Practise Discrete Distribution Problems

Source: Walpole, Ronald E. and Myers, Raymond H., *Probability and Statistics for Engineers and Scientists, 2nd Edition*, 1978.

1. A company is interested in evaluating its current inspection procedure on shipments of 50 identical items. The procedure is to take a sample of 5 and pass the shipment if no more than 2 are found to be defective. What proportion of 20% defective shipments will be accepted?
2. A baseball player's batting average is 0.250. What is the probability that he gets exactly one hit in his next four times at bat?
3. Find the probability that a person flipping a coin gets the third head on the seventh flip.
4. A manufacturing company uses an acceptance scheme on production items before they are shipped. The plan is a two-stage one. Boxes of 25 are readied for shipment and a sample of 3 are tested for defectives. If any defectives are found, the entire box is sent back for 100% screening. If no defectives are found,the box is shipped.
   a. What is the probability that a box containing three defectives will be shipped?
   b. What is the probability that a box containing only one defective will be sent back for screening?
5. On the average a certain intersection results in three traffic accidents per week. What is the probability that exactly five accidents will occur at this intersection in any given week?
6. If we define the random variable X to be equal to the number of heads that occur when a balanced coin is flipped once, find the probability distribution of X. What two well known distributions describe the values of X?
7. The probability that a person will install a black telephone in a residence is estimated to be 0.3. Find the probability that the tenth phone installed in a new subdivision is the fifth black phone.
8. Find the probability of being dealt a bridge hand of 13 cards containing 5 spades, 2 hearts, 3 diamonds, and 3 clubs.
9. A certain area of the eastern United States is, on the average, hit by six hurricanes per year. Find the probability that in a given year
   a. Fewer than four hurricanes will hit this area.
   b. Anywhere from six to eight hurricanes will hit the area.
10. In testing a certain kind of truck tire over a rugged terrain, it is found that 25% of the trucks fail to complete the test run without a blowout. What is the probability that from 5 to 10 of the next 15 trucks tested have flat tires?
11. The surface of a circular dart board has a small center circle called the bull's-eye and 20 pie-shaped regions numbered from 1 to 20. Each of the pie shaped regions is further divided into three parts such that a person throwing a dart that lands on a specified number scores the value of the number, double the number, or triple the number, depending on which of the three parts the dart falls. If a person hits the bull's-eye with probability 0.01, hits a double with probability 0.10, hits a triple with probability 0.05, and misses the dart board with probability 0.02, what is the probability that seven throws will result in no bull's-eyes, no triples, a double twice, and a complete miss once?

12. On the average a secretary makes two typing errors per page. What is the probability that she makes
    a. Four or more errors on the next page she types?
    b. No errors?

13. The probability that a person dies from a certain respiratory infection is 0.002. Find the probability that fewer than 5 of the next 2000 so infected will die.

14. A foreign student club lists as its members two Canadians, three Japanese, five Italians, and two Germans. Find the probability that all nationalities are represented if a committee of size 4 is selected at random.

15. From a lot of 10 missiles, 4 are selected at random and fired. If the lot contains 3 defective missiles that will not fire, what is the probability that
    a. All 4 will fire?
    b. At most 2 will not fire?

16. The probability that a patient recovers from a delicate heart operation is 0.9. What is the probability that exactly five of the next seven patients having this operation survive?

17. An annexation suit is being considered against a county subdivision of 1200 residences by a neighboring city. If the occupants of half the residences object to being annexed, what is the probability that in a random sample of 10 at least 3 favor the annexation suit?

18. A traffic control engineer reports that 75% of the vehicles passing through a check point are from within the state. What is the probability that at least three of the next five vehicles are from out of the state?

19. A survey of the residents in a United States city showed that 20% preferred a white telephone over any other color available. What is the probability that more than half of the next 20 telephones installed in this city will be white?

20. If a person is dealt 13 cards from an ordinary deck of 52 playing cards several times, how many hearts per hand can he expect? Between what two values would you expect the number of hearts to fall at least 75% of the time?

21. It is known that 75% of mice inoculated with a serum are protected from a certain disease. If three mice are inoculated, what is the probability that at most two of the mice contract the disease?

22. If X represents the number of out-of-state vehicles in Exercise 18 when five vehicles are checked, find the probability distribution of X. Using Chebychev's theorem, find an interpret $\mu \pm 2\sigma$.

23. Suppose that airplane engines operate independently in flight and fail with probability $q = 1/5$. Assuming that a plane makes a safe flight if at least half of its engines run, determine whether a four-engine plane or a two-engine plane has the highest probability for a successful flight. Repeat the exercise for $q = 1/2$ and $q = 1/3$.

