{240V}{8\Omega} = 30\,A$$



$$I_1 = I_s$$

$$I_3 = \frac{I_s}{2} = \frac{30\,A}{2} = 15\,A$$



$$I_6 = \frac{(6\Omega)\,I_3}{6\Omega + 3\Omega} = \frac{6}{9}(15\,A) = 10\ A$$

$$V_6 = I_6\,R_6 = (10\,A)(2\,\Omega) = 20\,V$$