Chapter 6

EMT1150
Introduction to Circuit Analysis

Department of Computer Engineering Technology

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Chapter6 Parallel DC Circuit

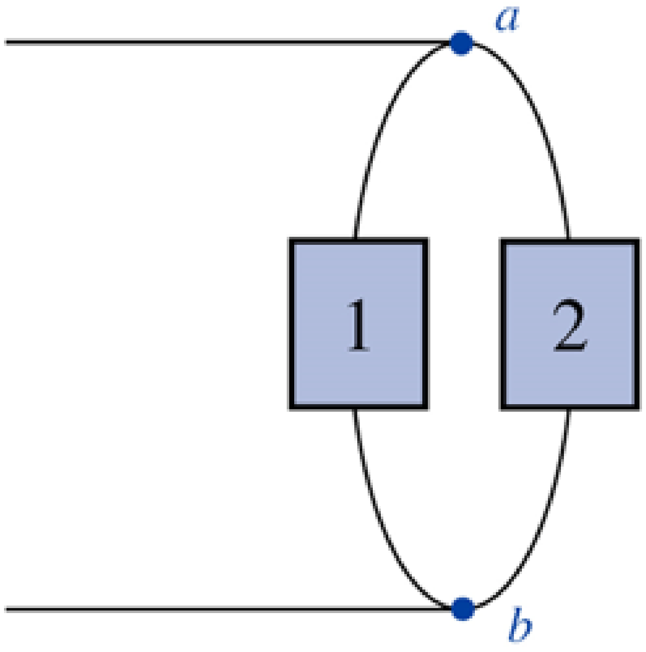
- Parallel connection and total resistance
- Parallel circuits analysis
- Kirchhoff's Current Law
- Current Divider Rule
- Voltage Sources in Parallel

Parallel connections

- Two elements are in parallel if they have two points in common.
- Elements 1 and 2 have

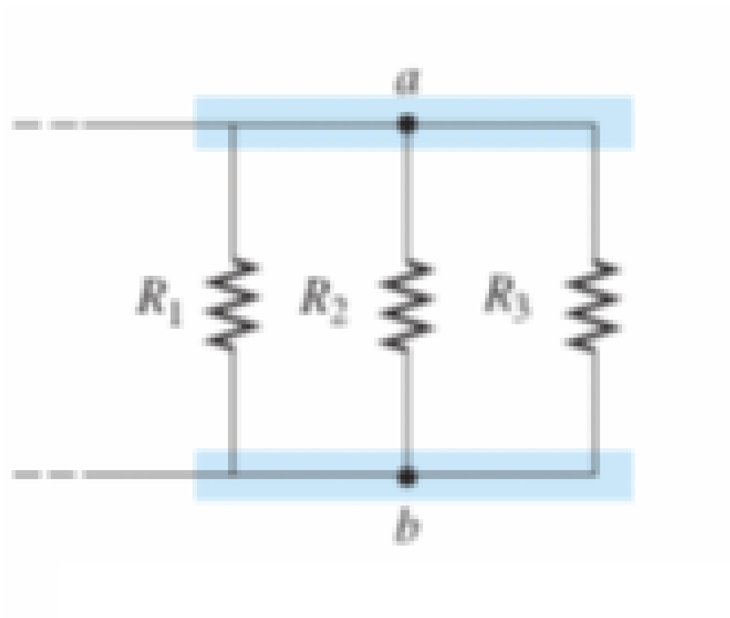
terminals a and b in common

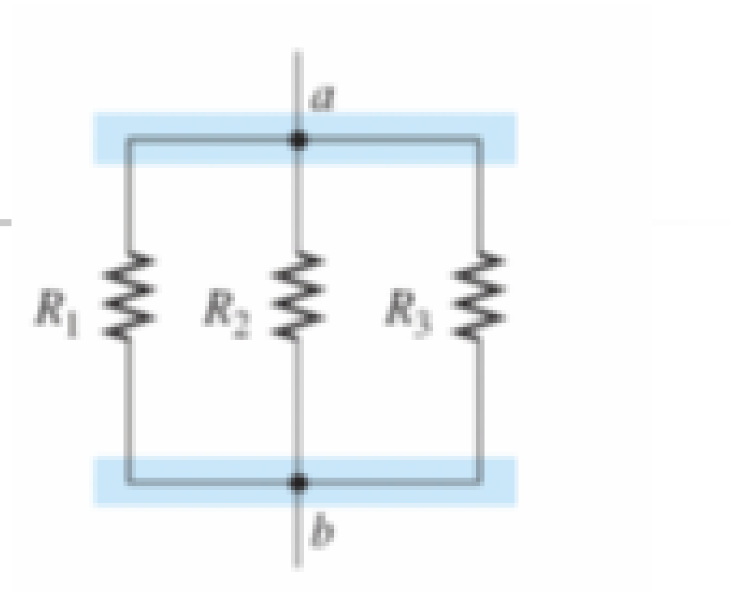
they are therefore in parallel.