24. A scientist inoculates several mice, one at a time, with a disease germ until he finds two that have contracted the disease. If the probability of contracting the disease is 1/6, what is the probability that eight mice are required?

25. In Exercise 10 how many of the 15 trucks tested would you expect to have a flat tire? Using Chebychev's theorem, find and interpret the interval $\mu \pm 2\sigma$.

26. A card is drawn from a well-shuffled deck of 52 playing cards, the result recorded, and the card replaced. If the experiment is repeated five times, what is the probability of obtaining two spades and one heart?

27. To avoid detection at customs, a traveler has placed six narcotic tablets in a bottle containing nine vitamin pills that are similar in appearance. If the customs official selects three of the tablets at random for analysis, what is the probability that the traveler will be arrested for illegal possession of narcotics?

28. Suppose that the manufacturing company in Exercise 4 decided to change its acceptance scheme. Under the new scheme an inspector takes one at random, inspects it, and then replaces it in the box; a second inspector does likewise. Finally a third inspector goes through the same procedure. The box is not shipped if any of the three find a defective. Answer part a) and b) of Exercise 20 under this new plan.

29. A homeowner plants six bulbs selected at random from a box containing five tulip bulbs and four daffodil bulbs. What is the probability that he planted two daffodil bulbs and four tulip bulbs?

30. Find the probability of obtaining 2 ones, 1 two, 1 three, 2 fours, 3 fives, and 1 six in 10 rolls of a balanced die?

31. Three people toss a coin and the odd man pays for the coffee. If the coins all turn up the same, they are tossed again. Find the probability that fewer than four tosses are needed.

32. A random committee of size 3 is selected from four doctors and two nurses. Write a formula for the probability distribution of the random variable X representing the number of doctors on the committee. Find $P(2 \leq X \leq 3)$.

33. In Exercise 15 how many defective missiles might we expect to be included among the 4 that are selected? Use Chebychev's theorem to describe the variability of the number of defective missiles when 4 are selected from several lots each of size 10 containing 3 defective missiles.

34. It is estimated that 4000 of the 10,000 voting residents of a town are against a new sales tax. If 15 eligible voters are selected at random and asked their opinion, what is the probability that at most 7 favor the new tax?

35. An urn contains three green balls, two blue balls, and four red balls. In a random sample of five red balls, find the probability that both blue balls and at least one red ball are selected.

36. In an inventory study it was determined that on the average demands for a particular item at a warehouse were made five times per day. What is the probability that on a given day this item is requested
    a. More than five times?
    b. Not at all?

37. Suppose that on the average 1 person in 1000 make a numerical error in preparing their income tax returns. If 10,000 forms are selected at random and examined, find the probability that 6, 7, or 8 of the forms will be in error.

38. Using Chebychev's theorem, find and interpret the interval $\mu \pm 2\sigma$ for Exercise 37.

39. The probability that a student pilot passes the written test for his private pilot's license is 0.7. Find the probability that a person passes the test
    a. On the third try.
    b. Before the fourth try.

40. According to the theory of genetics, a certain cross of guinea pigs will result in red, black, and white offspring in the ratio $8 : 4 : 4$. Find the probability that among eight offspring five will be read, two black, and one white.

Partial Solutions

1) $X \sim$ Hypergeometric $(M = 10, N = 50, n = 5)$; $P(X \leq 2)$

2) $X \sim$ Binomial $(n = 4, p = .25)$; $P(X = 1)$

3) $X \sim$ Negative Binomial $(r = 3, p = .5)$; $P(X = 4)$

4a) $X \sim$ Hypergeometric $(M = 3, N = 25, n = 3)$; $P(X = 0)$

4b) $X \sim$ Hypergeometric $(M = 1, N = 25, n = 3)$; $P(X = 1)$

5) $X \sim$ Poisson $(\lambda = 3)$; $P(X = 5)$

6) $X \sim$ Binomial $(n = 1, p = .5)$; $X \sim$ Uniform(2)

7) $X \sim$ Negative Binomial $(r = 5, p = .3)$; $P(X = 5)$

8) $X \sim$ Hypergeometric $(A_{1\ldots4} = 13, N = 52, n = 13)$; $P(X_1 = 5, X_2 = 2, X_3 = 3, X_4 = 3)$

9) $X \sim$ Poisson $(\lambda = 6)$; 1) $P(X \leq 3)$ $b)$ $P(6 \leq X \leq 8)$

10) $X \sim$ Binomial $(n = 15, p = .25)$; $P(5 \leq X \leq 10)$

11) $X \sim$ Multinomial $(n = 7, p_1 = .01, p_2 = .1, p_3 = .05, p_4 = .02, p_5 = .82)$;

