Part A

These problems are to practice stating null hypotheses and choosing the form of the alternate hypothesis. Each of the following situations requires a test of hypotheses about a population mean μ. State the appropriate form of the null hypothesis H_0 and the alternate hypothesis H_A in each case. [reference: *Introduction to the Practice of Statistics*, Moore and McCabe, Section 7.2 exercises.]

1. The mean area of the several thousand apartments in a new housing development is advertised to be 1250 square feet. A tenants group thinks that the apartments are smaller than advertised. They hire an engineer to measure a sample of apartments to test their suspicion.

2. Larry's car averages 32 miles per gallon on the highway. He now switches to a new motor oil that is advertised as increasing gas mileage. After driving 3000 miles with the new oil, he wants to determine if his gas milage has actually increased.

3. The diameter of a spindle in a small motor is supposed to be 5 mm. If the spindle is either too small or too large, the motor will not perform properly. The manufacturer measures the diameter in a sample of motors to determine whether the mean diameter has moved away from the target [5 mm.].

4. Experiments on learning in animals sometimes measure how long it takes a mouse to find its way through a maze. The mean time is 18 seconds for one particular maze. A researcher thinks that a loud noise will cause the mice to complete the maze faster. She measures how long each of 10 mice takes with a noise as stimulus.

5. The examinations in a large psychology class are scaled after grading so that the mean score is 50. A self-confident teaching assistant thinks that his students have a higher mean score than the class as a whole. His students this semester can be considered a sample from the population of all students he might teach, so he compares their mean score with 50.

6. A university gives credit in French language courses to students who pass a placement test. The language department wants to know if students who get credit in this way differ in their understanding of spoken French from students who actually take the French courses. Some faculty think the students who test out of the courses are better, but others argue that they are weaker in oral comprehension. Experience has shown that the mean score of students in the courses on a standard listening test is 24. The language department gives the same listening test to a sample of 40 students who passed the credit examination to see if their performance is different.

Part B

These problems are to practice interpreting the meaning of Type I and Type II errors. In each of the problems above, state [in words] what it would mean to make a Type I error, and what it would mean to make a Type II error. Also tell which type of error might be of more concern to the people doing the study, in each case, and give a reason for your answer.

Part C

These problems are to practice computing the probability of making Type I and Type II errors in some simplified hypothesis tests. Answers and some partial solutions are given here. [reference: our textbook, problems 12.3-12.6 on page 383 (page 339 in 8th ed.) and problems 12.29-12.36 on page 395 (pages 349-350 in 8th ed.). The remaining odd-numbered answers are in the back of the textbook. Details of the computations are omitted.]

12.3 Use hypergeometric distributions with $N = 7, n = 2$

We reject H_0 if $x = 2$ so $\alpha = P(X = 2|M = 2) = \frac{\binom{2}{2}\binom{5}{0}}{\binom{7}{2}} = \frac{1}{21}$

We accept H_0 if $x \neq 2$ so $\beta = P(X \neq 2|M = 4) = 1 - P(X = 2|M = 4) = 1 - \frac{\binom{4}{2}\binom{3}{0}}{\binom{7}{2}} = \frac{5}{7}$

12.4 Use binomial distributions with $n = 20$

We reject H_0 if $x \leq 16$ so $\alpha = b(X \leq 16; 20, 0.90) \approx 0.1329$

We accept H_0 if $x > 16$ so $\beta = b(X > 16; 20, 0.60) \approx 0.0159$

12.6 Use exponential distributions

We reject H_0 if $x \geq 3$ so $\alpha = \frac{1}{2}\int_3^\infty e^{-x/2}dx = e^{-3/2}$

We accept H_0 if $x < 3$ so $\beta = \frac{1}{5}\int_0^3 e^{-x/5}dx = 1 - e^{-3/5}$

12.29 Type I error: The executive is fit to take on the additional responsibilities, but the doctor says that he is not (based on the checkup).

Type II error: The executive is not fit, but the doctor says that he is (based on the checkup).

12.30 (a) $H_A : \mu < 20$

(b) $H_A : \mu > 20$

12.32 (a) He rejects H_0 (Type I error)

(b) He rejects H_0 (correct decision)

(c) He rejects H_0 (Type I error)

(d) He rejects H_0 (correct decision)

12.34 Assuming that the null hypothesis is that the device is effective at removing pollutants:

(a) Type I error: The device really is effective, but we conclude that it is not effective based on our study.

Type II error: The device really is not effective, but we conclude that it is effective based on our study.

(b) H_0: the device does not remove enough pollutants to be effective

12.36 The population distribution is a Poisson distribution.

$H_0 : \lambda = 10$, $H_A : \lambda > 10$

We reject H_0 if $\bar{x} > 12.5$ so $\bar{x} = 11.2$ means that we accept H_0

(a) If really $\lambda = 11.5$, it is a Type II error (since H_0 is actually false)

(b) If really $\lambda = 10.0$ it is a correct decision (since H_0 is actually true)