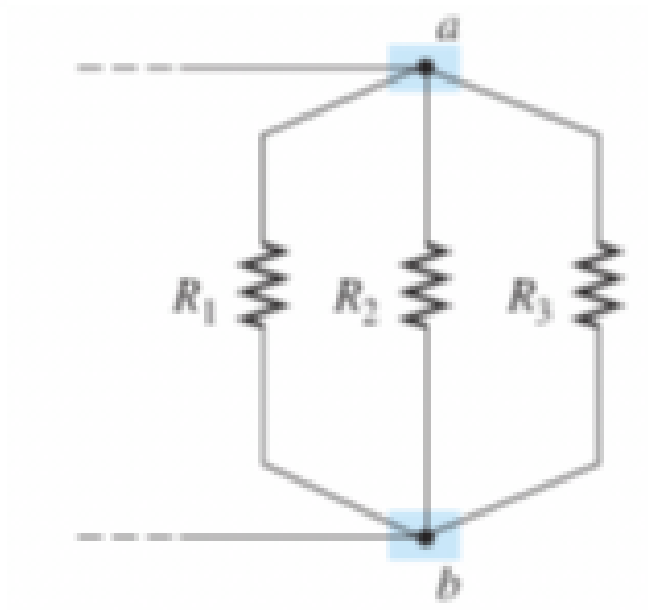


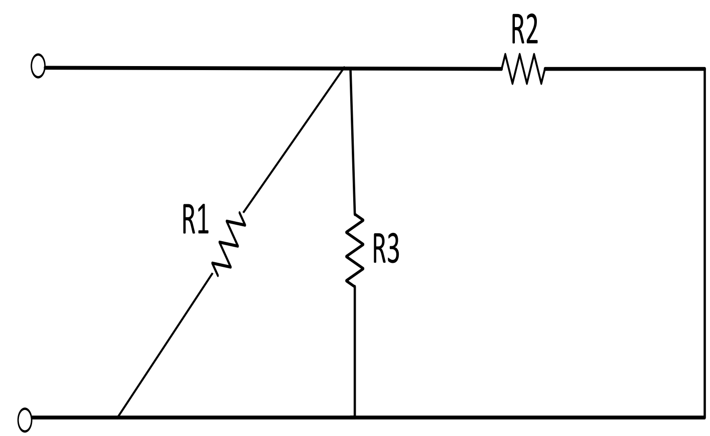
$R_1 // R_2$

Different representations of parallel connection

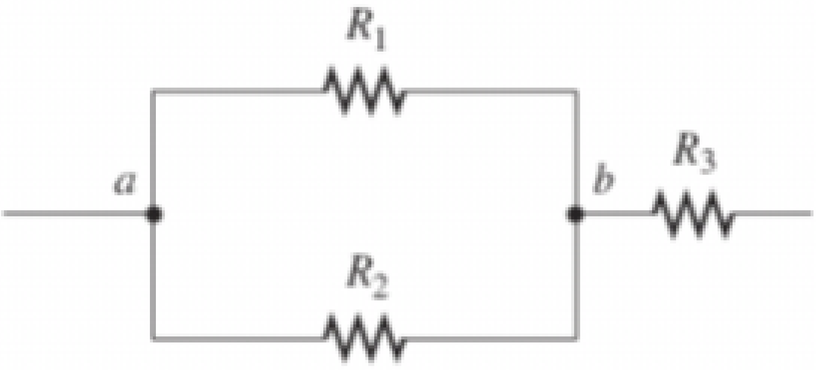




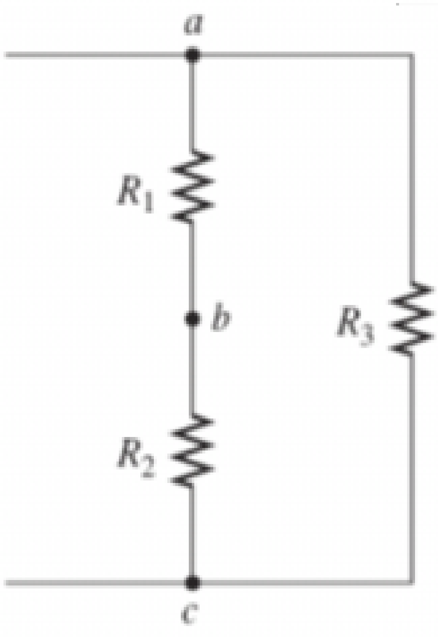




Mixtures of parallel and series connections



$R_1 // R_2 + R_3$

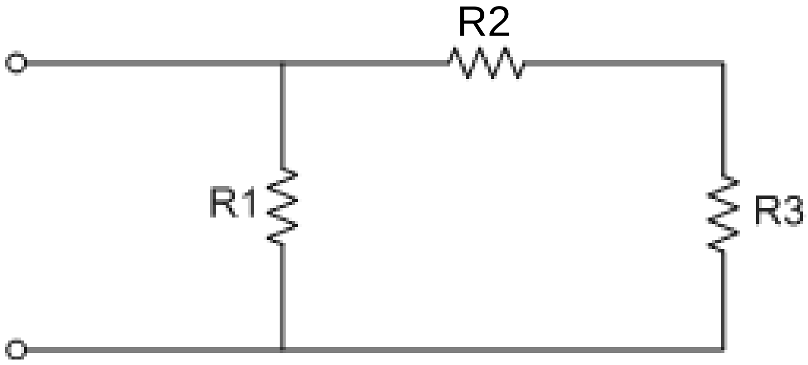


$(R_1 + R_2) // R_3$

Or $R_{1,2} // R_3$

Check understanding

- Exp1. What is the relationship between R1 and R3?
  - (a) Parallel
  - (b) Series
  - (c) Neither



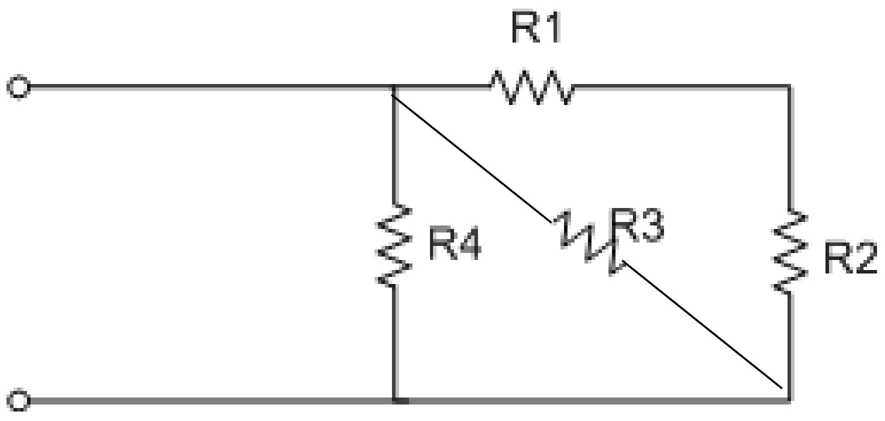
Exp2. Find the equivalent resistance of the net work.

(a) R1+R2+R3+R4

(b) (R1+R2)//R3//R4

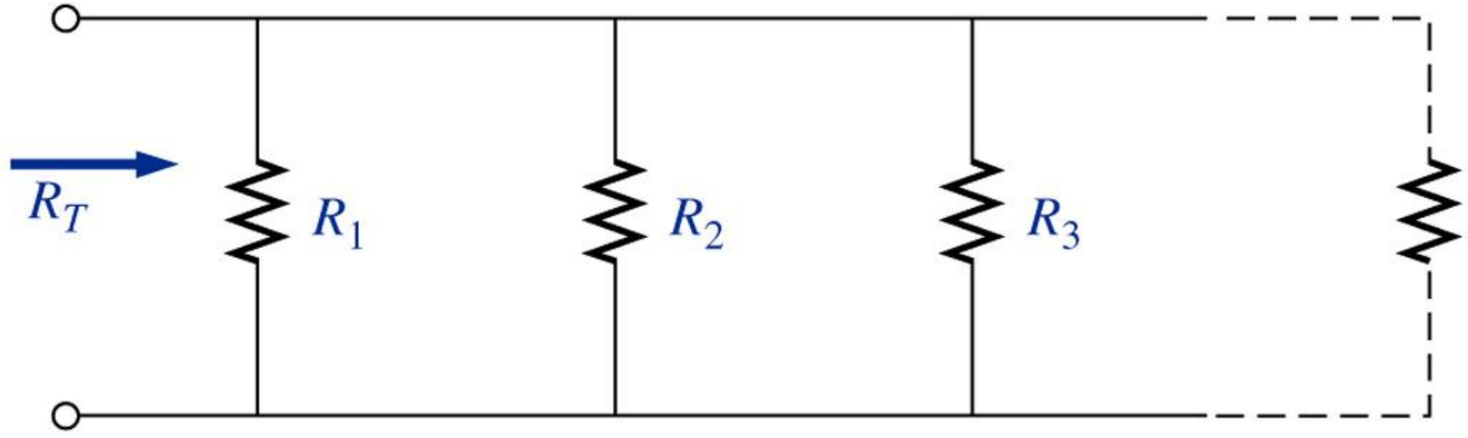
(c) (R1+R2)//R3+R4

(d) (R1+R2)//(R3+R4)



Total resistance of parallel connections

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_N}$$



$$G = \frac{1}{R}$$

$$G_T = G_1 + G_2 + G_3 + \cdots + G_N$$

For parallel elements, the total conductance is the sum of the individual conductance.

