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# Before you start

# Notes on hands-on exercises and functionality of Simulation

This book goes beyond a standard software manual. It takes a unique approach by bridging the theory of mechanical vibrations with examples showing the practical implementation of vibration analysis. This book builds on material covered in **"Engineering Analysis with SOLIDWORKS Simulation"**.

We recommend that you study the exercises in the order presented in the book. As you go through the exercises, you will notice that explanations and steps described in detail in earlier exercises are not repeated in later chapters. Each subsequent exercise assumes familiarity with software functions discussed in previous exercises and builds on the skills, experience, and understanding gained from previously presented problems.

Exercises in this book require **SOLIDWORKS Simulation**. The **SOLIDWORKS Simulation** Product Matrix document is available at:

www.SOLIDWORKS.com/sw/docs/SOLIDWORKS-simulation-matrix.pdf

Information on the SOLIDWORKS Simulation Packages is available at:

www.SOLIDWORKS.com/sw/products/10169\_ENU\_HTML.htm

All exercises in this book use **SOLIDWORKS** models, which can be downloaded from <u>www.SDCpublications.com</u>. Most of these exercises do not contain any **Simulation** studies; you are expected to create all studies, results plots, and graphs yourself. Animations of selected results may be found using YouTube link in <u>www.designgenerator.com</u>

All problems presented here have been solved with **SOLIDWORKS Simulation Premium** running on Windows 7 Professional.

Working on exercises you may notice that your results may be slightly different from results presented in this book. This is because numerical results may differ slightly depending on the operating system and software service pack.

We encourage you to explore each exercise beyond its description by investigating other options, other menu choices, and other ways to present results. You will soon discover that the same simple logic applies to all functions in **SOLIDWORKS Simulation**, be it structural, vibration or thermal analysis.

This book is not intended to replace software manuals. The knowledge acquired by the reader will not strictly be software specific. The same concepts, tools and methods apply to any FEA software.

# **Prerequisites**

"Vibration Analysis with SOLIDWORKS Simulation" is not an introductory text to SOLIDWORKS Simulation. Rather, it picks up Vibration Analysis from where it was left in the pre-requisite textbook "Engineering Analysis with SOLIDWORKS". If you are new to SOLIDWORKS Simulation we recommend reading "Engineering Analysis with SOLIDWORKS Simulation" to gain essential familiarity with Finite Element Analysis. At the very least go through chapters 4, 6, 18, 19 of this prerequisite textbook.

The following prerequisites are recommended:

- □ An understanding of Vibration Analysis
- □ An understanding of Structural Analysis
- □ An understanding of Solid Mechanics
- **□** Familiarity with **SOLIDWORKS**
- □ Familiarity with SOLIDWORKS Simulation to the extend covered in "Engineering Analysis with SOLIDWORKS" or equivalent experience
- **G** Familiarity with the Windows Operating System

# Selected terminology

The mouse pointer plays a very important role in executing various commands and providing user feedback. The mouse pointer is used to execute commands, select geometry, and invoke pop-up menus. We use Windows terminology when referring to mouse-pointer actions.

Item	Description		
Click	Self-explanatory		
Double-click	Self-explanatory		
Click-inside	Click the left mouse button. Wait a second, and then click the left mouse button inside the pop- up menu or text box. Use this technique to modify the names of folders and icons in <b>SOLIDWORKS Simulation</b> Manager.		
Drag and drop	Use the mouse to point to an object. Press and hold the left mouse button down. Move the mouse pointer to a new location. Release the left mouse button.		
Right-click	Click the right mouse button. A pop-up menu is displayed. Use the left mouse button to select a desired menu command.		

All **SOLIDWORKS** file names appear in CAPITAL letters, even though the actual file names may use a combination of capital and small letters. Selected menu items and **SOLIDWORKS Simulation** commands appear in **bold**, **SOLIDWORKS** configurations, **SOLIDWORKS Simulation** folders, icon names and study names appear in *italics* except in captions and comments to illustrations. **SOLIDWORKS** and **Simulation** also appear in bold font. Bold font is also used to draw reader's attention to particular term.