12) $X \sim$ Poisson $(\lambda = 2)$; 1) $P(X \geq 4)$ $b)$ $P(X = 0)$

13) $X \sim$ Binomial $(n = 2000, p = .002)$

approx. $X \sim$ Poisson $(\lambda = 4)$; $P(X \leq 4)$

14) $X \sim$ Hypergeometric $(A_1 = 2, A_2 = 3, A_3 = 5, A_4 = 2, N = 12, n = 4)$; $P(X_{1\ldots4} = 1)$

15) $X \sim$ Hypergeometric $(M = 3, N = 10, n = 4)$; $a)$ $P(X = 0)$ $b)$ $P(X \leq 2)$

16) $X \sim$ Binomial $(n = 7, p = .9)$; $P(X = 5)$

17) $X \sim$ Hypergeometric $(M = 600, N = 1200, n = 10)$

approx. $X \sim$ Binomial $(n = 10, p = .5)$; $P(X \geq 3)$

18) $X \sim$ Binomial $(n = 5, p = .25)$; $P(X \geq 3)$

19) $X \sim$ Binomial $(n = 20, p = .2)$; $P(X \geq 11)$

20) $X \sim$ Hypergeometric $(M = 13, N = 52, n = 13)$; $E(X) = 3.25$ $V(X) = 1.864$; $3.25 \pm 2(1.364)$

21) $X \sim$ Binomial $(n = 3, p = .25)$; $P(X \leq 2)$

22) $E(X) = 3.75$; $V(X) = 0.9375$; $3.75 \pm 2\sqrt{0.9375}$

23) $X \sim$ Binomial $(n = 4, p = .80)$; $P(X \geq 2)$

$X \sim$ Binomial $(n = 2, p = .80)$; $P(X \geq 1)$

24) $X \sim$ Negative Binomial $(r = 2, p = \frac{1}{6})$; $P(X = 6)$

25) $p = .25;\ E(X) = 1.25;\ V(X) = 0.9375;\ 1.25 \pm 2\sqrt{0.9375}$

26) $X \sim$ Multinomial $(n = 5, p_1 = .25, p_2 = .25, p_3 = .5)$; $P(X_1 = 2, X_2 = 1, X_3 = 2)$

$P(X_1 = 0, X_2 = 2, X_3 = 0, X_4 = 1, X_5 = 4)$

27) $X \sim$ Hypergeometric $(M = 6, N = 15, n = 3)$; $1 - P(X = 0)$

28a) $X \sim$ Binomial $(n = 3, p = \frac{3}{25})$; $P(X = 0)$

28b) $X \sim$ Binomial $(n = 3, p = \frac{1}{25})$; $P(X = 1)$

29) $X \sim$ Hypergeometric $(A_1 = 5, A_2 = 4, N = 9, n = 6)$; $P(X_1 = 4, X_2 = 2)$

30) $X \sim$ Multinomial $(p_{1\ldots6} = \frac{1}{6}, n = 10)$; $P(X_1 = 2, X_2 = 1, X_3 = 1, X_4 = 2, X_5 = 3, X_6 = 1)$

31) $X \sim$ Geometric $(p = .75)$; $P(X \leq 3)$

32) $X \sim$ Hypergeometric $(M = 4, N = 6, n = 3)$

33) $E(X) = 1.2$

34) $X \sim$ Hypergeometric $(M = 6000, N = 10000, n = 15)$

approx. $X \sim$ Binomial $(n = 15, p = .6)$; $P(X \leq 7)$

35) $X \sim$ Hypergeometric $(A_1 = 3, A_2 = 2, A_3 = 4, N = 9, n = 5)$;

$P(X_1 = 2, X_2 = 2, X_3 = 1) + P(X_1 = 1, X_2 = 2, X_3 = 2) + P(X_1 = 0, X_2 = 2, X_3 = 3)$

36) $X \sim$ Poisson $(\lambda = 5)$; 1) $P(X \geq 6)$ $b)$ $P(X = 0)$

37) $X \sim$ Binomial $(n = 10000, p = \frac{1}{1000})$

approx. $X \sim$ Poisson $(\lambda = 10)$; $P(6 \leq X \leq 8)$

38) $E(X) = 10;\ V(X) = 10;\ 10 \pm 2 \times \sqrt{10}$

39) $X \sim$ Geometric $(p = .7)$; $a)$ $P(X = 2)$ $b)$ $P(X \leq 2)$

40) $X \sim$ Multinomial $(n = 8, p_1 = .5, p_2 = .25, p_3 = .25)$; $P(X_1 = 5, X_2 = 2, X_3 = 1)$