Two Special cases

- Two parallel resistors

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_2 + R_1}{R_1 R_2}$$

$$R_T = \frac{R_1 R_2}{R_1 + R_2}$$

The total resistance of two parallel resistors is the product of the two divided by their sum.

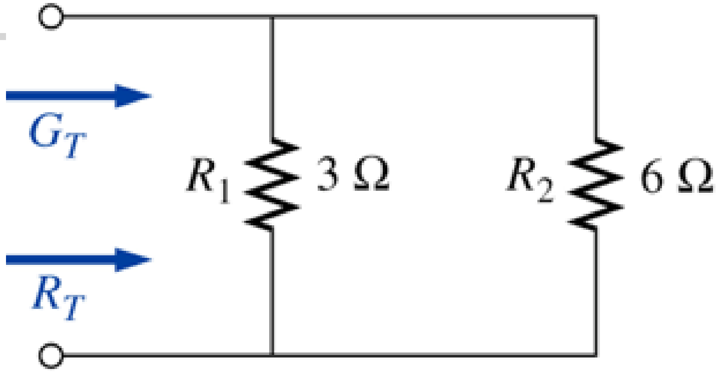
- Equal parallel resistors ($R_1 = R_2 = \ldots = R_N = R$)

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_N} = \frac{N}{R}$$

$$R_T = \frac{R}{N}$$

Exp3: Determine the total resistance for the parallel network

Method1:



$$G_T = G_1 + G_2 = \frac{1}{3\Omega} + \frac{1}{6\Omega}$$
$$= 0.333S + 0.167S = 0.5S$$

$$R_T = \frac{1}{G_T} = \frac{1}{0.5S} = 2\Omega$$

Method2:

$$R_T = \frac{R_1 R_2}{R_1 + R_2} = \frac{3\Omega \times 6\Omega}{3\Omega + 6\Omega} = 2(\Omega)$$

Exp4: Determine the total resistance for the parallel network

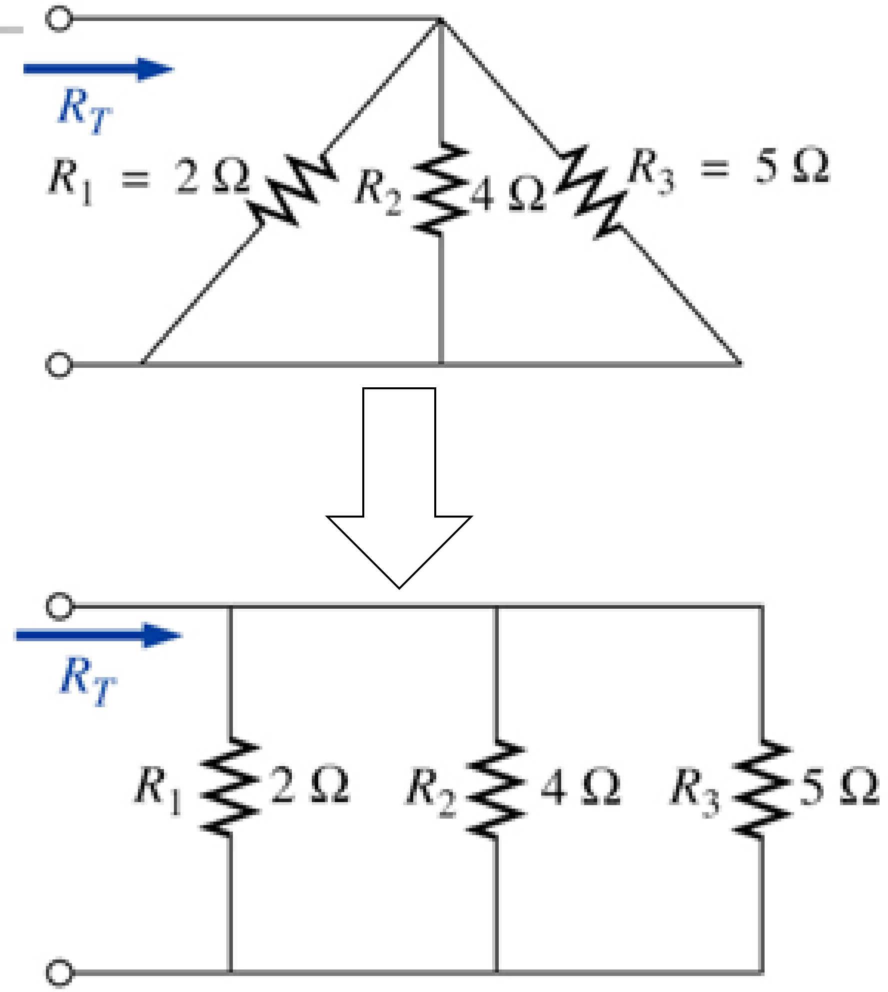
$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_2}$$

