New York City College of Technology
Department of Mathematics

MAT 1275 Final Exam Review Problems[1]

1. Evaluate (and find on a number line)

(a) $-2^{-4} + 3^0 - (2/3)^{-1}$

(b) $5 - 3(7 - 9) + 27^{\frac{2}{3}}$

(c) $7 - 4(5 - 6) + 9^{\frac{3}{2}}$

2. Simplify $\dfrac{9x^4y^3 - 12x^2y^4 + 3xy^3}{3xy^3}$.

3. Solve

(a) $-2(x - 1) - (4 - 3x) = -2(-1 + x) - 7 + x - 3$

(b) $3(x - 2) - 7(x + 3) = -2(x + 6) + x$

4. Solve for x and simplify the answer.

(a) $6x^2 - 2x - 3 = 0$

(b) $-2x^2 + 4x - 5 = 0$

(c) $2x^2 + 2x = -3$

5. Solve the following equation

(a) $4\sqrt{2x + 1} - 3 = 17$

(b) $5\sqrt{1 - 2x} + 3 = 18$

6. Given the following quadratic equation, state the x- and y-intercepts (solutions of the form $(x, 0)$ and $(0, y)$), the vertex, and then use this information to sketch the graph.

(a) $y = x^2 - 2x - 3$

(b) $y = x^2 + 2x$

(c) $y = -x^2 + 4x - 2$

7. Simplify the complex fraction.

(a) $\dfrac{\dfrac{2}{y^2} + \dfrac{1}{y}}{\dfrac{4}{y^2} - \dfrac{1}{y}}$

(b) $\dfrac{\dfrac{2}{x} + \dfrac{1}{y}}{\dfrac{3}{y} - \dfrac{4}{x}}$

(c) $\dfrac{3 - \dfrac{1}{2x}}{2 + \dfrac{1}{x^2}}$

[1]Revised by Profs. Benakli, Carley, Colucci, Li, Masuda, Niezgoda, Rozenblyum, et al (Spring 2024).

8. Simplify $\dfrac{2x^2+4x-30}{x^2-9}$ (for values of x for which the denominator is not zero).

9. Solve $\dfrac{2}{x^2-4x+3} = \dfrac{2x}{x-1} + \dfrac{1}{x–3}$.

10. Write the equation of the circle given below in standard form. Identify the center and radius of the circle and graph it. Label four points on the graph with coordinates.

(a) $x^2+y^2-6x+4y-4=0$

(b) $x^2+y^2+8x-2y-8=0$

(c) $x^2+y^2+10x+6y+18=0$

11. Perform the indicated operation and express the answer in the form $a+bi$.

(a) $(-2+3i)(5-7i)$

(b) $(4-9i)(3-2i)$

(c) $\dfrac{3-2i}{1-i}$

(d) $\dfrac{2-7i}{3+4i}$

12. Solve the system of equations.

(a) $\begin{cases} 3x+y=4 \\ x^2-3y=-32 \end{cases}$

(b) $\begin{cases} x^2+y^2=5 \\ x-y^2=-3 \end{cases}$

(c) $\begin{cases} 4x-y=2 \\ x^2+2y=5 \end{cases}$

(d) $\begin{cases} x^2-y^2=3 \\ 2x+y^2=5 \end{cases}$

13. For the given expressions,

1. identify the quadrant in which the angle is located,
2. find the reference angle,
3. calculate the exact value.

(a) $\tan(-135°)$

(b) $\cos(240°)$

(c) $\sin(330°)$

(d) $\cos(-120°)$

(e) $\cot\left(-\dfrac{5\pi}{4}\right)$

(f) $\csc\left(-\dfrac{4\pi}{3}\right)$

(g) $\sec\left(\dfrac{2\pi}{3}\right)$

14. For the given information, state the value of the five remaining trig functions of θ.

(a) $\sin\theta = \dfrac{2}{5}$ and $\cos\theta < 0$

(b) $\tan\theta = 2$ and $\sin\theta < 0$

(c) $\cos\theta = \dfrac{2}{3}$ and $\tan\theta < 0$

(d) $\cos\theta = \dfrac{3}{7}$ and $\sin\theta < 0$

15. Given a ΔABC , draw a picture of the triangle and label it with the information provided. Round each answer to the nearest tenth. You may use either the law of sines:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

or the law of cosines:

$$\begin{aligned} c^2 &= a^2 + b^2 - 2ab\cos(C) \\ a^2 &= c^2 + b^2 - 2cb\cos(A) \\ b^2 &= a^2 + c^2 - 2ac\cos(B). \end{aligned}$$

(a) If $a = 12$, $b = 8$ and $c = 5$, find $\angle C$.

(b) If $\angle A = 50°$, $\angle B = 75°$ and $a = 20$, find side b.

(c) If $b = 9$, $c = 6$ and $\angle A = 67°$, find side a.

(d) If $\angle C = 37°$, $\angle B = 79°$ and $b = 13$, find side a.