# 1: Introduction to vibration analysis

# **Topics** covered

- Differences between a mechanism and a structure
- Difference between dynamic analysis and vibration analysis
- **□** Rigid body motion and degrees of freedom
- □ Kinematic pairs
- Discrete and distributed vibration systems
- □ Single degree of freedom and multi degree of freedom vibration systems
- □ Mode of vibration
- □ Rigid Body Mode
- □ Modal superposition method
- Direct integration method
- □ Vibration Analysis with SOLIDWORKS Simulation and SOLIDWORKS Motion
- □ Functionality of SOLIDWORKS Simulation and SOLIDWORKS Motion
- Terminology issues

# Differences between a mechanism and a structure

A mechanism is not firmly supported and can move without having to deform; components of a mechanism can move as rigid bodies. On the contrary, any motion of a structure must involve deformation because a structure is, by definition, firmly supported. This motion may take the form of a one-time deformation when a static load is applied, or the structure may be oscillating about the position of equilibrium when a time varying load is present. In short, a mechanism may move without having to deform its components while any motion of a structure must be accompanied by deformation. If an object may move without experiencing deformation then it can be classified as a structure. Examples of mechanism and structure are shown in Figure 1-1.



A mechanism: ELLIPTIC TRAMMEL

A structure: TRUSS weldment

Figure 1-1: A mechanism and a structure.

ELLIPTIC TRAMMEL model is discussed in this chapter; TRUSS model comes from "Engineering Analysis with SOLIDWORKS Simulation", chapter 16.

An elliptic trammel is a mechanism; it is designed to trace an ellipse when it performs motion. This motion can be studied without considering deformation of its components. A weldment is a structure. It is designed to stand still and can move only as it deforms from its position of equilibrium. Any motion of a structure is always accompanied by deformation.

Depending on the objective of analysis, an object or its components may be treated either as a mechanism or a structure. A wind turbine rotor is a mechanism; it spins relative to the tower. Depending on the objective of analysis this mechanism may be treated as a rigid body mechanism or an elastic body mechanism. An individual blade may be treated as a rigid body component of a mechanism or as a structure if its vibration characteristics need to be analyzed (Figure 1-2).



Figure 1-2: A wind turbine may be considered either as a mechanism or as a structure. *Source of model: GrabCAD.* 

A wind turbine tower is a structure; a rotor can be considered as a mechanism composed of rigid bodies, a mechanism composed of elastic bodies or as a structure.

## Difference between Dynamic Analysis and Vibration Analysis

**Dynamic Analysis** can be performed on a mechanism or a structure. The analyzed object can be an airplane wing, a car, a building subjected to an earthquake, an engine or a door bell making sound. Dynamic Analysis can deal with rigid and/or elastic bodies. **Vibration Analysis** is more specific, it deals only with elastic bodies vibrating about the position of equilibrium.

# Rigid body motion and degrees of freedom

If an object moves without deforming, then this object is called a **Rigid Body** and the motion is classified as a **Rigid Body Motion**. The number of independent variables necessary to define the position of a rigid body with respect to a reference coordinate system equals the number of degrees of freedom of that body. The number of degrees of freedom of a rigid body is equal to the number of rigid body motions of that body.

Consider an unsupported rigid body; to define its position with respect to a Cartesian coordinate system we need to provide six pieces of information: linear coordinates x, y, z along any point belonging to this body, and three angles defining angular orientation of the body (Figure 1-3).



Figure 1-3: A flying plane considered as a rigid body has six degrees of freedom or six rigid body modes.

Source of model: GrabCAD.

In the case of a flying airplane, three linear coordinates are required to define the position of the center of mass and three angular coordinates are required to define the angular position.

# Kinematic Pairs

A kinematic pair is comprised of two rigid bodies; one rigid body is fully restrained meaning it has no degrees of freedom. The other rigid body can move and is connected to the fully restrained body it in such a way that the number of degrees of freedom of the moving body is limited.

We will review basic kinematic pairs and their Degrees of Freedom (DOF) using the assembly models listed in Figure 1-4. Notice that all assemblies are not fully defined to allow for relative motion of assembly components.