$$= \frac{1}{2\Omega} + \frac{1}{4\Omega} + \frac{1}{5\Omega}$$

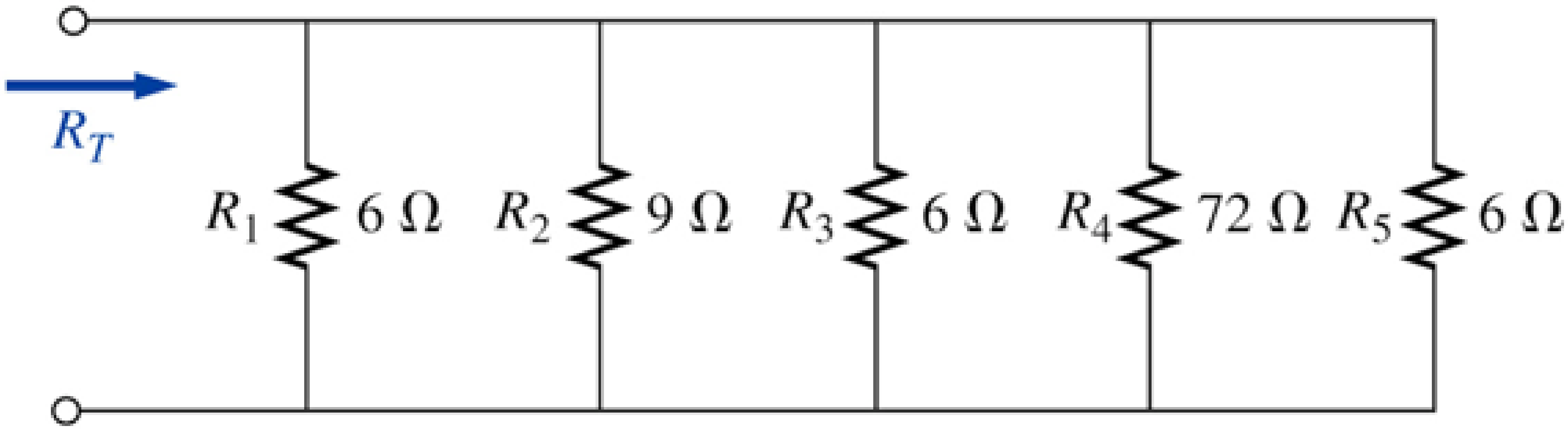
$$= 0.5\,S + 0.25\,S + 0.2\,S$$

$$= 0.95\,S$$

$$R_T = \frac{1}{0.95\,S} = 1.053\,\Omega$$

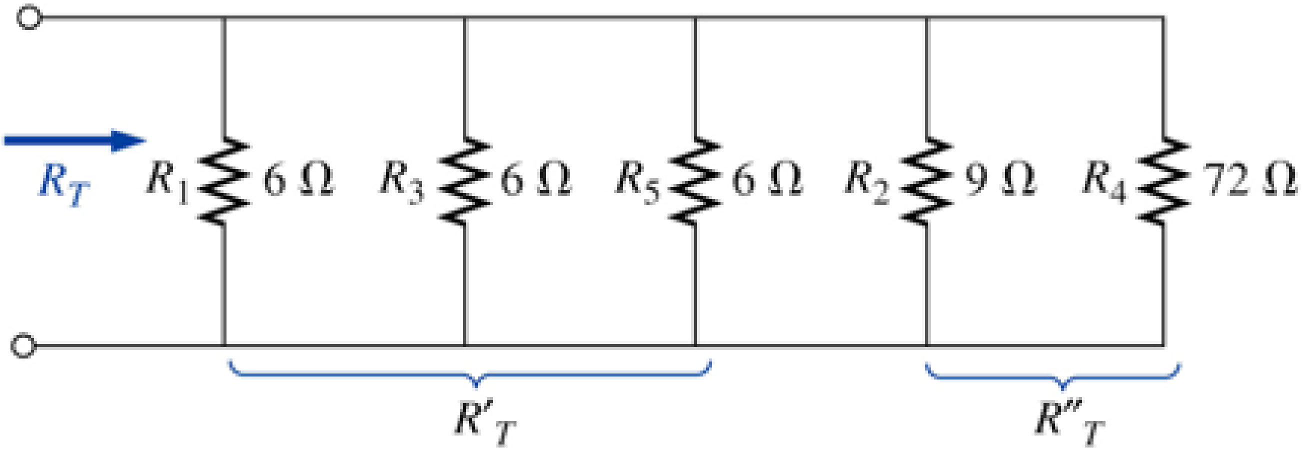


Exp5: Determine the total resistance for the parallel network



$$R_T = \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_5}}$$

$$= \frac{1}{\frac{1}{6\Omega}+\frac{1}{9\Omega}+\frac{1}{6\Omega}+\frac{1}{72\Omega}+\frac{1}{6\Omega}} = 1.6\,\Omega$$



$$R_T' = \frac{R}{N} = \frac{6\,\Omega}{3} = 2\,\Omega$$

$$R_T'' = \frac{R_2 R_4}{R_2 + R_4} = \frac{(9\,\Omega)(72\,\Omega)}{9\,\Omega + 72\,\Omega} = \frac{648\,\Omega}{81} = 8\,\Omega$$

$$R_T = R_T' \,//\, R_T'' = \frac{R_T' R_T''}{R_T' + R_T''}$$

$$= \frac{(2\,\Omega)(8\,\Omega)}{2\,\Omega + 8\,\Omega} = \frac{16\,\Omega}{10} = 1.6\,\Omega$$

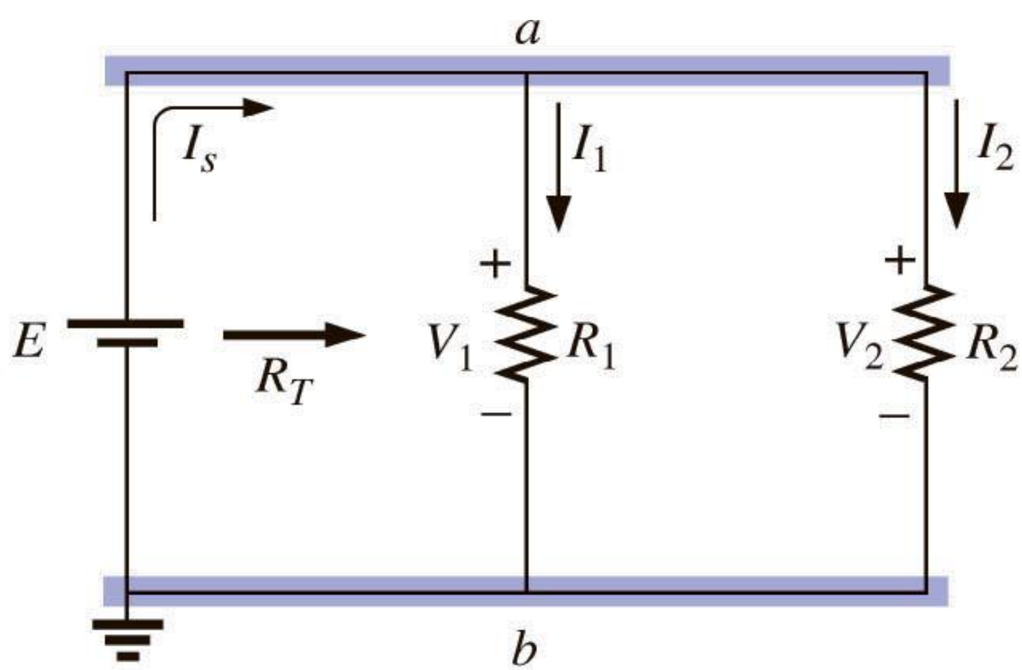
Parallel circuits

If two elements are in parallel, the voltage across them must be the same.