16. Find the exact solutions to the trigonometric equations for $x \in [0, 2\pi)$.

(a) $4\sin x + 2 = 0$

(b) $6\cos x - 3 = 0$

(c) $2\tan x - 2 = 0$

(d) $6\tan x = -2\sqrt{3}$

17. Suppose that you are are asked to find the exact solutions to equations in problems 4, 12, and 16.

(a) What kind of problem is this?

(b) What does x represent?

(c) Are there values that you can see without computation can not possibly be a solution? Explain.

(d) Describe your strategy for solving this problem and explain why you think this will work. Provide a picture as part of your explanation where appropriate.

18. Evaluate the logarithm without using a calculator.

(a) $\log_2 \sqrt[3]{2}$

(b) $\log_3 \dfrac{1}{9}$

19. Consider the equation $0 = -1 + x + 3x^2 + x^3$. Note that $x = -1$ is a solution. Find all other solutions exactly. Hint: identify a factor and use long division to find the other factor.

20. Write a cubic expression with one variable whose leading coefficient 4, and whose roots are 0, -2, and 7.

21. Write down a polynomial with one variable, x, which when evaluated at each of $x = 2$, $x = -3$, and $x = 1$ gives 0.

22. Use the Binomial Theorem to find the coefficient of x^3 in the expression $(2x - 1)^5$.

23. Assuming the variables take on positive values, simplify and write your answer as a simple fraction using only positive exponents with each variable appearing at most once:

(a) $\left(\dfrac{4x^3y^{-2}}{16x^{-3}y^4}\right)^{-1/2}$

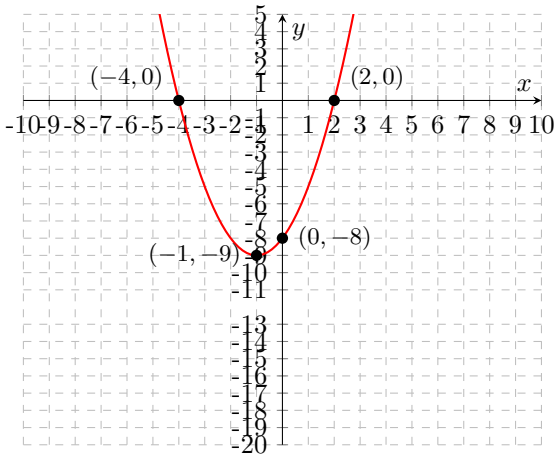
(b) $\left(\dfrac{4x^3}{x^{-3}}\right)^{-\frac{1}{2}}$

(c) $\left(4y^{-3}y^2\right)^{-\frac{1}{2}}$

24. Write an equation of a line which is perpendicular to the line $y - 3 = 2(x + 4)$ which passes through $(2, -1)$. Graph both lines.

25. Write an equation of a line passing through $(-2, 3)$ and $(3, 4)$.

26. Given the graph of $y = ax^2 + bx + c$,



(a) How many solutions of the form $(a, -4)$ does this equation have? Explain.

(b) How many solutions of the form $(\sqrt{3}, b)$ does this equation have? Explain.

(c) How many solutions with an y-coordinate of -15 does the equation have? Explain

(d) Find the roots of $0 = ax^2 + bx + c$.

(e) What is the value of the constant coefficient c? Explain.

(f) Is the coefficient a, positive, negative or zero? Explain.

(g) How many solutions does the system $\begin{cases} y &= ax^2 + bx + c \\ y &= 3 \end{cases}$ have? Explain.

(h) Find the equation for the line of symmetry.

27. A 10-foot ladder is leaning up against a wall. If the base of the ladder is situated 4 feet away from the base of the wall, what is the angle of elevation of the ladder? Draw a picture that depicts the situation and label the relevant information. Round your answer to the nearest tenth of a degree.

28. Solve for x and round your answer to the nearest tenth:

$$3^x = 12.$$

Answers:

1. (a) $-\dfrac{9}{16}$, which is between -1 and 0 on the number line

(b) 20, to the right of zero

(c) 38, to the right of zero

2. $3x^3 - 4xy + 1$

3. (a) $x = -3$

(b) $x = -5$

4. **(a)** $\dfrac{1 \pm \sqrt{19}}{6}$

(b) $\dfrac{2 \pm i\sqrt{6}}{2}$

(c) $\dfrac{-1 \pm i\sqrt{5}}{2}$

5. (a) $x = 12$

(b) $x = -4$

6. **(a)** Vertex: $(1, -4)$, x-intercepts: $(3, 0)$ and $(-1, 0)$, y-intercept: $(0, -3)$

(b) Vertex: $(-1, -1)$, x-intercepts: $(0, 0)$ and $(-2, 0)$, y-intercept: $(0, 0)$

(c) Vertex: $(2, 2)$, x-intercepts: $(2 - \sqrt{2}, 0)$ and $(2 + \sqrt{2}, 0)$, y-intercept: $(0, -2)$