Kinematic pair	Number of translational DOF	Number of rotational DOF	SOLIDWORKS Assembly file name
Planar	2	1	PLANAR
Prismatic	1	0	PRISMATIC
Revolute	0	1	REVOLUTE
Cylindrical	1	1	CYLINDRICAL
Spherical	0	3	SPHERICAL
Screw	1/2	1/2	SCREW

Figure 1-4: A summary of basic kinematic pairs and their number of degrees of freedom.

If opened in SOLIDWORKS, you will find all assemblies consist of two components: the blue part is fixed (fully restrained) and the yellow part can move relative to the blue part.

To review the above models, use assembly commands **Move Component** and **Rotate Component**. No **SOLIDWORKS Simulation** study is required. In all examples featuring kinematic pairs, the fixed bodies are blue, and the moving bodies are yellow. Animation of the above kinematic pairs can be easily done in **SOLIDWORKS Motion**.

### A planar kinematic pair is shown in

Figure 1-5.



PLANAR.SLDASM

Figure 1-5: A planar kinematic pair removes three degrees of freedom from the moving body.

Two linear coordinates are required to define the position of the center of the sliding block and one angular coordinate is required to define its angular position. Therefore, the pair has two translational degrees of freedom and one rotational degree of freedom.

### A prismatic kinematic pair is shown in

Figure1-6.



PRISMATIC.SLDASM

Figure 1-6: A prismatic kinematic pair removes five degrees of freedom from the moving body.

One linear coordinate is required to define the position of the sliding block. This pair has one translational degree of freedom.

A revolute kinematic pair is shown in Figure 1-7.



**REVOLUTE.SLDASM** 

Figure 1-7: A revolute kinematic pair removes five degrees of freedom from the moving body.

One angular coordinate is required to define the position of the moving part. The pair has one rotational degree of freedom.

A cylindrical kinematic pair is shown in Figure 1-8.



CYLINDRICAL.SLDASM

Figure 1-8: A cylindrical kinematic pair removes four degrees of freedom from the moving body.

One linear coordinate and one angular coordinate are required to define the position of the moving part. The pair has one translational and one rotational degree of freedom. A spherical kinematic pair is shown in Figure 1-9.



Figure 1-9: A spherical kinematic pair removes three degrees of freedom from the moving body.

Three angular coordinates are required to define the position of the moving part. The pair has three rotational degrees of freedom.

A screw kinematic pair is shown in Figure 1-10.



SCREW.SLDASM

Figure 1-10: A screw kinematic pair removes five degrees of freedom from the moving body.

Either linear or angular coordinates fully define the position of nut. The pair has one degree of freedom that couples translation and rotation. When you review this model, notice that the thread is not modeled explicitly. Instead, its presence is modeled by a mechanical mate. Cosmetic thread is added to the model appearance.

## Discrete and distributed vibration systems

All that is required for vibration is a mass and stiffness. Therefore any real life object can be treated as a vibration system.

Depending on how mass and stiffness are distributed we can classify vibration systems as discrete or distributed systems. A discrete system is one where mass and stiffness are separated meaning that certain portions of the system are responsible for inertial properties while others are responsible for stiffness properties. Figure 1-11 shows two discrete systems: a single degree of freedom system and a two degree of freedom system. Both systems perform linear vibrations; the base and masses are treated as rigid bodies; deformations are limited to the springs which have no mass. Review the assembly model DISCRETE LINEAR in configurations *1DOF* and *2DOF* as shown in Figure 1-11.



Figure 1-11: A single degree of freedom system (left) and two degree of freedom system (right) have mass and stiffness separated.

The cylindrical masses can move only in the vertical direction, they can't rotate. The base is fixed in both models.

One linear coordinate is required to define the position of the vibrating mass in the 1DOF model; two linear coordinates are required to define the positions of two vibrating masses in the 2DOF model.

A single degree of freedom (SDOF) discrete vibration system is a "workhorse" of the theory of vibration. Vibration responses of a SDOF have simple analytical solutions and many real life problems can be successfully studied by representing the system as a SDOF. We will study a SDOF in the second chapter to introduce essential concepts of vibration analysis with **SOLIDWORKS Simulation** and **SOLIDWORKS Motion**.

A discrete vibrating system can also perform rotary (angular) vibration. An example is the SWING ARM assembly model shown in Figure 1-12. The base and arm are treated as rigid bodies; deformation is limited to the massless spring.