$$V_1 = V_2 = E$$

$$I_1 = \frac{V_1}{R_1} = \frac{E}{R_1}$$

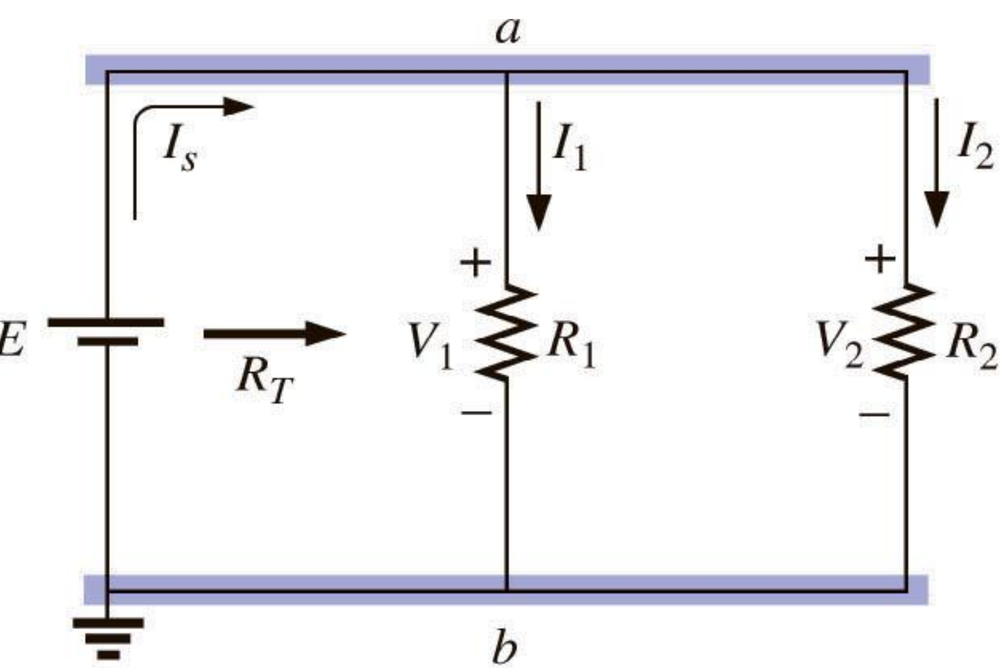
$$I_2 = \frac{V_2}{R_2} = \frac{E}{R_2}$$



Current & Power distribution

For single–source parallel networks, the source current (I_s) is equal to the sum of the individual branch current.

$$I_s = I_1 + I_2$$



The power dissipated by the resistors is equal to the power delivered by the source.

$$P_1 = V_1 I_1 = I_1^2 R_1 = \frac{V_1^2}{R_1}$$

$$P_1 = V_2 I_2 = I_2^2 R_2 = \frac{V_2^2}{R_2}$$

$$P_s = E I_s = I_s^2 R_T = \frac{E^2}{R_T}$$

Review

	Series	Parallel
Definition	One node in common and same current	Two nodes in common
Total resistance	$R_T=R_1+R_2+\ldots+R_N$	$\frac{1}{R_T}=\frac{1}{R_1}+\frac{1}{R_2}+\cdots+\frac{1}{R_N}$ $G_T=G_1+G_2+G_3+\cdots+G_N$
V & I	same current, voltage depends Ohm's law	same voltage, current depends Ohm's law

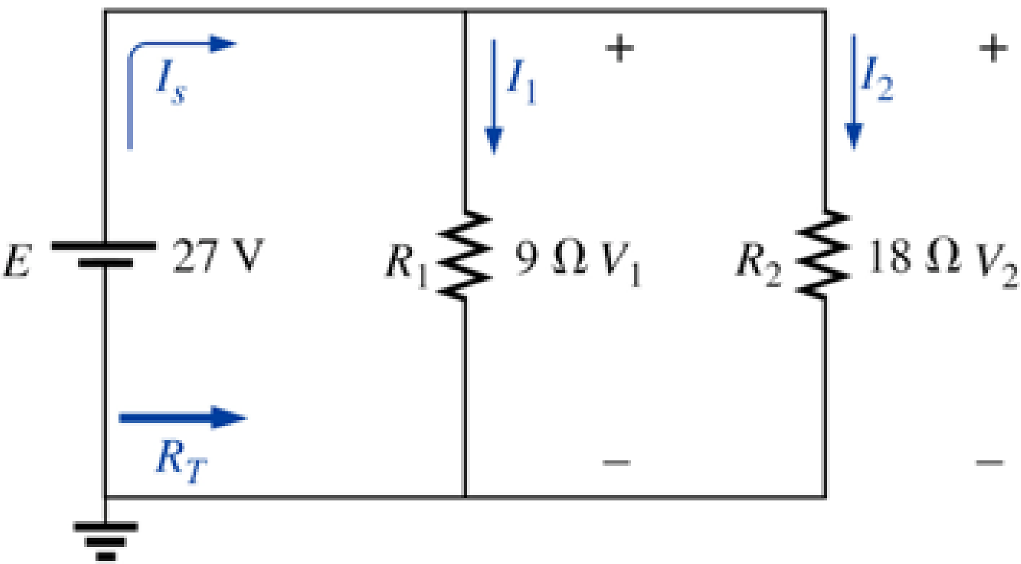
Exp6. a. Calculate R_T.

b. Determine I_s.

c. Calculate I_1 and I_2, and demonstrate that $I_s = I_1 + I_2$.

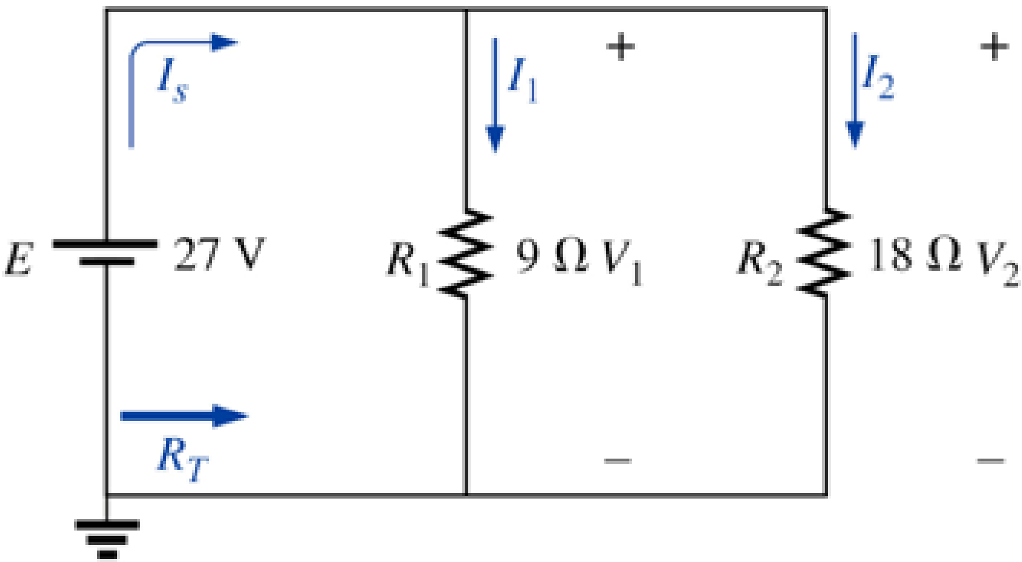
d. Determine the power to each resistive load.