7. **(a)** $\dfrac{2 + y}{4 - y}$

(b) $\dfrac{2y + x}{3x - 4y}$

(c) $\dfrac{6x^2 - x}{4x^2 + 2}$ or $\dfrac{x(6x - 1)}{2(2x^2 + 1)}$

8. $\dfrac{2(x + 5)}{x + 3}$

9. $x = \dfrac{-1}{2}$

10. **(a)** Center: $(3, -2)$, radius: $r = \sqrt{17}$

(b) Center: $(-4, 1)$, radius: $r = 5$

(c) Center: $(-5, -3)$, radius: $r = 4$

11. **(a)** $11 + 29i$

(b) $-6 - 35i$

(c) $\dfrac{5}{2} + \dfrac{1}{2}i$

(d) $-\dfrac{22}{25} - \dfrac{29}{25}i$

12. **(a)** $(-5, 19)$, $(-4, 16)$

(b) $(-2, 1)$, $(1, 2)$, $(1, -2)$, $(-2, -1)$

(c) $(-9, -38)$, $(1, 2)$

(d) $(2, 1)$, $(2, -1)$, $(-4, \sqrt{13})$, $(-4, -\sqrt{13})$

13. **(a)** III, $45°$, 1

(b) III, $60°$, $-\frac{1}{2}$

(c) IV, $30°$, $-\frac{1}{2}$

(d) III, $60°$, $-\frac{1}{2}$

(e) II, $\frac{\pi}{4}$, -1

(f) II, $\frac{\pi}{3}$, $\frac{2\sqrt{3}}{3}$

(g) II, $\frac{\pi}{3}$, -2

14. **(a)** $\cos\theta = -\frac{\sqrt{21}}{5}$, $\tan\theta = -\frac{2}{\sqrt{21}} = -\frac{2\sqrt{21}}{21}$, $\sec\theta = -\frac{5}{\sqrt{21}} = -\frac{5\sqrt{21}}{21}$, $\csc\theta = \frac{5}{2}$, $\cot\theta = -\frac{\sqrt{21}}{2}$

(b) $\cos\theta = -\frac{1}{\sqrt{5}} = -\frac{\sqrt{5}}{5}$, $\sin\theta = -\frac{2}{\sqrt{5}} = -\frac{2\sqrt{5}}{5}$, $\sec\theta = -\sqrt{5}$, $\csc\theta = -\frac{\sqrt{5}}{2}$, $\cot\theta = \frac{1}{2}$

(c) $\sin\theta = -\frac{\sqrt{5}}{3}$, $\tan\theta = -\frac{\sqrt{5}}{2}$, $\sec\theta = \frac{3}{2}$, $\csc\theta = -\frac{3}{\sqrt{5}} = -\frac{3\sqrt{5}}{5}$, $\cot\theta = -\frac{2}{\sqrt{5}} = -\frac{2\sqrt{5}}{5}$

(d) $\sin\theta = -\frac{2\sqrt{10}}{7}$, $\tan\theta = -\frac{2\sqrt{10}}{3}$, $\sec\theta = \frac{7}{3}$, $\csc\theta = -\frac{7}{2\sqrt{10}} = -\frac{7\sqrt{10}}{20}$,
$\cot\theta = -\frac{3}{2\sqrt{10}} = -\frac{3\sqrt{10}}{20}$

15. **(a)** $17.6°$

(b) 25.2

(c) 8.6

(d) 11.9

16. **(a)** $\frac{7\pi}{6}$, $\frac{11\pi}{6}$

(b) $\frac{\pi}{3}$, $\frac{5\pi}{3}$

(c) $\frac{\pi}{4}$, $\frac{5\pi}{4}$

(d) $\frac{5\pi}{6}$, $\frac{11\pi}{6}$

17.

18. **(a)** $\frac{1}{3}$

(b) -2

19. $-1+\sqrt{2}, -1-\sqrt{2}$

20. $4x(x+2)(x-7)$

21. Answers vary.

22. 80

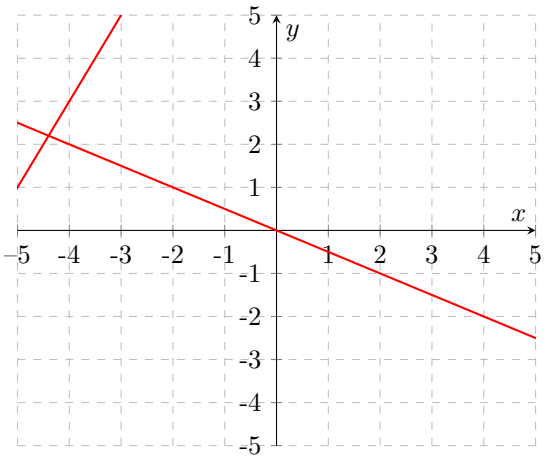
23. (a) $\frac{2y^3}{x^3}$

(b) $\dfrac{1}{2x^3}$

(c) $\dfrac{\sqrt{y}}{2}$

24. The equation is not unique but is equivalent to $y + 1 = -\dfrac{1}{2}(x - 2)$

25. The equation is not unique but is equivalent to $y - 3 = \dfrac{1}{5}(x + 2)$ or $y = \frac{1}{5}x + \dfrac{17}{5}$



26. (a) 2

(b) 1

(c) 0

(d) -4 and 2

(e) -8

(f) positive

(g) 2

(h) $x = -1$

27. $\cos^{-1}\dfrac{4}{10} \approx 66.4°$

28. 2.3