Figure 1-12: A single degree of freedom rotary vibration system has mass and stiffness separated.

One angular coordinate is required to define the position of this system.

Referring to Figure 1-12, notice that the arm performs angular vibration, therefore vibration properties are defined by its mass moment of inertia about the axis of rotation. The spring experiences linear deformation, therefore its vibration properties are defined by linear stiffness. The swing arm could also perform angular vibration if a torsional spring was installed at the hinge.

Discrete systems don't exist in real life and discrete representation is only a modeling simplification. We use discrete system representations because many problems can be described with sufficient accuracy by discrete models, usually with a few degrees of freedom. Degrees of freedom of discrete systems are clearly defined because deformation is limited to springs (linear and angular) present in the model. Analytical solutions of discrete systems are simple and results are often intuitive.

In a distributed vibrating system, mass and stiffness are not separated. All portions of an object are responsible for both mass and stiffness properties. Figure 1-13 shows the model CLIP which represents a distributed vibration system.



Figure 1-13: In a distributed vibration system, all portions of the object are responsible for both mass and stiffness properties.

Mass and stiffness are not separated.

# Mode of vibration

The mode of vibration is the most fundamental property of any vibrating system. Its central role in vibration analysis requires us to discuss it in this introductory chapter. We will refine this definition later.

The mode of vibration can be defined as the preferred way of a structure to vibrate. It is characterized by its frequency of vibration, mass participating in the vibration, and shape of vibration. The number of modes of vibration is equal to the number of degrees of freedom of the vibrating system.

Shapes of vibration in the first (and only) mode of the discrete vibration system DISCRETE LINEAR in the *1DOF* configuration is shown in Figure 1-14. This model is available with **Frequency** studies set up and ready to run. Please run the solutions before proceeding.



Figure 1-14: The shape of vibration of the 1DOF model. The moving part translates as a rigid body; it does not experience any deformation. The only deforming element is the spring.

The mass moves as a rigid body, its motion is restricted to translation. Motion of the moving part is not restricted by any kinematic pair in this model. Distributed spring connector and restraints are defined in Frequency study: it is shown connecting components in the undeformed position.

Notice that the "Deformed shape" shown in Figure 1-14 refers to the deformation of the spring even though the spring deformation can't be seen on this illustration. The cylinder itself is not deforming, it performs linear oscillations as a rigid body.

The SWING ARM model in the *1DOF* configuration also has a **Frequency** study defined; please run the solution before proceeding.



Figure 1-15: Shape of vibration of the SWING ARM model.

The arm rotates as a rigid body; it does not experience any deformation. The only deforming element is the spring. Motion of the arm is controlled by a hinge.

Review assembly model SWING ARM and notice that the vertical post in the base serves only to locate the hinge position of the swing arm. The hinge itself is not modeled. The hinge is simulated in the **Frequency** study using a **Fixed Hinge** restraint.

A two degree of freedom discrete system has two modes of vibration, each one characterized by its own unique shape as demonstrated by the assembly model DOUBLE PENDULUM, shown in Figure 1-16. Run the **Frequency** study before proceeding.



### Figure 1-16: Shapes of vibration of the DOUBLE PENDULUM model.

The shape of mode 1 and mode 2 are result plots in the Frequency study. The undeformed shape is superimposed on the deformed model.

Review the **Frequency** study in the DOUBLE PENDULUM model. Notice that **Global Contact** is set to **Allow Penetration** in order to disconnect the touching faces which otherwise would be bonded. The three links are connected by two **Pin Connectors** which have torsional stiffness and therefore act as torsional springs

Distributed systems, where deformation is not limited to discrete springs have an infinite number of degrees of freedom and consequently, an infinite number of modes of vibration. Consider CLIP (Figure 1-13) to be unsupported. CLIP treated as a rigid body has six degrees of freedom: three translations and three rotations; we may say it has six rigid body motions. When treated as an elastic body, CLIP will have an infinite number of degrees of freedom associated with elastic deformation in addition to those six degrees of freedom associated with **Rigid Body Motions** (**RBMs**). Consequently is has an infinite number of modes of vibration. When this CLIP is analyzed using FEA, the number of elastic degrees of freedom becomes finite due to discretization which is an inherent part of FEA (Figure 1-17).