e. Determine the power delivered by the source, and compare it to the total power dissipated by the resistive elements.



$$a.\quad R_T = \frac{R_1 R_2}{R_1 + R_2} = \frac{(9\Omega)(18\Omega)}{9\Omega + 18\Omega} = \frac{162\Omega}{27} = 6\Omega$$

$$b.\quad I_s = \frac{E}{R_T} = \frac{27V}{6\Omega} = 4.5\ A$$



$$c.\quad I_1 = \frac{V_1}{R_1} = \frac{E}{R_1} = \frac{27V}{9\Omega} = 3\,A$$

$$I_2 = \frac{V_2}{R_2} = \frac{E}{R_2} = \frac{27V}{18\Omega} = 1.5\,A$$

$$I_s = I_1 + I_2$$

$$4.5\ A = 3\ A + 1.5\ A$$

$$= 4.5\ A\ (checks)$$

$$d.\quad P_1 = V_1\,I_1 = E\,I_1 = (27V)(3\,A) = 81\,W$$

$$P_2 = V_2\,I_2 = E\,I_2 = (27V)(1.5\,A) = 40.5\,W$$

$$e.\ P_S = EI_S = (27V)(4.5A) = 121.5W$$

$$P_1 + P_2 = 81W + 40.5W = 121.5W$$

$$P_d = P_c$$

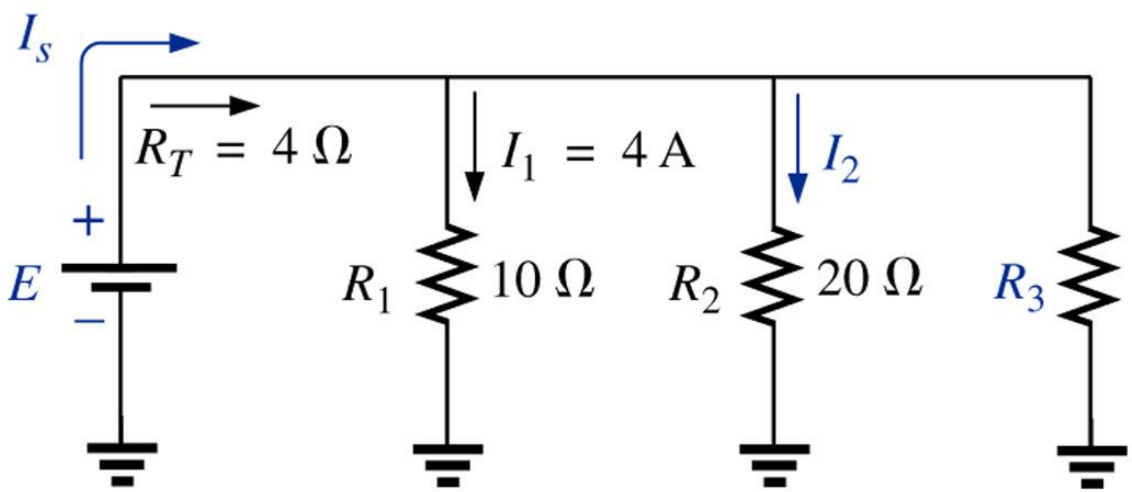
Exp7. Given the information provided in the Figure

a. Determine R_3.

b. Calculate E.

c. Find I_s & I_2.

d. Determine P_2.



$$(a)\ \frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\frac{1}{4\Omega} = \frac{1}{10\Omega} + \frac{1}{20\Omega} + \frac{1}{R_3}$$

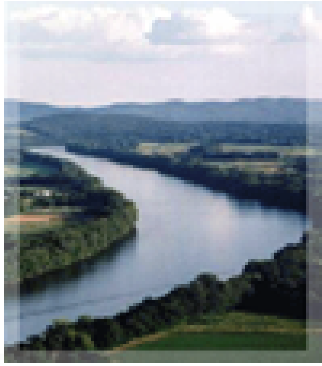
$R_3 = 10\ \Omega$

(b) $E = V_1 = I_1R_1 = (4A)(10\Omega) = 40V$

$$(c)\ I_s = \frac{E}{R_T} = \frac{40V}{4\Omega} = 10\ A$$

$$I_2 = \frac{V_2}{R_2} = \frac{E}{R_2} = \frac{40V}{20\Omega} = 2\ A$$

$$(d)\ P_2 = I_2^2\, R_2 = \left(2\ A\right)^2\left(20\Omega\right) = 80\,W$$

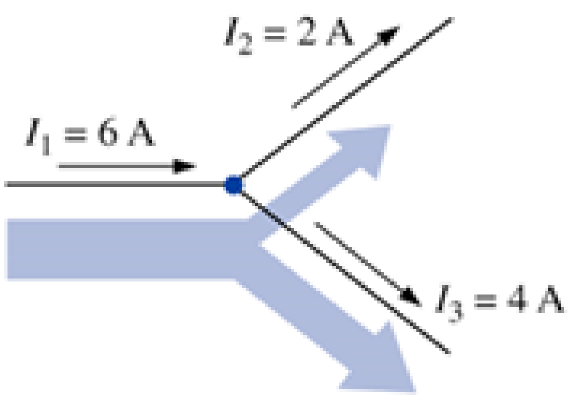


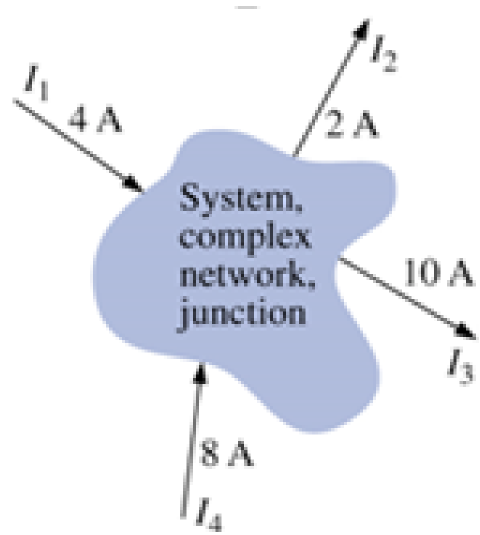
Kirchhoff's current law

- Current can only flow in a closed circuit.
- Current must not be lost as it flows around the circuit:
  - net charge cannot accumulate within the circuit
  - charge must be conserved

Kirchhoff's current law (KCL) states that the sum of the currents entering a node (junction) must equal to the sum of the currents leaving the node.

$$\sum I_{in} = \sum I_{out}$$



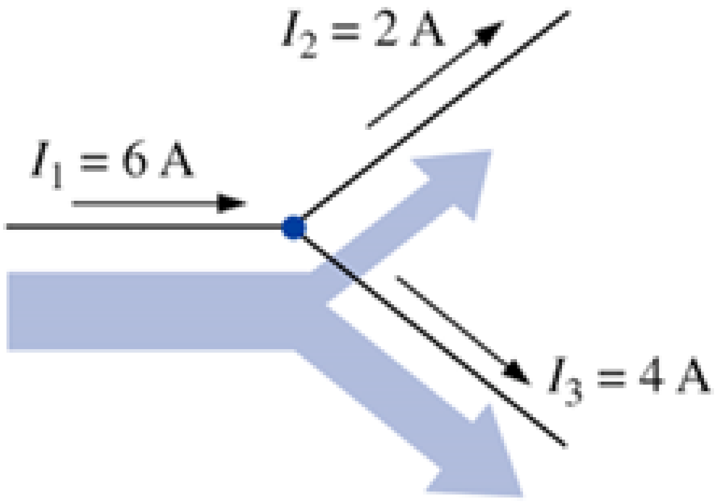


Different ways to write KCL

- The sum of the currents entering a node (junction) must equal to the sum of the currents leaving the node.

$$\sum I_{in} = \sum I_{out}$$

$$I_1 = I_2 + I_3$$



- The algebraic sum of the current entering and leaving a node (or junction) of a network is zero. Give negative sign when current come in, give positive sign when current leave the node.

$$\Sigma I_i = 0 \qquad -I_1 + I_2 + I_3 = 0$$

Exp8. Determine the currents I_1, I_3, I_4, and I_5 for the network

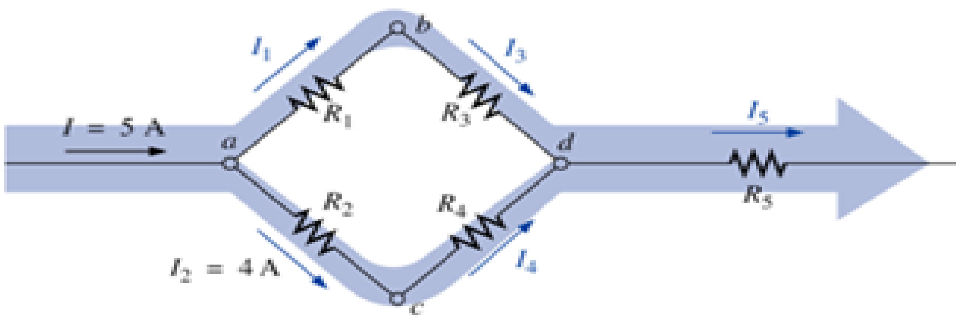
KCL @ node a:

$\Sigma I_{in} = \Sigma I_{out}$

$I = I_1 + I_2$

$5A = I_1 + 4A$

$I_1 = 1A$



KCL @ node b:

$\Sigma I_{in} = \Sigma I_{out}$

$I_1 = I_3 = 1A$

KCL @ node c:

$\Sigma I_{in} = \Sigma I_{out}$

$I_2 = I_4 = 4A$

KCL @ node d:

$\Sigma I_{in} = \Sigma I_{out}$

$I_1 + I_2 = I_5$

$I_5 = 1A + 4A$

$I_1 = 5A$

Exp9. Find the magnitude and direction of the currents I_3, I_4, I_6, and I_7 for the network

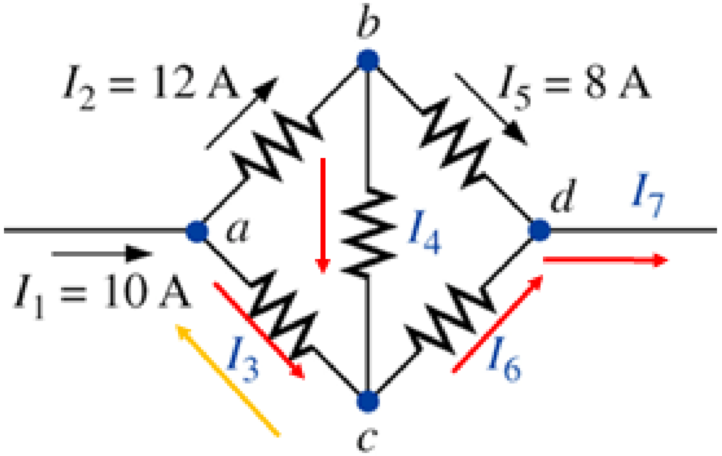
KCL @ node a:

$\Sigma I_{in} = \Sigma I_{out}$

$I_1 = I_2 + I_3$

$10A = 12A + I_3$

$I_3 = -2A$



KCL @ node b:

$\Sigma I_{in} = \Sigma I_{out}$

$I_2 = I_4 + I_5$

$12A = I_4 + 8A$

$I_4 = 4A$

KCL @ node c:

$\Sigma I_{in} = \Sigma I_{out}$

$I_3 + I_4 = I_6$

$-2A + 4A = I_6$

$I_6 = 2A$

KCL @ node d:

$\Sigma I_{in} = \Sigma I_{out}$

$I_5 + I_6 = I_7$

$8A + 2A = I_7$

$I_7 = 10A$

Apply KCL to parallel resistor network to find equivalent resistance

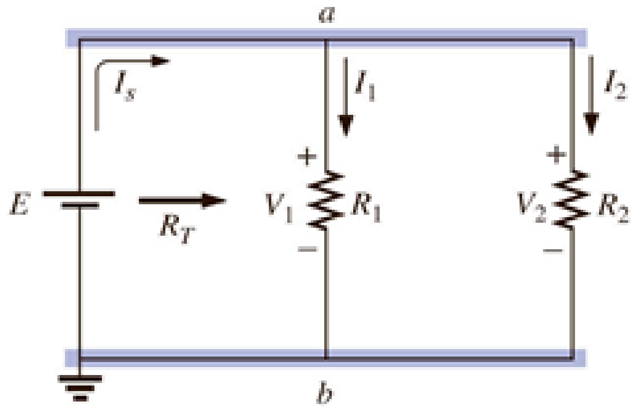
- Parallel connection: $V_1 = V_2 = E$

KCL @ node a

$$I_s = I_1 + I_2$$

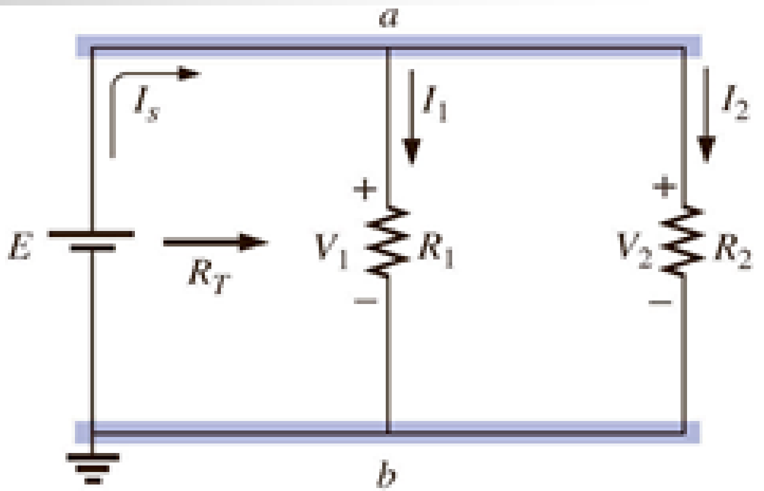
$$I_s = \frac{E}{R_1} + \frac{E}{R_2} = E\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$R_T = \frac{E}{I_s} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}$$