CAD model

Finite element mesh and mesh details

## Figure 1-17: CLIP model before and after discretization.

Meshing was performed using second order tetrahedral solid elements with a default element size. This produced a mesh with 13326 nodes.

Remembering that solid elements have three degrees of freedom per node, the number of degrees of freedoms in the finite element mesh is 39978.

The total number of degrees of freedom is found by adding:

	Number of associated degrees of freedom
Rigid body motions	6
Deformation	39978
Total:	39984

#### Vibration Analysis with SOLIDWORKS Simulation 2015

The CLIP model after discretization has 39978 DOFs associated with deformation, most often this will be elastic deformation. Theoretically, this gives the model 39978 modes of vibration. However, all those higher modes are never excited in the real life. Only the first few modes with frequencies low enough to be excited are important in the vibration response.

When the model is unsupported (as is the CLIP model) **Simulation** detects six **Rigid Body Modes** and assigns them frequencies of 0Hz. The first three modes are rigid body translations and the second three are rigid body rotations (Figure 1-18).



Figure 1-18: Six Rigid Body Modes of the unsupported CLIP model are in this case aligned with the directions of the global coordinate system.

The dark shaded shape shows the undeformed model.

The directions of RBMs happen to be aligned with the global coordinate system in the CLIP model example, but this is not always the case.

Notice that the **Rigid Body Motion** and **Rigid Body Mode** terms may be considered interchangeable. In vibration theory, **Rigid Body Mode** is more frequently used.

Mode 7 and all higher modes are elastic modes of vibration. In most cases we don't count Rigid Body Modes (if they are present at all) and mode numbering starts from the first elastic mode. Figure 1-19 shows the first four elastic modes of vibration which we will simply call the modes of vibration. The frequencies of the first four modes are within the range of 0-2600Hz.



Figure 1-19: The first four modes of vibration of the unsupported CLIP model. The undeformed model is superimposed on the deformed plots.

How many modes of vibration are of practical importance depends on the expected frequency range of the expected excitation. Most often, higher modes of vibration can't be excited and therefore have no importance in vibration analysis of mechanical systems (Figure 1-20).



Figure 1-20: Higher modes have complex shapes and high frequencies. *The undeformed model is not shown.* 

# Modal Superposition Method

The restrained and meshed with a default element size CLIP model has 39800 degrees of freedom. The exact number depends on how are the restraints defined. Solving this vibration problem would require finding a solution to the set of some 39800 equations. Considering that vibration problems are time dependent, those equations would have to be solved for all time steps where the number of time steps may be into the thousands. In the view of very high numerical effort required to solve vibration problems. We will describe this process in steps.

## Step 1

Let's summarize what we already know about modes of vibration. Each mode is characterized by frequency, shape of deformation, and the mass participating in the vibration. Furthermore, the direction of deformation is associated with certain stiffness. Collecting these facts we may represent an object vibrating in a given mode of vibration by a single degree of freedom oscillator with mass, stiffness and direction that correspond to the properties of the mode under consideration.

## Step 2

Now we decide how many modes are important in the vibration response of the system under investigation. For example, if only three modes are important, we can represent the system with three Single Degree of Freedom oscillators (SDOF), each one corresponding to one mode. This way the number of degrees of freedom is reduced to just three, no matter how complex the model is.

## Step 3

Instead of studying the original model, we may now study a simple 3DOF system where each mode is represented by a SDOF.

### Step 4

We assume that the system is linear and we find its vibration response as a superposition of vibration responses of the three SDOFs.





Figure 1-21: A vibration system represented by three single degree of freedom oscillators.

It is assumed that three modes of vibration are sufficient to model the vibration response. Each mode is represented by a SDOF oscillator. Each oscillator is characterized by its mass, stiffness and direction of linear displacement.

The **Modal Superposition Method** is universally used in vibration analysis with FEA. The most important condition that must be satisfied in order to use the **Modal Superposition Method** is that the problem must be **linear**. The second important issue is how many modes should be considered? This is most often decided by considering the frequency range of excitation. A convergence process may also be conducted where the same problem is solved a number of times, each time with more modes calculated to find the sensitivity of the data of interest to the number of modes considered in the **Modal Superposition Method**.