Current Divider Rule

- This rule is used to calculate the current through any resistor connected in a parallel circuit.
  - Consider finding I_1 in the parallel circuit :



$$I_1 = \frac{E}{R_1}$$

$$E = I_s R_T$$

$$R_T = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}$$

$$I_1 = \frac{E}{R_1} = \frac{I_s R_T}{R_1} = \frac{I_s\left(\frac{1}{R_1}\right)}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{R_T}{R_1} I_s$$

- This same analysis can be used to find I_2:

$$I_2 = \frac{E}{R_2} = \frac{I_s R_T}{R_2} = \frac{I_s}{R_2}\left(\frac{1}{\frac{1}{R_1}+\frac{1}{R_2}}\right) = I_s\left(\frac{\frac{1}{R_2}}{\frac{1}{R_1}+\frac{1}{R_2}}\right) = I_s\frac{R_T}{R_2}$$

- In general, if a circuit contains N resistors in parallel, the current through any one of the resistors R_i is:

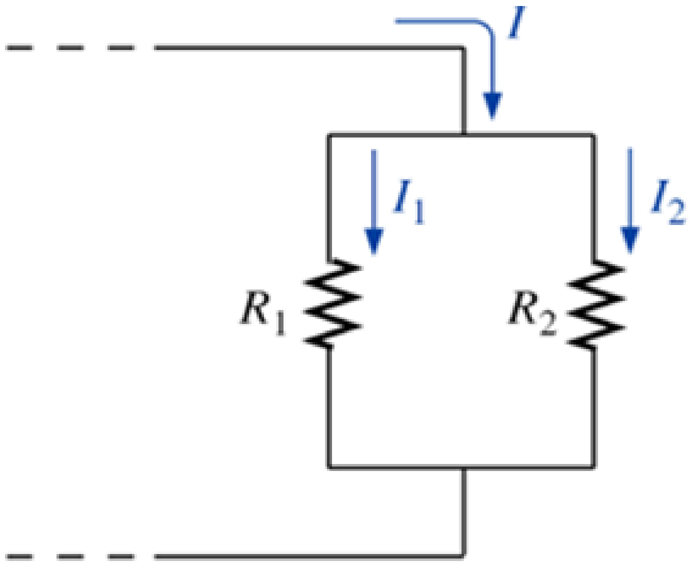
$$I_i = I_s\left(\frac{\frac{1}{R_i}}{\frac{1}{R_1}+\frac{1}{R_2}+\ldots+\frac{1}{R_N}}\right) = I_s\frac{R_T}{R_i}$$

Note: we don't need to know the voltage, V

In other words, in a parallel resistive circuit, the current will split with a ratio equal to the inverse of their resistor values

$$\frac{I_1}{I_2} = \frac{R_2}{R_1}$$

Special case: Two parallel resistors



$$R_T = \frac{R_1 R_2}{R_1 + R_2}$$

and

$$I_1 = \frac{R_T}{R_1} I = \frac{\frac{R_1 R_2}{R_1 + R_2}}{R_1} I$$

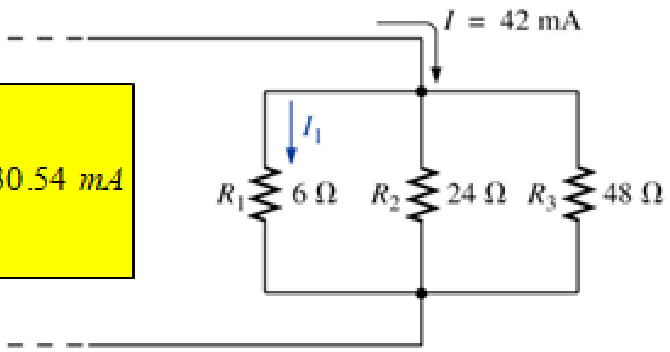
$$= \frac{R_2 I}{R_1 + R_2}$$

$$I_2 = \frac{R_1 I}{R_1 + R_2}$$

For two parallel branches, the current through either branch is equal to the product of the other parallel resistor and the input current divided by the sum of the two parallel resistances.

Exp11: Find the current I_1 for the network

$$I_1 = \frac{\frac{1}{R_1}}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}} I_T = \frac{\frac{1}{6\,\Omega}}{\frac{1}{6\,\Omega}+\frac{1}{24\,\Omega}+\frac{1}{48\,\Omega}} 42\ mA = 30.54\ mA$$



Current seeks the path of least resistance.

1. More current passes through the smaller of two parallel resistors.
2. The current entering any number of parallel resistors divides into these resistors as the inverse ratio of their resistance values.

Exp12: Determine the resistance R_1 to effect the division of current in figure.

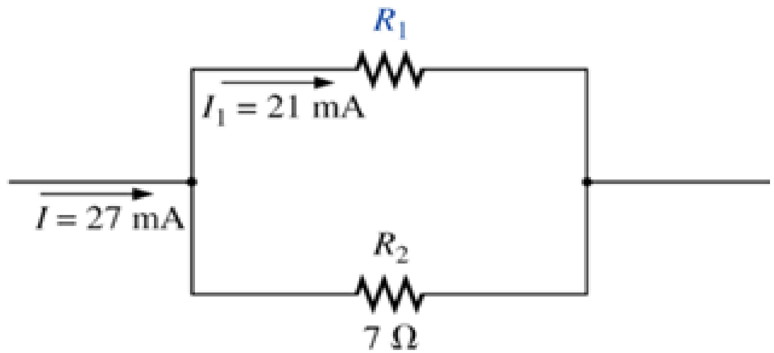
$$I_1 = \frac{R_2\, I}{R_1 + R_2}$$

$$(R_1 + R_2) I_1 = R_2\, I$$

$$R_1\, I_1 + R_2\, I_1 = R_2\, I$$

$$R_1\, I_1 = R_2\, I - R_2\, I_1$$

$$R_1 = \frac{R_2\, I - R_2\, I_1}{I_1} = \frac{R_2 (I - I_1)}{I_1}$$



$$R_1 = \frac{7\,\Omega (27\,mA - 21\,mA)}{21\,mA}$$

$$= 7\,\Omega \left(\frac{6}{21} \right) = 2\,\Omega$$

$$21mA = \frac{7\Omega \times 27mA}{R_1 + 7\Omega}$$

$$R_1 = 2\Omega$$

Voltage source in parallel

- Voltage sources can be placed in parallel only if they have the same voltage.
- The primary reason for placing two or more batteries or supplies in parallel is to increase the current rating above that of a single supply.